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SIMULTANEOUS TUNING OF CONTROLLER AND COMPENSATOR PARAMETERS FOR SYSTEMS WITH BACKLASH

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Abstract. *This paper presents a controller tuning method for systems with backlash by using the so-called fictitious reference iterative tuning (FRIT) method which is a kind of direct controller parameter tuning method. The key idea of the presented method is to use an inverse backlash model as a backlash compensator and is to tune simultaneously controller and compensator parameters by the FRIT method. The effectiveness of the presented FRIT method is demonstrated with an experimental motor system.*
Keywords: Controller parameter tuning, Backlash, PID control, FRIT

1. **Introduction.** Backlash is a typical nonlinearity which appears as undesirable factor in many precision positioning applications such as industrial machines and robots. To overcome this difficulty of backlash, various control methods have been investigated [1, 2, 3, 4]. However, most of the control methods are based on complicated control structures and processes, so that they often need heavy burdens to tune the controllers.

On the other hand, fictitious reference iterative tuning (FRIT) has received significant attention as a simple and practical controller tuning method. FRIT is based on input and output data obtained from an only one-shot experiment and does not require any mathematical model of a plant [5]. Because FRIT is, in theory, primarily developed for linear systems, its control performance tends to be deteriorated for systems with nonlinearities. To cope with this problem, modified frameworks of FRIT have been recently developed for systems with nonlinearities of dead zone, saturation, and hysteresis [6, 7, 8, 9]. To the best knowledge of the authors, however, an application of FRIT to systems with backlash has not been fully investigated. Therefore, we often encounter situations where it is difficult to obtain acceptable control performance for systems with backlash by the existing frameworks of FRIT.

In this paper, we present a controller tuning method for systems with backlash by using FRIT. In the control system configuration, an inverse backlash model is used as a backlash compensator, and controller and compensator parameters are simultaneously tuned according to an idea of FRIT. After describing the procedure of the presented FRIT method, we illustrate its effectiveness by using an experimental direct-current (DC) motor system.

2. **Backlash Model.** We consider a typical discrete-time backlash model $B_{\boldsymbol{C}}$ [2, 3] as shown by

$$w(k) = B_{\boldsymbol{C}}(v(k)) = \begin{cases} v(k) - c_l & \text{if } v(k) \le w_l \\ v(k) - c_r & \text{if } v(k) \ge w_r \\ w(k-1) & \text{if } w_l < v(k) < w_r \end{cases} \tag{1}$$

$$w_l = w(k-1) + c_l, \quad w_r = w(k-1) + c_r,$$

where $v(k)$ and $w(k)$ denote the input and output, respectively, of the backlash model, and $\boldsymbol{c} = [c_l, c_r]^T$ denotes a vector of parameters in the backlash model. Let $\tilde{B}_{\boldsymbol{c}}$ denote the right inverse of the backlash model $B_{\boldsymbol{c}}$. Then we can express $\tilde{B}_{\boldsymbol{c}}$ as

$$v(k) = \tilde{B}_{\boldsymbol{c}}(w(k)) = \begin{cases} w(k) + c_l & \text{if } w(k) < w(k-1) \\ w(k) + c_r & \text{if } w(k) > w(k-1) \\ v(k-1) & \text{if } w(k) = w(k-1), \end{cases} \tag{2}$$

where $w(k)$ and $v(k)$ denote the input and output of the right inverse model $\tilde{B}_{\boldsymbol{c}}$. The input and output characteristics of the backlash model and its right inverse model are shown in Figure 1. We here give a brief explanation of the backlash model in Figure 1. Roughly speaking, when the input is constantly increasing, the output is moving along the below diagonal line. Conversely, when the input is constantly decreasing, the output is moving along the above diagonal line. In contrast, when the input changes direction, the output remains constant until the input passes through the gap of the backlash, i.e., $c_r - c_l$.

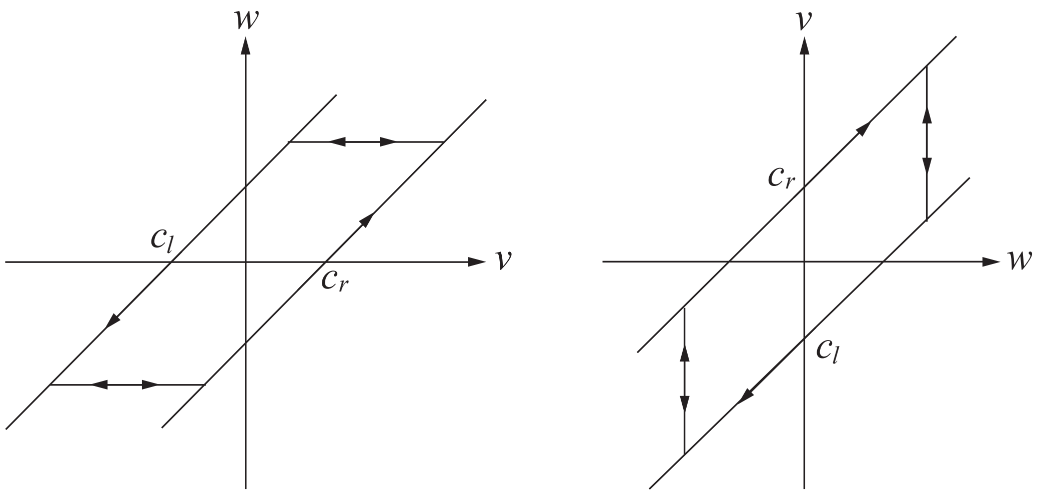

FIGURE 1. Backlash model (left) and its right inverse model (right)

3. **Control System Configuration.** Consider the closed-loop system as shown in Figure 2. In the figure, $u(k)$, $y(k)$, $r(k)$, and $e(k)$ denote the control input, control output, reference signal, and control error, respectively. For example, geared DC motors can be often modeled as a linear system with output backlash as in the plant of the figure.

As a typical controller, we deal with the following proportional-integral-derivative (PID) controller $C(z, \boldsymbol{\theta})$:

$$C(z, \boldsymbol{\theta}) = \frac{K_P(1 - z^{-1}) + K_I + K_D(1 - z^{-1})^2}{1 - z^{-1}}, \tag{3}$$

where K_P, K_I, and K_D are the proportional, integral, and derivative gains, respectively, and these gains are denoted by a vector $\boldsymbol{\theta} = [K_P, K_I, K_D]^T$. It should be noted that the right inverse of the backlash model is placed at the output part of the controller as a backlash compensator in Figure 2.

In [6, 7, 8], a linear system and a nonlinear element are assumed to be switched in the plant of Figure 2, and FRIT-based methods are proposed to compensate for nonlinear elements such as dead-zone, saturation, and hysteresis. The basic idea of these methods is to neutralize the nonlinear elements by means of their inverse nonlinearities which are used as nonlinear compensators, and to tune the controller and compensator parameters simultaneously. Therefore, the same idea is also applicable and effective to the system

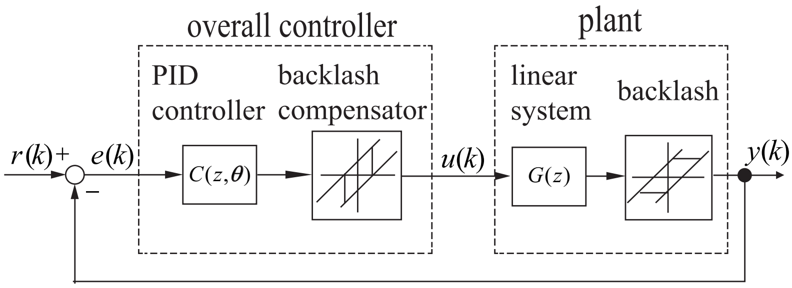

FIGURE 2. Proposed system configuration

with backlash as shown in Figure 2 if the linear part and the backlash element in the plant are approximately commutable.

In fact, we can often observe that the linear property and the backlash property are approximately commutable as shown in some simulation results and the experimental DC motor system which is described later. We can expect from this observation that an FRIT method for tuning parameters of the controller and backlash compensator in the system configuration of Figure 2 provides a better control performance than the conventional FRIT in which only a controller is tuned without a backlash compensator.

4. **FRIT Algorithm for Systems with Backlash.** In FRIT, we need to calculate the so-called fictitious reference signal $\tilde{r}$ based on input and output data obtained from a closed-loop experiment. By introducing the backlash compensator, the fictitious reference signal is different from that in the conventional FRIT and is calculated as follows:

$$\tilde{r}(\boldsymbol{x},k) = C(z,\boldsymbol{\theta})^{-1}B_{\boldsymbol{c}}(u_0(k)) + y_0(k), \tag{4}$$

where $\boldsymbol{x} = \left[\boldsymbol{\theta}^T, \boldsymbol{c}^T\right]^T$ is the parameter vector to be tuned. It should be noted that the backlash model $B_{\boldsymbol{c}}$ is included in the calculation of the fictitious reference signal.

The FRIT algorithm for systems with backlash is summarized as follows.

Step 1 (Initial settings): Set the initial PID gains $\boldsymbol{\theta}_0$, the reference signal $r(k)$, $k = 1,\ldots,N$, and the reference model $M(z)$.

Step 2 (Initial experiment): Implement the PID controller $C(z,\boldsymbol{\theta}_0)$ with the initial PID gains $\boldsymbol{\theta}_0$ and give the reference signal $r(k)$ in the closed-loop experiment without backlash compensation to obtain initial control input and output data $u_0(k)$ and $y_0(k)$.

Step 3 (Minimization of the performance index): Calculate the fictitious reference signal $\tilde{r}(\boldsymbol{x},k)$ based on (4) and the obtained initial control input and output data $u_0(k)$ and $y_0(k)$. Then, find optimal (suboptimal) parameters $\boldsymbol{x}^*$ minimizing the performance index

$$J(\boldsymbol{x}) = \sum_{k=1}^{N} \left(y_0(k) - M(z)\tilde{r}(\boldsymbol{x},k)\right)^2$$

by using a certain optimization method[1].

5. **Experimental Results.** We deal with a DC motor system as a plant with backlash and apply the proposed FRIT to this system. The experimental system overview and its system configuration are shown in Figure 3. As shown in the figure, the DC motor and encoder are connected to the same shaft. We measure the rotation angle of the DC motor through the encoder and calculate the control input which is transmitted to the drive circuit.

[1]For example, a global optimization method proposed in [10] is useful in this case because, in general, the optimization problem to be solved is nonconvex and nondifferentiable.

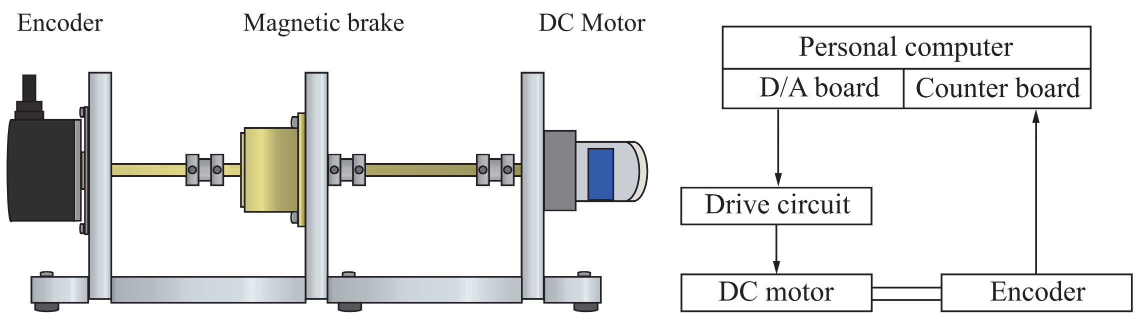

FIGURE 3. DC motor system (left) and its system configuration (right)

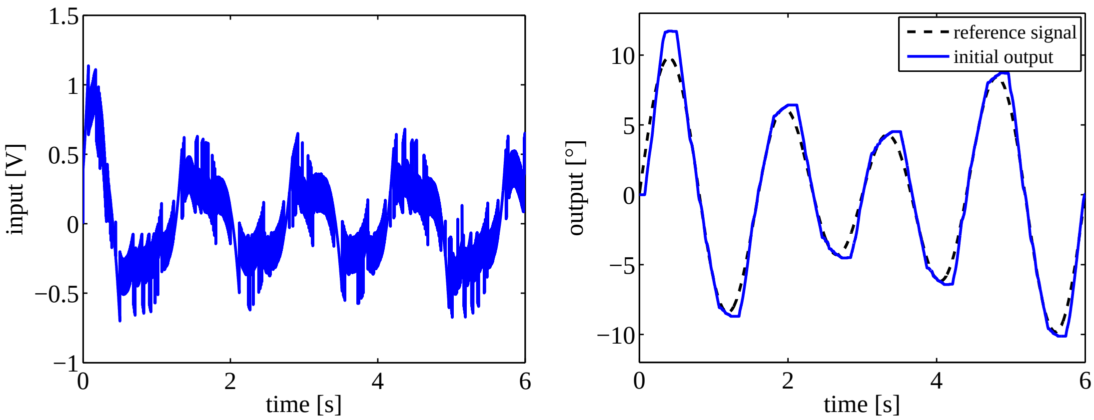

FIGURE 4. Initial control input and output

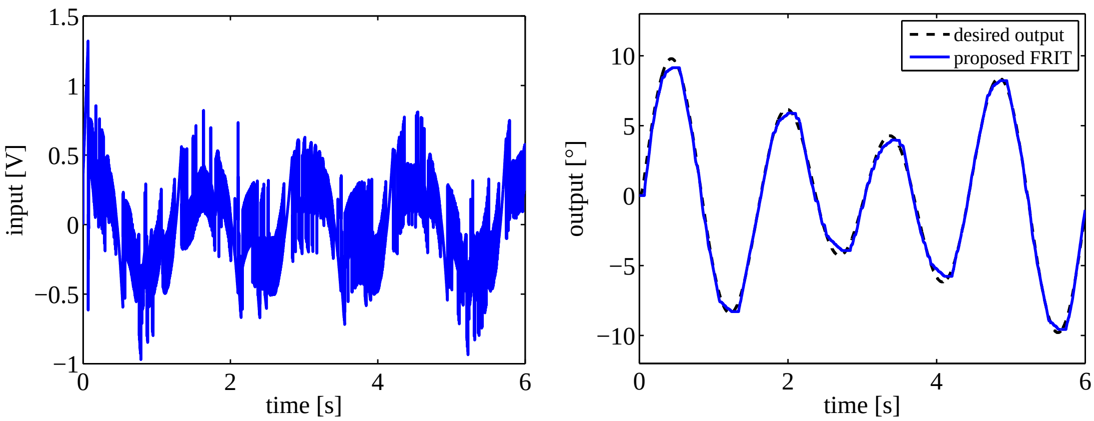

FIGURE 5. Control input and output of the proposed FRIT

The reference model $M(z)$ is set to the discrete-time model obtained by discretizing $M(s) = 1/(0.008s + 1)^4$ with sampling time 1 ms. We let $N = 6000$ and set the reference signal $r(k)$ as a waveform consisting of multiple sinusoidal signals with the maximum amplitude 10°. In Figure 4, we show the initial control input and output when the initial PID gains $\boldsymbol{\theta}_0 = [0.2, 0.002, 10.0]^T$ are implemented.

We obtained the parameter $\boldsymbol{x}^* = [0.3641, 0.0005, 16.0742, -0.1726, 0.2737]^T$ by means of the proposed FRIT. We implemented the obtained parameter $\boldsymbol{x}^*$ to the PID controller and backlash compensator and carried out the closed-loop experiment. As a result, we obtained the control input and output as shown in Figure 5. For comparison, we conducted the conventional FRIT in which the backlash compensator is not used, and consequently obtained the PID gains $\boldsymbol{\theta}^* = [0.3538, 0.0009, 40.9683]^T$. The control input and output

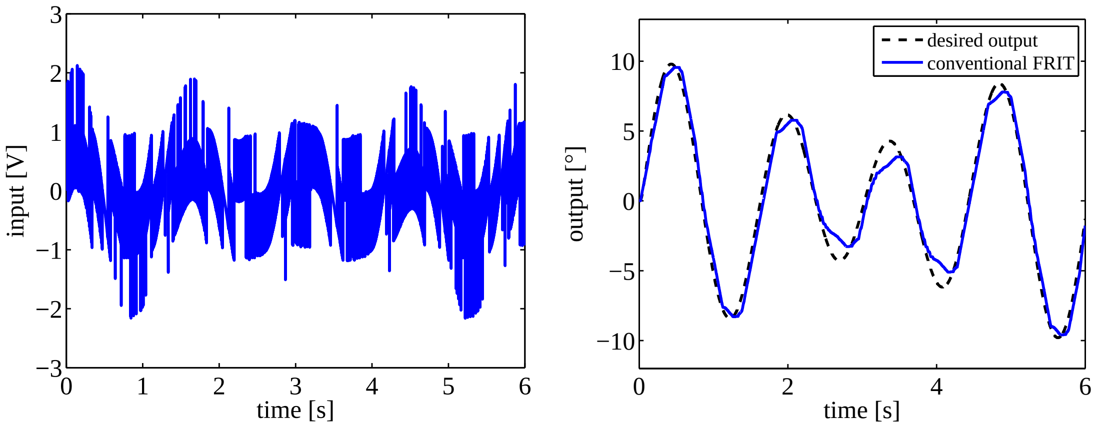

FIGURE 6. Control input and output of the conventional FRIT

of the conventional FRIT are shown in Figure 6. We can see from the figures that the proposed FRIT provides a better control performance than the conventional one.

To verify the effectiveness of the proposed FRIT in more detail, we conducted experiments under various conditions. We let the case of the abovementioned experiment be "Case 1" and consider the following cases by changing the conditions of Case 1.

Case 2: The initial PID gains are changed to $\boldsymbol{\theta}_0 = [0.1, 0.001, 5.0]^T$.
Case 3: The initial PID gains are changed to $\boldsymbol{\theta}_0 = [0.3, 0.003, 15.0]^T$.
Case 4: The time constant of the reference model is changed from 0.008 to 0.01.
Case 5: The time constant of the reference model is changed from 0.008 to 0.006.

The control performances of Cases 1-5 are shown in Table 1. In the table, we show the control performances of all the cases with the performance index $J_{yd} = \sum_{k=1}^{N}(M(z)r(k) - y_0(k))^2$. For the initial experiment, we also evaluate the control performance with $J_r = \sum_{k=1}^{N}(r(k) - y_0(k))^2$ related to the reference signal $r(k)$. We see from the table that, in Cases 2, 4, and 5, the performance index values of the conventional FRIT are larger than those of the initial experiment, which means that the controller and compensator parameters cannot be tuned appropriately by the conventional FRIT due to harmful effects of backlash. In contrast, the proposed FRIT provides smaller performance index values than the initial experiment and the conventional FRIT for all the cases. Therefore, this apparently confirms that the proposed FRIT is effective for systems with backlash.

TABLE 1. Control performance

	Case 1	Case 2	Case 3	Case 4	Case 5
Initial experiment (J_r)	4328.8	5620.0	1918.6	4328.8	4328.8
Initial experiment (J_{yd})	4632.2	4435.5	2894.9	5713.5	3945.0
Conventional FRIT	3927.8	5598.8	2204.2	4817.5	2.3×10^8
Proposed FRIT	652.0	989.4	970.0	799.5	730.1

6. **Conclusion.** We have proposed an FRIT method for systems with backlash in which parameters of not only a controller but also a backlash compensator are tuned simultaneously. Compared with the existing control methods for systems with backlash, the proposed method is extremely simple and practical. This should be emphasized as one of the important academic contributions. We also have shown the effectiveness of the proposed method through an experimental system with a geared DC motor. The same idea as in this method may be applied to another controller tuning method, e.g., virtual reference feedback tuning, which is one of the future research directions.

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