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STABILIZATION OF THE FRACTIONAL-ORDER ECONOMICAL SYSTEM VIA ADAPTIVE FUZZY CONTROL

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Abstract. *This paper is concerned with the problem of robust control of nonlinear fractional-order economical systems in the presence of uncertainties and external disturbance is investigated. Fuzzy logic systems are used for estimating the unknown nonlinear functions. Based on the fractional Lyapunov direct method and some proposed lemmas, an adaptive fuzzy controller is designed. The proposed method can guarantee all the signals in the closed-loop systems remain bounded and the tracking errors converge to an arbitrarily small region of the origin. Finally, an illustrative simulation result is given to demonstrate the effectiveness of the proposed scheme.*
Keywords: Fractional-order system, Lyapunov direct method, Adaptive control

1. **Introduction.** In the 1980s, economist Stutzer revealed the chaotic phenomena in economic system for the first time [1], which aroused the human's reflection on the traditional economics theory and after that the issue on nonlinear economics, chaotic economics has become a hot topic [2-5]. The modern research has shown that economic system can exhibit not only stable, unstable and periodic behavior but also chaotic phenomenon. In fact, financial crisis is just a chaotic phenomenon of the economic system [6]. Moreover, economists have noticed the fact that uncertainties in the economic development, such as the impact of non-economic factors, and the sudden change of economy in frequency are increasing [7]. Financial risks come from uncertainties, and therefore it has an important theoretical and practical significance by introducing uncertainties into economic system [8]. Taking into consideration the chaotic behaviors and uncertainties in the economic system, it is essential to investigate the chaos control strategies for economic and financial systems in order to solve financial crisis and the related problems. The aim of chaos control is to suppress or eliminate the chaotic behavior of the nonlinear system. Some techniques have been proposed to control chaos in economic systems, such as time-delayed feedback method [9], sliding mode control method [10], linear control [11], and lag projective synchronization [12].

On the other hand, fractional calculus is a more than 300 years old topic. During those days, it was considered that this technique is only a mathematical concept. Lately, this technique has been introduced to physics and engineering science [13] and many real phenomena are modeled with the fractional-order equations. Very recently, fractional modeling has gained much attention in life science and economics [14]. One of these models is the model proposed for financial systems in [15]. Having a memory is the aspect that makes this model distinct from its integer-order one. Memory (i.e., a history of the system) has a very important role in financial systems. So, the fractional-order financial model has more extended range of applications. However, the same as the integer-order financial models, this model shows a chaotic behavior which should be quenched. As a fundamental tool to analyze the stability of nonlinear systems, the Lyapunov method has been introduced in [16]. However, how to construct the simple direct Lyapunov functions remains an open problem [17]. The stability of fractional-order nonlinear systems by applying the Lyapunov direct method with the same fractional-order operators is firstly investigated in [18]. Some authors have proposed Lyapunov functions to prove the stability of fractional-order nonlinear systems, for example, a new property for Caputo fractional derivative which allows finding a simple Lyapunov candidate function for many fractional-order systems is presented in [19]. However, the effects of both system uncertainties and external noises are neglected; on the other hand, the fractional Lyapunov stability theory is not applied to guaranteeing the stability of the overall system. To date and to the best of our knowledge, the problem of robust control of nonlinear fractional-order systems whose model uncertainty and external noises are unknown has not been fully investigated, which motivates the study of this paper.

In this paper, an adaptive fuzzy control method for fractional-order nonlinear economical systems in the presence of model uncertainty and external noises is proposed. Fuzzy logic systems are used for estimating the unknown nonlinear functions. Based on the fractional Lyapunov direct method, an adaptive fuzzy controller is designed. Fractional adaptation laws are proposed to update the parameters of the fuzzy systems. The proposed method can guarantee all the signals in the closed-loop systems remain bounded and the tracking errors converge to an arbitrary small region of the origin.

This paper is organized as follows. In Sections 2-4, the problem is stated and some useful definitions and lemmas are given, and then the main results of this paper are proposed in Section 5. Section 6 provides a numerical example to illustrate the effectiveness of our results. Finally, Section 7 gives some concluding remarks.

2. **Preliminaries.** The fractional-order integrodifferential operator is the extended concept of integer-order integrodifferential operator. The commonly used definitions in literature are Grunwald-Letnikov, Riemann-Liouville, and Caputo definitions. Due to taking on the same form as integer-order differential on the initial conditions, which have well-understood physical meanings and have more applications in engineering, in this paper, the Caputo derivative is only considered. The Caputo fractional derivative is defined as follows:

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau \tag{1}$$

where Γ is the Gamma function, α is the fractional order, and t is function argument.

3. **System Description and Problem Formulation.** Consider the following uncertain nonlinear economical system:

$$\begin{aligned} D^{\alpha} x &= z + (y-a)x + \Delta f_1(x,y,z,t) + d_1(t) + u_1(t) \\ D^{\alpha} y &= 1 - by - x^2 + \Delta f_2(x,y,z,t) + d_2(t) + u_2(t) \\ D^{\alpha} z &= -x - cz + \Delta f_3(x,y,z,t) + d_3(t) + u_3(t) \end{aligned} \tag{2}$$

where the three state variables x, y, z stand for the interest rate, the investment demand, and the price index, respectively. Constant a is the saving amount, constant b is the cost per investment, and constant c is the elasticity of demand of the commercial markets. $\alpha \in (0,1)$ is the order of the system, $\Delta f_i(x,y,z,t)$, $i = 1,2,3$ and $d_i(t)$, $i = 1,2,3$ represent unknown model uncertainty and external disturbances of the system, respectively, and $u_i(t)$, $i = 1,2,3$ is the control input. Denote

$$X = [x(t), y(t), z(t)]^T$$
$$F(X) = \left[z + (y-a)x, 1 - by - x^2, -x - cz\right]^T$$
$$D(t) = [d_1(t), d_2(t), d_3(t)]^T$$

$$\Delta F(X,t) = [\Delta f_1(x,y,z,t), \Delta f_2(x,y,z,t), \Delta f_3(x,y,z,t)]^T$$
$$U(t) = [u_1(t), u_2(t), u_3(t)]^T$$

Then, system (2) can be rewritten as

$$D^\alpha X = F(X) + \Delta F(X,t) + D(t) + U(t) \tag{3}$$

The main objective is to construct an adaptive fuzzy controller $U(t)$ such that the state vector $X(t)$ tracks the following referenced signal with all involved signals keeping bounded in the closed-loop system.

$$X_d(t) = [x_d(t), y_d(t), z_d(t)] \tag{4}$$

The tracking error vector is defined as

$$E(t) = X_d(t) - X(t) \tag{5}$$

Thus, the dynamic of the tracking error can be written as

$$D^\alpha E(t) = D^\alpha X_d(t) - F(X) - \Delta F(X,t) - D(t) - U(t) \tag{6}$$

Lemma 3.1. (*see* [20]). *If $x(t)$ is continuous and derivable, then*

$$\frac{1}{2} D^\alpha X^T(t) P X(t) \leq X^T(t) P D^\alpha X(t) \tag{7}$$

where P is an $n \times n$ positive definite constant matrix.

Lemma 3.2. (*see* [21]). *Consider the following fractional-order system*

$$D^\alpha Y(t) \leq -aY(t) + b \tag{8}$$

then there exists a constant $t_0 > 0$ such that for all $t \in (t_0, \infty)$

$$\|Y(t)\| \leq \frac{2b}{a} \tag{9}$$

where $Y(t)$ is the state variable, and a, b are two positive constants.

4. **Description of the Fuzzy Logic System.** The basic configuration of a fuzzy logic system consists of a fuzzifier, some fuzzy IF-THEN rules, a fuzzy inference engine and a defuzzifier. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input vector $X = [X_1, X_2, \cdots, X_n]^T \in R^n$ to an output $\zeta(X) \in R$. The ith fuzzy rule is written as

Rule i: if X_1 is F_1^i and $\cdots$ and X_n is F_n^i then $\zeta(X)$ is α_i

where $F_1^i, F_2^i, \cdots$ and F_n^i are fuzzy sets and α_i is the fuzzy singleton for the output in the ith rule. By using the singleton fuzzifier, product inference and the center of gravity defuzzification, the output of the fuzzy system can be expressed as follows:

$$\zeta(X) = \frac{\sum_{j=1}^N \alpha_j \prod_{i=1}^n \mu_{F_i^j}(X_i)}{\sum_{j=1}^N \left[\prod_{i=1}^n \mu_{F_i^j}(X_i)\right]} = \theta^T \psi(X) \tag{10}$$

where $\mu_{F_i^j}(X_i)$ is the degree of membership of X_i to F_i^j, N is the number of fuzzy rules, $\theta = [\alpha_1, \cdots, \alpha_N]^T$ is the adjustable parameter vector, and $\psi(X) = [p_1(X), p_2(X), \cdots, p_N(X)]^T$, where

$$p_j(X) = \frac{\prod_{i=1}^n \mu_{F_i^j}(X_i)}{\sum_{j=1}^N \left[\prod_{i=1}^n \mu_{F_i^j}(X_i)\right]} \tag{11}$$

is the fuzzy basis function. It is assumed that fuzzy basis functions are selected so that there is always at least one active rule.

5. **Adaptive Fuzzy Controller Design.** In this section, we will design an adaptive fuzzy controller, such that not only all the signals of the closed-loop system (6) are bounded, but also the tracking error tends to the origin asymptotically. Denote

$$\xi(X(t)) = D^\alpha X_d(t) - F(X) - \Delta F(X,t) - D(t) \tag{12}$$

Then (6) can be written as

$$D^\alpha E(t) = \xi(X(t)) - U(t) \tag{13}$$

Since the model uncertainty $\Delta F(X,t)$ and the external perturbations $D(t)$ are unknown, which lead to the nonlinear function $\xi(X(t))$ is unknown. Thus, we need to design an adaptive fuzzy controller, precisely, we will apply the fuzzy system (10) to approximate the unknown nonlinear functions $\xi(X(t))$ in the following manner:

$$\hat{\xi}_i(\theta_i(t), X(t)) = \theta_i^T(t)\psi_i(X(t)), \quad i = 1, 2, \cdots, n \tag{14}$$

where $\xi_i(X(t))$ is the ith element of the nonlinear function $\xi(X(t))$. Let us define the ideal parameters of θ_i as

$$\theta_i^* = \arg\min_{\theta_i} [\sup |\xi_i(X(t)) - \hat{\mu}_i(X(t))|] \tag{15}$$

Define the parameter estimation errors and the fuzzy approximation errors as follows:

$$\tilde{\theta}_i = \theta_i - \theta_i^* \tag{16}$$

$$\varepsilon_i(x) = \mu_i(X(t)) - \hat{\mu}_i(\theta_i^*, X(t)) \tag{17}$$

with $\hat{\xi}_i(\theta_i^*, X(t)) = \theta_i^* \psi_i(X(t))$. We can assume that the fuzzy approximation error is bounded for all X, i.e., $|\varepsilon_i(X)| < \bar{\varepsilon}_i$, where $\bar{\varepsilon}_i$ is an unknown constant. Let $\varepsilon = [\varepsilon_1(X), \cdots, \varepsilon_n(X)]^T$, $\bar{\varepsilon} = [\bar{\varepsilon}_1, \cdots, \bar{\varepsilon}_n]^T$. Then we can get $|\varepsilon(X)| \le \bar{\varepsilon}$. From the above analysis, we have

$$\begin{aligned}\hat{\xi}(\theta_i(t), X(t)) - \xi(X(t)) &= \hat{\xi}(\theta_i(t), X(t)) - \hat{\xi}(\theta^*, X(t)) + \hat{\xi}(\theta^*, X(t)) - \xi(X(t)) \\ &= \hat{\xi}(\theta_i(t), X(t)) - \hat{\xi}(\theta^*, X(t)) - \varepsilon(X(t)) \\ &= \tilde{\theta}^T(t)\psi(X(t)) - \varepsilon(X(t))\end{aligned} \tag{18}$$

Then the adaptive fuzzy controller can be constructed as

$$U(t) = \theta^T(t)\psi(X(t)) + kE(t) + bsign(E(t)) \tag{19}$$

where k and b are free positive constants to be designed. Substituting the proposed controller (19) into the tracking error dynamics (13) gives

$$PD^\alpha e(t) = \xi(X(t)) - \theta^T(t)\psi(X(t)) - kE(t) - bsign(E(t)) \tag{20}$$

Multiplying $E^T(t)$ to both sides of (20) and applying (18) yield

$$\begin{aligned}E^T(t)D^\alpha E(t) = &- kE^T(t)E(t) + \sum_{i=1}^n E_i(t)\varepsilon_i(X(t)) \\ &- b\sum_{i=1}^n E_i(t)\tilde{\theta}_i^T(t)\psi_i(X(t)) - b\sum_{i=1}^n |E_i(t)|\end{aligned} \tag{21}$$

The fractional adaptation laws for updating the fuzzy parameters $\theta_i(t)$ are designed as the following fractional-order differential equations

$$D^\alpha \theta_i(t) = \gamma_i E_i(t)\psi_i(X(t)) - \gamma_i\sigma_i\theta_i(t), \quad i = 1, 2, \cdots, n, \tag{22}$$

where σ_i and γ_i are positive design parameters.

Theorem 5.1. *Suppose that the controller is designed as (19) and the fractional adaptation laws are defined as (22). Then all signals in the closed-loop system will keep bounded, and the tracking error will eventually be arbitrarily small if appropriate control parameters are chosen.*

Proof: Choose the following quadratic Lyapunov function

$$V(t) = \frac{1}{2}E^T(t)E(t) + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\gamma_i}\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) \tag{23}$$

By using Lemma 3.1, we can obtain

$$D^\alpha V(t) \leq E^T(t)D^\alpha E(t) + \sum_{i=1}^{n}\frac{1}{\gamma_i}\tilde{\theta}_i^T(t)D^\alpha\tilde{\theta}_i(t) \tag{24}$$

Noting that the Caputo derivative of a constant function is 0, we have

$$D^\alpha\tilde{\theta}_i(t) = D^\alpha\theta_i(t) \tag{25}$$

Thus, we have

$$D^\alpha V(t) \leq E^T(t)D^\alpha E(t) + \sum_{i=1}^{n}\frac{1}{\gamma_i}\tilde{\theta}_i^T(t)D^\alpha\theta_i(t) \tag{26}$$

Substituting (21) and the fractional adaptation laws (22) into (26), we have

$$D^\alpha V(t) \leq -kE^T(t)E(t) - (b-\bar{\varepsilon})\sum_{i=1}^{n}|E_i(t)| - \sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\theta_i(t) \tag{27}$$

If b is taken from $(\bar{\varepsilon}, +\infty)$, then

$$D^\alpha V(t) \leq -kE^T(t)E(t) - \sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\theta_i(t) \tag{28}$$

Note that

$$-\sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\theta_i^* \leq \frac{1}{2}\sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) + \frac{1}{2}\sum_{i=1}^{n}\sigma_i\theta_i^{*T}\theta_i^* \tag{29}$$

Thus, we have

$$\begin{aligned}
D^\alpha V(t) &\leq -kE^T(t)E(t) - \sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\theta_i(t) \\
&= -kE^T(t)E(t) - \sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) - \sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\theta_i^* \\
&\leq -kE^T(t)E(t) - \frac{1}{2}\sum_{i=1}^{n}\sigma_i\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) + \frac{1}{2}\sum_{i=1}^{n}\sigma_i\theta_i^{*T}\theta_i^* \\
&\leq -kE^T(t)E(t) - \frac{\sigma}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) + \frac{1}{2}\sum_{i=1}^{n}\sigma_i\theta_i^{*T}\theta_i^* \\
&\leq -kE^T(t)E(t) - \frac{\sigma\gamma}{2}\sum_{i=1}^{n}\frac{1}{\gamma_i}\tilde{\theta}_i^T(t)\tilde{\theta}_i(t) + \frac{1}{2}\sum_{i=1}^{n}\sigma_i\theta_i^{*T}\theta_i^* \\
&\leq -k_0V(t) + \frac{1}{2}\sum_{i=1}^{n}\sigma_i\theta_i^{*T}\theta_i^*
\end{aligned} \tag{30}$$

where

$$\sigma = \min\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$$
$$\gamma = \min\{\gamma_1, \gamma_2, \cdots, \gamma_n\}$$
$$k_0 = \min\{2k, \sigma\gamma\}$$

Applying Lemma 3.2, there exists a $t_0 > 0$ such that

$$\|V(t)\| \leq \frac{\sum_{i=1}^{n} \sigma_i \theta_i^{*T} \theta_i^*}{k_0} \tag{31}$$

which yields that

$$\|E(t)\| \leq \sqrt{\frac{2\sum_{i=1}^{n} \sigma_i \theta_i^{*T} \theta_i^*}{k_0}} \tag{32}$$

which means $\|E(t)\|$ can be arbitrarily small in (t_0, ∞) if the parameters k and γ_i are chosen large enough. Besides, it can be easily seen that all the signals in the closed-loop system will remain bounded.

Remark 5.1. *In* [22]*, the parameter uncertainties* $\Delta f_i(x, y, z, t)$*,* $i = 1, 2, \cdots, n$ *and the external noise perturbations* $d_i(t)$*,* $i = 1, 2, \cdots, n$ *are bounded.*

6. **Numerical Simulations.** In this section, an illustrative example is presented to illustrate the effectiveness and applicability of the proposed adaptive fuzzy control approach and to confirm the theoretical results. Consider the following fractional-order economic system with model uncertainties and external disturbances [22].

$$\begin{aligned} D^\alpha x &= z + (y-3)x + \Delta f_1(x, y, z, t) + d_1(t) \\ D^\alpha y &= 1 - 0.1y - x^2 + \Delta f_2(x, y, z, t) + d_2(t) \\ D^\alpha z &= -x - z + \Delta f_3(x, y, z, t) + d_3(t) \end{aligned} \tag{33}$$

In the simulation, the uncertainty term and external noise of the system are selected as follows

$$\begin{aligned} \Delta f_1(x, y, z, t) + d_1(t) &= -0.15\sin(2t)x + 0.15\sin(3t) \\ \Delta f_2(x, y, z, t) + d_2(t) &= 0.25\cos(4t)y + 0.1\cos(t) \\ \Delta f_3(x, y, z, t) + d_3(t) &= 0.2\sin(3t)z + 0.2\sin(3t) \end{aligned} \tag{34}$$

Numerical simulations are made with the initial value $x_0 = 2$, $y_0 = -1$, and $z_0 = 3$, the fractional order $\alpha = 0.9$. The referenced signal is set to be $X_d(t) = [0, 0, 0]^T$. Throughout the simulation, the model of the fractional-order nonlinear system (33) is fully unknown. The proposed control methods do not need to the knowledge of the system. The parameters of the controller are chosen as $k = 1$, $b = 1$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.001$, γ_1, γ_2, γ_3. The initial conditions of the fuzzy systems $\theta_1(0)$, $\theta_2(0)$, and $\theta_3(0)$ are chosen randomly.

The simulation results are shown in Figures 1 and 2. Figure 1 gives the system states without control input, and Figure 2 gives the track performance of the state variables $x(t)$, $y(t)$, and $z(t)$, respectively. From the simulations results, we can see that the theoretical results obtained in this paper are feasible to achieve good performance for the controlled fractional order economic system. It should be pointed out that although the theoretical result we derived in this paper is aimed at commensurate order case, in fact, the designed controllers can also be generalized to the control of incommensurate fractional order system.

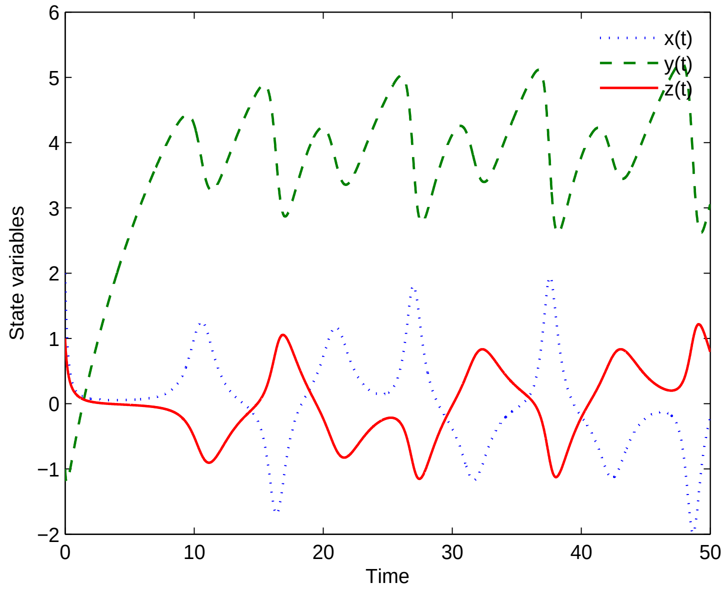

FIGURE 1. Responses of the system state without control inputs

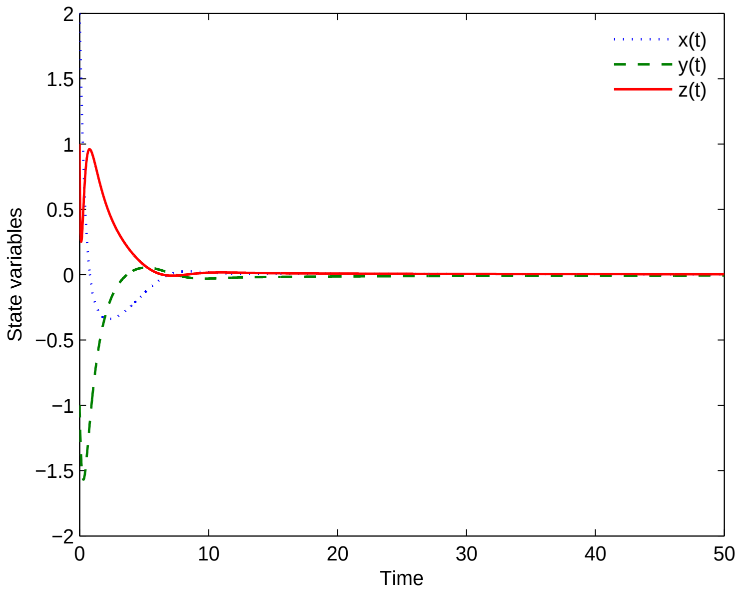

FIGURE 2. Responses of the system state with control inputs

7. **Conclusions.** In this paper, an adaptive fuzzy control method for fractional-order nonlinear economical systems in the presence of model uncertainty and external noises is proposed. Fuzzy logic systems are used for estimating the unknown nonlinear functions. Based on the fractional Lyapunov direct method, an adaptive fuzzy controller is designed. The proposed method can guarantee all the signals in the closed-loop systems remain bounded and the tracking errors converge to an arbitrarily small region of the origin. Lastly, an illustrative example is given to demonstrate the effectiveness of the proposed results. Our future research direction is finite-time control of nonlinear economical systems.

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