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PERFORMANCE LIMITATION OF DISCRETE NETWORKED CONTROL SYSTEMS BASED ON PACKET DROPOUTS AND CHANNEL NOISE

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Abstract. *The performance limitation of single-input and single-output (SISO) discrete networked control systems (NCSs) with packet dropouts and channel noise is studied in this paper. The white noise in the forward channel and the packet dropouts in the feedback channel are considered. A new result is derived and it is shown that the performance limitation depends on the intrinsic properties of a given plant such as non-minimum phase zeros, unstable poles, as well as the packet dropouts and the white noise. The result shows how the packet dropouts and white noise of communication channel may fundamentally constrain the tracking capability of NCSs. A typical example is given to illustrate the theoretical results.*

Keywords: Performance limitation, Packet dropouts, White noise, Unstable poles, Non-minimum phase zeros

1. **Introduction.** In recent years, the network control system has been successfully used in various fields [1, 2]. Compared with conventional point-to-point communication system, networked control system has many advantages in installation, wiring and maintenance. In networked control systems, the data is transmitted to the controller and the controlled object through the channel. The analysis of the control system will be more complicated, due to the introduction of the communication network into the forward channel and the feedback control loop. The performance of the control system will be declined by the influence such as the communication bandwidth, quantization, encoding, time delay, packet dropouts and white noise, which may even lead to instability of the system. Currently, the theory about modeling and stability analysis in NCSs has been quite mature [3]. However, from the application point of view, it is not enough just to consider the stability of networked control systems, the performance limitation should also be considered in NCSs. Therefore, it is very important to study the optimal design of discrete networked control systems. At the same time, it is necessary to research the influence of communication parameters on the performance limitation of the systems.

For many years, the performance limitation of the design in control system has been an important research field of the control science and engineering subjects [4]. At present, many domestic and foreign scholars have made a lot of research results in this area. The optimal tracking performance of the SISO networked control systems based on network input energy constraint is studied in [5]. The tracking performance limitation of the multi-input and multi-output (MIMO) networked control systems with two parameter controllers is studied in [6]. The optimal tracking performance of tracking step signal in NCSs is studied in [7].

In this paper, the performance limitation of tracking unit step signal in SISO discrete networked control systems is studied, with the main consideration based on data packet dropouts in the feedback channel and white noise in the forward channel. The performance index is characterized by the energy of the tracking error, and the limit value is obtained by the technique of spectral decomposition. The results show that regardless of compensation, the tracking performance limitation of the systems is determined by the intrinsic properties of a given plant and the communication parameters, which will provide theoretical guidance for the design of networked control systems. Furthermore, the tracking performance limitation of NCSs depends on the intrinsic properties of a given plant such as non-minimum phase zeros, unstable poles, as well as the packet dropouts and the white noise. Finally, the simulation results verify the correctness of the theory.

This paper is organized as follows. Section 2 introduces the problem formulation. The performance limitation with packet dropouts and white noise is studied in Section 3. A typical example is given to illustrate the results in Section 4. The paper conclusions and future research directions are presented in Section 5.

2. **Problem Formulations.**

The symbols used in this paper are standard. $\bar{z}$ denotes the conjugate of a complex number z, and $x(z)$ denotes the z-transformation of discrete time series $x(z)$. Define $D := \{z : |z| < 1\}$, $\bar{D} := \{z : |z| \le 1\}$, $\bar{D}^c := \{z : |z| > 1\}$ and $\partial D := \{z : |z| = 1\}$ as an open unit circle, a closed unit circle, an exterior of the closed unit circle and a unit circle, respectively. $\mathcal{L}_2$ represents Hilbert space:

$$\|F\|_2^2 := \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\|F\left(e^{j\theta}\right)\right\|_F^2 d\theta < \infty$$

The inner product in the $\mathcal{L}_2$ Hilbert space is:

$$\langle F, G\rangle := \frac{1}{2\pi}\int_{-\infty}^{\infty} tr\left[F^H\left(e^{j\theta}\right)G\left(e^{j\theta}\right)\right]d\theta$$

For $\forall F$, $G \in \mathcal{L}_2$, if $\langle F, G\rangle = 0$, then they are orthogonal. $\mathcal{L}_2$ can be decomposed into two orthogonal subspaces, and they are defined as $\mathcal{H}_2$ and $\mathcal{H}_2^{\perp}$. Finally, define $\mathcal{RH}_\infty$ is all stable, regular transfer function.

We establish the SISO discrete networked control systems as depicted in Figure 1, where the problem is to investigate the performance limitation of the systems based on packet dropouts and white noise.

In Figure 1, G represents the given plant and K is denoted as one-parameter compensator, whose transfer functions are $G(z)$ and $K(z)$, respectively. d_r and n represent the data packet dropouts in the feedback channel and the white noise in the forward channel, respectively. Among them, d_r is a random process of Bernoulli distribution, used to simulate the process of data packet dropouts. The parameter d_r represents whether or not a

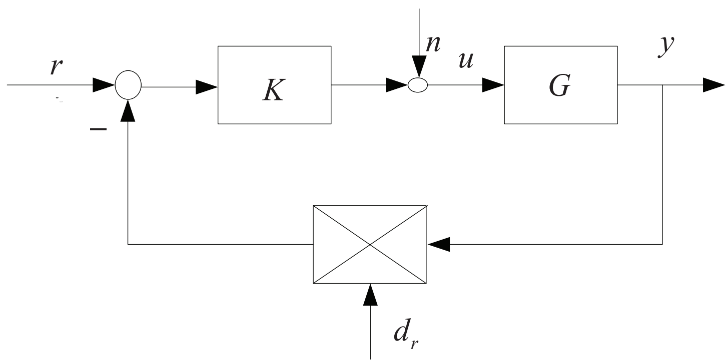

FIGURE 1. NCSs based on packet dropouts and white noise constraints

packet is dropped.

$$d_r = \begin{cases} 0 & \text{if the systems output is not successfully transmitted to the controller} \\ 1 & \text{if the systems output is successfully transmitted to the controller} \end{cases}$$

And the distribution probability for d_r is: $P\{d_r = 1\} = 1 - q$, $P\{d_r = 0\} = q$, $0 \leq q < 1$, and q represents the packet dropout probability. Signals r, y and u represent the reference input, the system output and control input signals, respectively. The signals $\tilde{r}$, $\tilde{y}$, $\tilde{u}$ and $\tilde{n}$ denote the z-transformation of signals r, y, u and n, respectively.

According to Figure 1, it is easy to obtain:

$$\tilde{u} = \tilde{n} + K(\tilde{r} - d_r\tilde{y}), \quad \tilde{y} = G\tilde{u} \tag{1}$$

According to (1), we can get:

$$\tilde{y} = \frac{KG}{1 + Kd_rG}\tilde{r} + \frac{G}{1 + Kd_rG}\tilde{n} \tag{2}$$

For a given reference input r, the tracking error of the networked control system is:

$$e = r - y \tag{3}$$

According to (2) and (3), we can get the z-transformation of e as follows:

$$\tilde{e} = \tilde{r} - \tilde{y} = \left(1 - \frac{KG}{1 + Kd_rG}\right)\tilde{r} - \frac{G}{1 + Kd_rG}\tilde{n} \tag{4}$$

According to [8], we can obtain:

$$\tilde{e} = T_1\tilde{r} + T_2\tilde{n} \tag{5}$$

where $T_1 = \left(1 - \frac{KG}{1+(1-q)KG}\right)$, $T_2 = -\frac{G}{1+(1-q)KG}$.

The tracking performance index of NCSs is defined as:

$$J := E\|\tilde{e}\|_2^2 = E\|\tilde{r} - \tilde{y}\|_2^2 \tag{6}$$

Define $\mathcal{K}$ as a set of controllers that stabilize the control system. The objective of this paper is to find the optimal controller in the set $\mathcal{K}$, which makes network control systems achieve optimal tracking performance, and the exact expression of the optimal tracking performance will be derived as follows:

$$J^* = \inf_{K \in \mathcal{K}} J \tag{7}$$

3. **Performance Limitation with Packet Dropouts and White Noise.** For any transfer function G, consider a coprime factorization of $(1-q)G$ as:

$$(1 - q)G = \frac{N}{M} \tag{8}$$

where $N, M \in \mathcal{RH}_\infty$, and it meets:

$$MX - NY = 1 \tag{9}$$

where $X, Y \in \mathcal{RH}_\infty$.

According to [9], it is well known that all controllers which make the control system stable can be expressed by Youla parameters.

$$\mathcal{K} := \left\{K : K = -\frac{(Y - MQ)}{X - NQ}, Q \in \mathcal{RH}_\infty\right\} \tag{10}$$

It is well known that a non-minimum phase transfer function can be decomposed into a minimum phase part and an all-pass factor. Then,

$$N(z) = (1 - q)L_zN_z, \quad M = B_pM_m \tag{11}$$

where L_z and B_p are all-pass factors, and N_z and M_m are the minimum phase parts. And L_z includes all non-minimum phase zeros $s_i \in \bar{D}^c$, $i = 1, \ldots, n_s$ of the given plant, and B_p includes all unstable poles $p_j \in \bar{D}^c$, $j = 1, \ldots, m$ of the given plant.

According to [10], L_z and B_p can be expressed as:

$$L_z(z) = \prod_{i=1}^{n_s} \frac{z - s_i}{1 - s_i} \frac{1 - \bar{s}_i}{1 - \bar{s}_i z}, \quad B_p(s) = \prod_{j=1}^{m} \frac{z - p_j}{1 - \bar{p}_j z} \tag{12}$$

Assume that the reference input signal r and the channel white noise n are independent of each other, and consider the reference input signal r as a unit step signal, whose z-transformation is: $\tilde{r} = \frac{z}{z-1}$. The tracking performance of NCSs can be rewritten as:

$$J = \left\| T_1 \frac{1}{z-1} \right\|_2^2 + \|T_2\|_2^2 \sigma^2 \tag{13}$$

where σ^2 is the variance of the white noise n in the forward channel.

According to (5), (8), (9) and (10), we can obtain:

$$T_1 = 1 + \frac{1}{1-q} N(Y - MQ), \quad T_2 = -\frac{1}{1-q} N(X - NQ) \tag{14}$$

According to (13) and (14), J can be rewritten as:

$$J = \left\| \left(1 + \frac{1}{1-q} N(Y - MQ)\right) \frac{1}{z-1} \right\|_2^2 + \left\| \frac{1}{1-q} N(X - NQ) \right\|_2^2 \sigma^2 \tag{15}$$

According to (6) and (15), we can get the optimal tracking performance J^* as follows:

$$J^* = \inf_{Q \in \mathcal{RH}_\infty} \left\| \left(1 + \frac{1}{1-q} N(Y - MQ)\right) \frac{1}{z-1} \right\|_2^2 + \inf_{Q \in \mathcal{RH}_\infty} \left\| \frac{1}{1-q} N(X - NQ) \right\|_2^2 \sigma^2 \tag{16}$$

Define J^* as:

$$J^* = J_1^* + J_2^* \tag{17}$$

where $J_1^* = \inf_{Q \in \mathcal{RH}_\infty} \left\| \left(1 + \frac{1}{1-q} N(Y - MQ)\right) \frac{1}{z-1} \right\|_2^2$, $J_2^* = \inf_{Q \in \mathcal{RH}_\infty} \left\| \frac{1}{1-q} N(X - NQ) \right\|_2^2$.

Theorem 3.1. *For NCSs as shown in Figure 1, assuming that the plant has many unstable poles $p_j \in \mathbb{C}_+$, $j = 1, \ldots, m$, and non-minimum phase zeros $z_i \in \mathbb{C}_+$, $i = 1, \ldots, n$, if $(1-q)G(z)$ can be decomposed as (8) and (11), the tracking performance limitation of network control systems is:*

$$\begin{aligned} J^* \geq & \sum_{i=1}^{n_s} \frac{|s_i|^2 - 1}{|s_i - 1|^2} + \sum_{i,j=1}^{m} \frac{\left(|p_i|^2 - 1\right)\left(|p_j|^2 - 1\right)}{\bar{b}_j b_i (1 - \bar{p}_j)(1 - p_i)} \frac{\gamma_j^H \gamma_j}{\bar{p}_j p_i - 1} \\ & + \sum_{j,i=1}^{n_s} \frac{(1 - s_i)(1 - s_j)\left(|s_i|^2 - 1\right)\left(|s_j|^2 - 1\right)}{\bar{D}_i D_j (1 - \bar{s}_i)(1 - \bar{s}_j)} \frac{\phi^H \phi}{\bar{s}_i s_j - 1} \sigma^2 \end{aligned}$$

where $\gamma_j = 1 - \frac{1}{1-q} L_z^{-1}(p_j)$, $\phi = N_n(z_i) X(z_i)$, $b_j = \prod_{\substack{i \in N \\ i \neq j}} \frac{p_j - p_i}{1 - p_j \bar{p}_i}$, $D_i = \prod_{\substack{j \in N \\ j \neq i}} \frac{s_i - s_j}{1 - s_j} \frac{1 - \bar{s}_j}{1 - \bar{s}_j s_i}$.

Proof: Firstly, calculating J_1^*, according to (11) and (17), we can obtain:

$$J_1^* = \inf_{Q \in \mathcal{RH}_\infty} \left\| (1 + L_z N_n (Y - MQ)) \frac{1}{z-1} \right\|_2^2$$

Because L_z is an all-pass factor, then

$$\begin{aligned} J_1^* &= \inf_{Q\in\mathcal{RH}_\infty} \left\| (1 + L_z N_n (Y - MQ)) \frac{1}{z-1} \right\|_2^2 \\ &= \inf_{Q\in\mathcal{RH}_\infty} \left\| ((L_z^{-1} - 1) + (1 + N_n (Y - MQ)) \frac{1}{z-1} \right\|_2^2 \end{aligned}$$

Because of $L_z^{-1} - 1 \in \mathcal{H}_2^\perp$, $1 + N_n(Y - MQ) \in \mathcal{H}_2$. Furthermore, J_1^* can be expressed as:

$$J_1^* = \left\| \left(L_z^{-1} - 1\right) \frac{1}{z-1} \right\|_2^2 + \inf_{Q\in\mathcal{RH}_\infty} \left\| (1 + N_n Y - N_n MQ) \frac{1}{z-1} \right\|_2^2$$

Define J_1^* as:

$$J_1^* = J_{11}^* + J_{12}^* \tag{18}$$

where $J_{11}^* = \left\| (L_z^{-1} - 1) \frac{1}{z-1} \right\|_2^2$, $J_{12}^* = \inf\limits_{Q\in\mathcal{RH}_\infty} \left\| (1 + N_n Y - N_n MQ) \frac{1}{z-1} \right\|_2^2$.

By a simple calculation, we can get:

$$J_{11}^* = \sum_{i=1}^{n_s} \frac{|s_i|^2 - 1}{|s_i - 1|^2} \tag{19}$$

According to (11), we can get:

$$J_{12}^* = \inf_{Q\in\mathcal{RH}_\infty} \left\| ((1 + N_n Y) - N_n B_p M_m Q) \frac{1}{z-1} \right\|_2^2$$

Because B_p is an all-pass factor, then

$$J_{12}^* = \inf_{Q\in\mathcal{RH}_\infty} \left\| \left(\frac{1 + N_n Y}{B_p} - N_n M_m Q \right) \frac{1}{z-1} \right\|_2^2$$

According to the partial fraction decomposition, we can obtain:

$$\frac{1 + N_n Y}{B_p} = \sum_{j=1}^{m} \frac{1 - \bar{p}_j z}{z - p_j} \frac{1 + N_n(p_j) Y(p_j)}{b_j} + R_1$$

where $b_j = \prod\limits_{\substack{i\in N \\ i\neq j}} \frac{p_j - p_i}{1 - p_j \bar{p}_i}$, $R_1 \in \mathcal{RH}_\infty$.

Therefore, J_{12}^* can be expressed as:

$$\begin{aligned} J_{12}^* &= \inf_{Q\in\mathcal{RH}_\infty} \left\| \left(\sum_{j=1}^{m} \frac{1 - \bar{p}_j z}{z - p_j} \frac{1 + N_n(p_j) Y(p_j)}{b_j} + R_1 - N_n M_m Q \right) \frac{1}{z-1} \right\|_2^2 \\ &= \inf_{Q\in\mathcal{RH}_\infty} \left\| \left[\sum_{j=1}^{m} \left(\frac{1 - \bar{p}_j z}{z - p_j} - \frac{1 - \bar{p}_j}{1 - p_j} \right) \frac{1 + N_n(p_j) Y(p_j)}{b_j} + R_2 - N_n M_m Q \right] \frac{1}{z-1} \right\|_2^2 \\ &= \inf_{Q\in\mathcal{RH}_\infty} \left\| \sum_{j=1}^{m} \frac{|p_j|^2 - 1}{z - p_j} \frac{1 + N_n(p_j) Y(p_j)}{(1 - p_j) b_j} + (R_2 - N_n M_m Q) \frac{1}{z-1} \right\|_2^2 \end{aligned}$$

where $R_2 \in \mathcal{RH}_\infty$, and

$$R_2 = R_1 + \sum_{j=1}^{m} \frac{1 - \bar{p}_j}{1 - p_j} \frac{1 + N_n(p_j) Y(p_j)}{b_j}$$

Because of $\sum_{j=1}^{m} \frac{|p_j|^2-1}{z-p_j}\frac{1+N_n(p_j)Y(p_j)}{(1-p_j)b_j} \in \mathcal{H}_2^\perp$, $R_2 - N_n M_m Q \in \mathcal{H}_2$. Furthermore, J_{12}^* can be expressed as:

$$J_{12}^* = \left\| \sum_{j=1}^{m} \frac{|p_j|^2-1}{z-p_j}\frac{1+N_n(p_j)Y(p_j)}{(1-p_j)b_j} \right\|_2^2 + \inf_{Q\in\mathcal{RH}_\infty} \left\| (R_2 - N_n M_m Q)\frac{1}{z-1} \right\|_2^2$$

Because N_n and M_m represent the external functions and minimum phase, respectively, we can obtain:

$$\inf_{Q\in\mathcal{RH}_\infty} \left\| (R_2 - N_n M_m Q)\frac{1}{z-1} \right\|_2^2 = 0$$

Then,

$$J_{12}^* = \left\| \sum_{j=1}^{m} \frac{|p_j|^2-1}{z-p_j}\frac{1+N_n(p_j)Y(p_j)}{(1-p_j)b_j} \right\|_2^2$$

Meanwhile, according to $M(p_j) = 0$, we can get:

$$N_n(p_j)Y(p_j) = -\frac{1}{1-q}L_z^{-1}(p_j)$$

By a simple calculation, we can get:

$$J_{12}^* = \sum_{i,j=1}^{m} \frac{\left(|p_i|^2-1\right)\left(|p_j|^2-1\right)}{\bar{b}_j b_i \left(1-\bar{p}_j\right)\left(1-p_i\right)} \frac{\gamma_j^H\gamma_j}{\bar{p}_j p_i - 1} \tag{20}$$

where $\gamma_j = 1 - \frac{1}{1-q}L_z^{-1}(p_j)$.

According to (19) and (20), we can obtain:

$$J_1^* = \sum_{i=1}^{n_s} \frac{|s_i|^2-1}{|s_i-1|^2} + \sum_{i,j=1}^{m} \frac{\left(|p_i|^2-1\right)\left(|p_j|^2-1\right)}{\bar{b}_j b_i (1-\bar{p}_j)(1-p_i)} \frac{\gamma_j^H\gamma_j}{\bar{p}_j p_i - 1} \tag{21}$$

According to J_1^*, similarly, we can get:

$$J_2^* = \sum_{j,i=1}^{n_s} \frac{(1-s_i)(1-s_j)\left(|s_i|^2-1\right)\left(|s_j|^2-1\right)}{\bar{D}_i D_j (1-\bar{s}_i)(1-\bar{s}_j)} \frac{\phi^H\phi}{\bar{s}_i s_j - 1} \tag{22}$$

where $\phi = N_n(z_i)X(z_i)$.

The proof is completed.

4. **Illustrative Example.**

The unstable system model is considered as follows:

$$G(z) = \frac{z-2}{(z-k)(z+1)}$$

For the given plant, the non-minimum phase zero is located at $z = 2$, and the unstable pole is located at $p = k$. Here, the value of the correlation value is: $\sigma = 0.2$, and then the performance limitation of network control systems is:

$$J^* = 3 + \frac{k+1}{k-1}\left(1 - \frac{1}{1-q}\frac{1-2k}{k-2}\right)^2 - \frac{3}{2-k}\sigma$$

The performance limitation of NCSs based on the influence of different packet dropouts and poles is shown in Figure 2.

As can be seen from Figure 2, different packet dropouts, probability affects the tracking performance of networked control systems, the greater the probability of packet dropouts is, the worse the tracking performance of the system is; at the same time, it can also be seen that the performance of NCSs becomes worse when the unstable poles of the given plant are close to the non-minimum phase zeros.

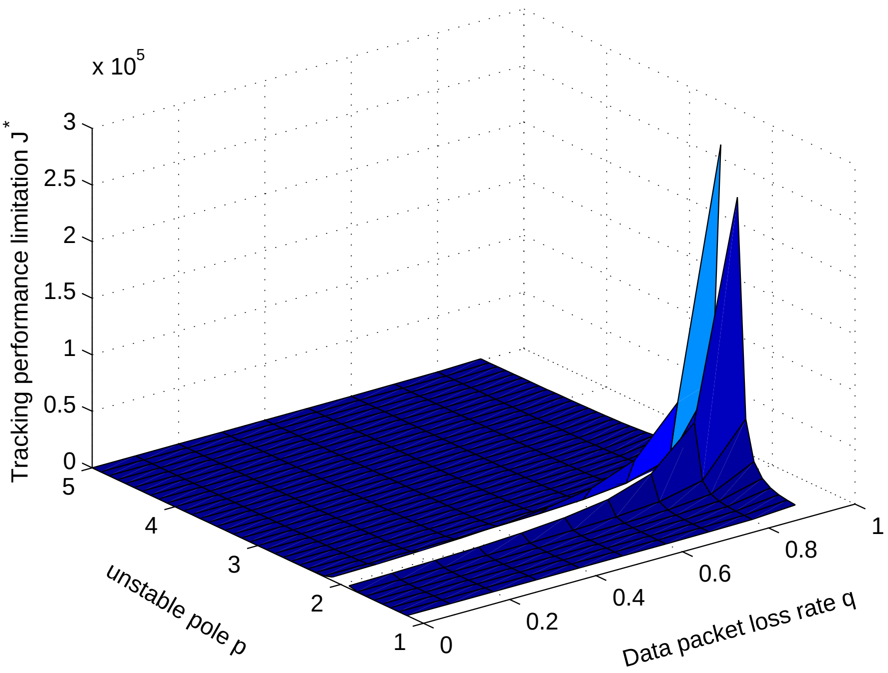

Figure 2. The performance limitation of NCSs based on packet dropouts constraint

5. **Conclusion.** This paper studies SISO discrete networked control systems tracking unit step signal based on packet dropouts and white noise constraints. In the systems, the data packet dropouts in the feedback channel and the white noise in the forward channel are mainly considered. The tracking performance index is characterized by the difference between the output of the plant and the reference signal. The results show that the performance limitation depends on the intrinsic properties of a given plant such as non-minimum phase zeros, unstable poles, as well as the packet dropouts and additive white noise, furthermore, how the packet dropouts and white noise affect the tracking ability of NCSs, which will provide theoretical guidance for the design of NCSs. The simulation example shows the correctness of the conclusion.

Possible future research extensions to this work include studying more general plants such as MIMO complex systems, and more parameters of communication channel constraints such as the bandwidth effect, the quantization effect, and the encoding effect.

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