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THE CONTROL OF GRID-CONNECTED PHOTOVOLTAIC INVERTER BASED ON PORT-CONTROLLED HAMILTONIAN SYSTEM THEORY

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Abstract. *The whole model of the boost circuit and the inverter circuit is established to eliminate the error caused by the separate modeling in this paper. The modeling and controller of grid-connected photovoltaic (PV) inverter system are presented based on energy-shaping and port-controlled Hamiltonian (PCH) theory. Firstly, a nonlinear mathematical model of grid-connected inverter system is established. Then, the control problem can be recast as finding a controller and an interconnection pattern such that the overall energy function takes the desired form. Using the energy-shaping and interconnection and damping assignment, the feedback stabilization theory of inverter system is given. The desired equilibrium of the system is determined according to the output power of the solar cell. In addition, the stability of the system is also analyzed. The simulation results show that the Hamiltonian controller of grid-connected inverter system possesses good performance and application prospect.*
Keywords: Inverter system, Energy-shaping, Grid-connected, Photovoltaic

1. **Introduction.** In recent years, mankind is forced to develop new energy sources to solve the problem of global warming and energy exhaustion caused by increasing energy consumption [1]. Solar energy is expected to play an important role in the future because of improved technology and reduction in photovoltaic (PV) cost [2]. A suitable grid-connected inverter system is required to transfer the energy to the grid. The grid-connected inverter system is usually composed of two stages. The first is used to boost the PV array voltage; the second converts the direct current (DC) power into alternating current (AC) power for grid injection [3]. The inverter continuously needs information about amplitude, phase angle and frequency of the grid voltages both in the steady-state and during the transient periods. For this reason, many estimation and synchronization methods, like phase locked loop (PLL) systems, are proposed in [4]. Some traditional inverter control methods have been reported. The proportional-integral (PI) control is easy to implement, but its dynamic performance is poor. It is difficult to achieve the desired control effect [5]. The sliding mode control has fast response, but its essence is switching system. It is difficult to achieve the ideal switching in practical control system [6]. Recently, the energy-shaping (ES) and port-controlled Hamiltonian (PCH) systems method has been used in many control systems [7-9]. The PCH system with dissipation has become an important tool in nonlinear control system research. In this paper, the whole model of the boost circuit and the inverter circuit is established to eliminate the error caused by the separate modeling. In view that the mathematical model of the inverter is nonlinear, therefore, a nonlinear control strategy from the nonlinear structure of the system achieves better control performance. A new control method of inverter system based on PCH control algorithm is proposed. The modeling and controller of grid-connected inverter system are presented based on a novel PCH and ES control principle. The scheme has the advantage that the closed-loop energy function can be used as

Lyapunov function rendering the stability analysis more transparent and controller design simpler. The control strategy is proved feasible by simulation. Moreover, the results show that the proposed control algorithm has good control performance.

The remainder of this paper is organized as follows. In Section 2, the mathematical model of grid-connected inverter system is presented. In Section 3, the controller of the inverter system is designed based on port-controlled Hamiltonian system theory. In addition, the stability of the system is also analyzed in this section. In Section 4, the theoretical analysis is verified to be right by simulation results. Finally, some conclusions are presented in Section 5.

2. Mathematical Model of Grid-Connected Inverter System.

The following figure is the topology of two level grid-connected inverter system. The output voltage of the photovoltaic power generation device is E, the capacitance voltage is u_{dc}, i_L is the current in the boost circuit, i_a, i_b, i_c are the three-phase inverter output current, e_a, e_b, e_c are the three-phase grid voltage, and their amplitude is U_m, L and R mean the filter parameters, L_1 and C mean inductance and capacitance of the boost circuit, $VT_1 \sim VT_6$ and S are IGBT switching devices, and VD is diode.

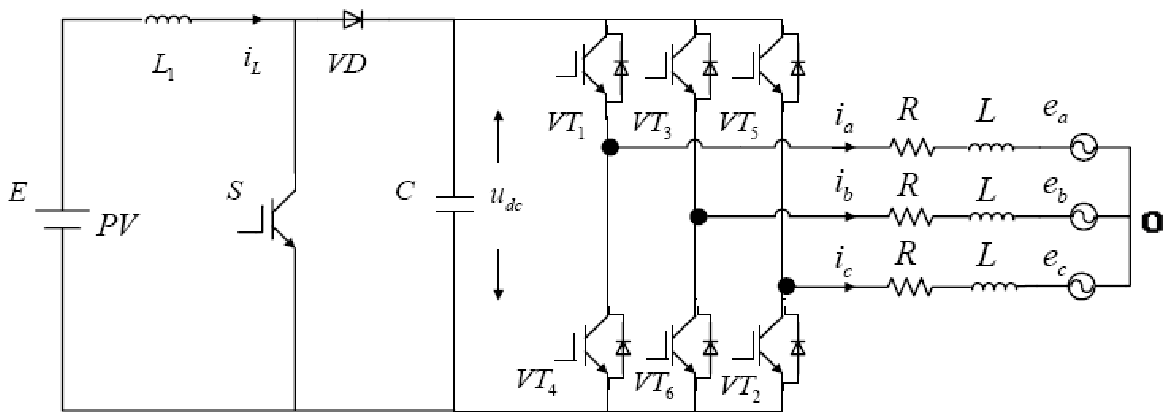

FIGURE 1. The topology of grid-connected inverter system

The system model is established according to the Kirchhoff's law

$$\begin{cases} L\dfrac{di_d}{dt} = \mu_d u_{dc} - Ri_d + \omega L i_q - e_d \\ L\dfrac{di_q}{dt} = \mu_q u_{dc} - Ri_q - \omega L i_d - e_q \\ L\dfrac{di_L}{dt} = E - (1-\mu)u_{dc} \\ C\dfrac{du_{dc}}{dt} = (1-\mu)i_L - \mu_d i_d - \mu_q i_q \end{cases} \tag{1}$$

where μ_d and μ_q are the duty ratio functions in the synchronous rotating dq coordinates, e_d, e_q and i_d, i_q are the voltages and currents of the d axis and the q axis of the ac side respectively, and μ is the duty ratio function of IGBT in the boost circuit.

3. The Hamilton Controller of Inverter System.

3.1. Hamilton model.

The PCH systems can be described as [8]

$$\begin{cases} \dot{\boldsymbol{x}} = [\boldsymbol{J}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})]\dfrac{\partial \boldsymbol{H}}{\partial \boldsymbol{x}}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{g}^T(\boldsymbol{x})\dfrac{\partial \boldsymbol{H}}{\partial \boldsymbol{x}}(\boldsymbol{x}) \end{cases} \tag{2}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ and $\boldsymbol{y}$ are the input and output vector respectively, $\boldsymbol{R}(\boldsymbol{x}) = \boldsymbol{R}^T(\boldsymbol{x}) \geq 0$ represents the dissipation, the interconnection structure is captured in matrix $\boldsymbol{g}(\boldsymbol{x})$ and the skew-symmetric matrix $\boldsymbol{J}(\boldsymbol{x}) = -\boldsymbol{J}^T(\boldsymbol{x})$, and $\boldsymbol{H}(\boldsymbol{x})$ is the Hamiltonian function of the system.

According to Equation (1) and Equation (2), we define the state vector of inverter control system as $\boldsymbol{x} = \begin{bmatrix} Li_d & Li_q & L_1 i_L & Cu_{dc} \end{bmatrix}^T$ and let the Hamilton function of control system be

$$\boldsymbol{H}(\boldsymbol{x}) = \left(\boldsymbol{x}^T \boldsymbol{D}^{-1} \boldsymbol{x}\right)/2 \tag{3}$$

where $\boldsymbol{D} = diag\begin{pmatrix} L & L & L_1 & C \end{pmatrix}$. Equation (1) can be rewritten as Equation (2) with

$$\boldsymbol{J}(\boldsymbol{x}) = \begin{bmatrix} 0 & \omega L & 0 & \mu_d \\ -\omega L & 0 & 0 & \mu_q \\ 0 & 0 & 0 & -(1-\mu) \\ -\mu_d & -\mu_q & 1-\mu & 0 \end{bmatrix}, \quad \boldsymbol{R}(\boldsymbol{x}) = \begin{bmatrix} R & 0 & 0 & 0 \\ 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} e_d \\ e_q \\ E \end{bmatrix} \tag{4}$$

3.2. **Hamilton controller design.** The target of the grid connected inverter system is that the inverter output current has the same frequency and phase as the grid voltage. In addition, the harmonic wave of the output current must meet the requirements of the grid. Assuming losses power transmission between solar array and grid line, the instantaneous active power exchanged between the PV array and the grid is given by

$$P_{pv} = P = e_d i_d \tag{5}$$

The reference current can be obtained as $i_{d0} = P_{pv}/e_d$. Thus, the variation of the irradiance will result in a variation of i_{d0} in the control. In order to let the inverter unity power factor operation when the system reaches the steady state, the i_{q0} is obtained as $i_{q0} = 0$.

In addition, V_{dc} is the desired value of the voltage. Substituting i_{d0}, i_{q0} and V_{dc} into Equation (1), i_{L0} can be obtained as $i_{L0} = \left(e_d i_{d0} + R i_{d0}^2\right)/E$. The desired equilibrium point of the system is defined as $\boldsymbol{x}_0 = \begin{bmatrix} Li_{d0} & Li_{q0} & L_1 i_{L0} & CV_{dc} \end{bmatrix}^T$.

In order to stabilize the control system at the desired equilibrium $\boldsymbol{x_0}$, assign a closed-loop system desired energy function $\boldsymbol{H}_d(\boldsymbol{x}) = (\boldsymbol{x} - \boldsymbol{x}_0)\boldsymbol{D}^{-1}(\boldsymbol{x} - \boldsymbol{x}_0)/2$ and $\boldsymbol{H}_d(\boldsymbol{x}) \geq 0$. With $\boldsymbol{J}_d(\boldsymbol{x}) = \boldsymbol{J}(\boldsymbol{x}) + \boldsymbol{J}_a(\boldsymbol{x}) = -\boldsymbol{J}_d{}^T(\boldsymbol{x})$, $\boldsymbol{R}_d(\boldsymbol{x}) = \boldsymbol{R}(\boldsymbol{x}) + \boldsymbol{R}_a(\boldsymbol{x}) = \boldsymbol{R}_d{}^T(\boldsymbol{x}) \geq 0$, and feedback control $\boldsymbol{\alpha} = \boldsymbol{\alpha}(\boldsymbol{x})$ can be found to satisfy

$$[\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})] \frac{\partial \boldsymbol{H}_d(\boldsymbol{x})}{\partial \boldsymbol{x}} = [\boldsymbol{J}(\boldsymbol{x}) - \boldsymbol{R}(\boldsymbol{x})] \frac{\partial \boldsymbol{H}(\boldsymbol{x})}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \tag{6}$$

System (2) can be described in the following form

$$\dot{\boldsymbol{x}} = [\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})] \frac{\partial \boldsymbol{H}_d}{\partial \boldsymbol{x}}(\boldsymbol{x}) \tag{7}$$

The closed-loop system (7) will be asymptotically stable at the desired equilibrium. Define $\boldsymbol{J}_a(\boldsymbol{x})$ and $\boldsymbol{R}_a(\boldsymbol{x})$ as follows

$$\boldsymbol{J}_a(\boldsymbol{x}) = \begin{bmatrix} 0 & j_{12} & 0 & 0 \\ -j_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{R}_a(\boldsymbol{x}) = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{bmatrix} \tag{8}$$

where j_{12} and $r_1 \sim r_4$ are interconnection and damping parameters to be designed, respectively. Equations (4), (8) and $\boldsymbol{x}_0$ are substituted into (6), and controllers of the system can be gotten as

$$\begin{cases} \mu_d = \dfrac{1}{V_{dc}}\left[j_{12}i_q + Ri_{d0} - r_1\left(i_d - i_{d0}\right) + e_d\right] \\ \mu_q = \dfrac{1}{V_{dc}}\left[\left(\omega L + j_{12}\right)i_{d0} - j_{12}i_d - r_2 i_q + e_q\right] \\ \mu = 1 - \dfrac{E}{V_{dc}\left(e_d + Ri_{d0}\right)}\left[\left(Ri_{d0} - r_2\left(i_d - i_{d0}\right) + e_d\right) + \dfrac{r_4}{i_{d0}}\left(V_{dc} - u_{dc}\right)\right] \\ \mu = 1 - \dfrac{r_3\left(i_L - i_{L0}\right) + E}{V_{dc}} \end{cases} \tag{9}$$

To ensure the controller's existence, the duty ratio functions of IGBT in the boost circuit must be equal to each other. The relationship between r_3 and r_4 is

$$r_3 = \frac{[r_4(V_{dc} - u_{dc}) + \mu_d i_{d0}]V_{dc} - Ei_{L0}}{(i_L - i_{L0})i_{L0}} \tag{10}$$

It is known that one of damping parameters can be expressed by the other, and the limit of the expressed parameter exists at the desired equilibrium state of system. Hence, the controller can be obtained by only choosing one parameter, here r_4 is chosen to set parameters based on the conditions $0 \le \mu \le 1$, $r_3 \ge 0$ and $r_4 \ge 0$, namely the duty ratio is chosen as

$$\mu = 1 - \frac{E}{V_{dc}(e_d + Ri_{d0})}\left[(Ri_{d0} - r_2\left(i_d - i_{d0}\right) + e_d) + \frac{r_4}{i_{d0}}(V_{dc} - u_{dc})\right] \tag{11}$$

Therefore, the PCH control of the grid-connected inverter system can be implemented using the block diagram of Figure 2.

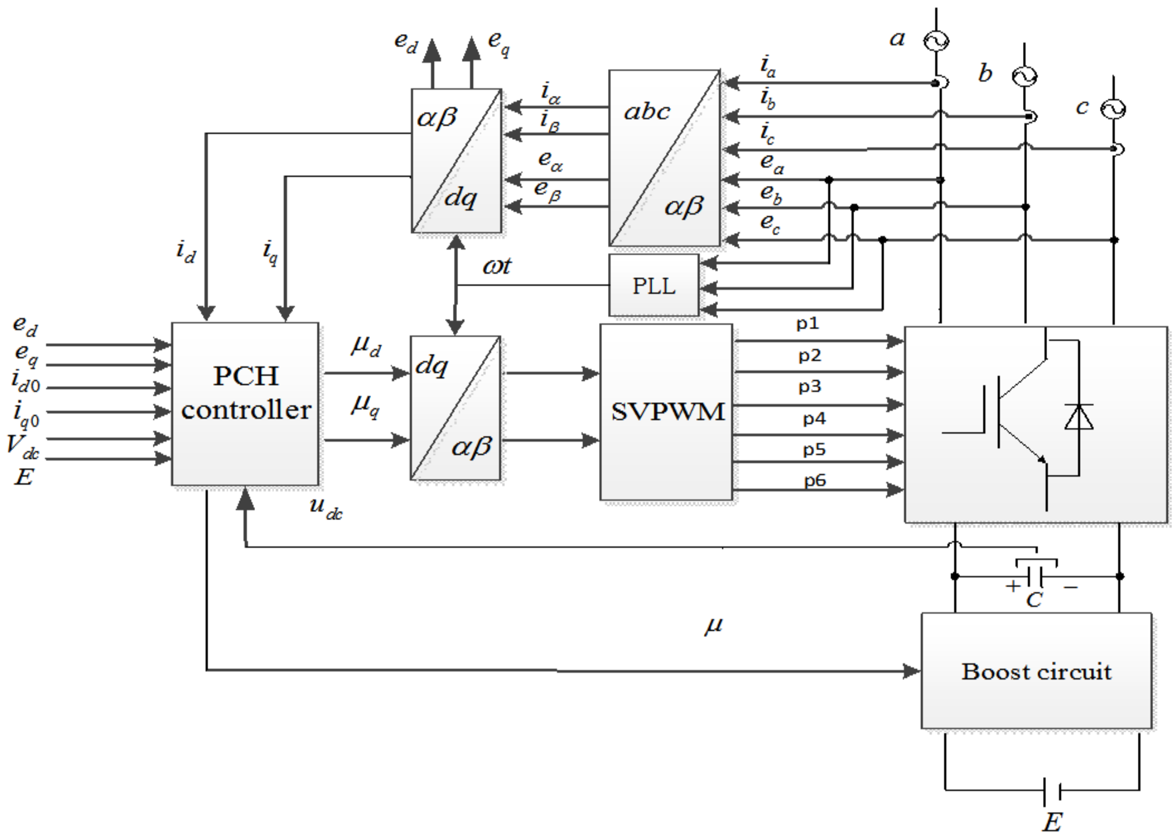

FIGURE 2. Block diagram of PCH control system for inverter system

3.3. **The stability analysis of the system.** The Lyapunov function of system can be defined as $V = \boldsymbol{H}(\boldsymbol{x}) = (\boldsymbol{x}^T\boldsymbol{D}^{-1}\boldsymbol{x})/2$. Obviously $V > 0$. Because the $\boldsymbol{J}_d(\boldsymbol{x})$ is skew-symmetric matrix and the $\boldsymbol{R}_d(\boldsymbol{x})$ is a positive semi-definite matrix, along the trajectory of system (7), the time derivative of V is

$$\begin{aligned}\dot{V} = \frac{\partial \boldsymbol{H}_d(\boldsymbol{x})}{\partial t} = \left[\frac{\partial \boldsymbol{H}_d(\boldsymbol{x})}{\partial \boldsymbol{x}}\right]^T \dot{\boldsymbol{x}} &= \left[\frac{\partial \boldsymbol{H}_d(\boldsymbol{x})}{\partial \boldsymbol{x}}\right]^T [\boldsymbol{J}_d(\boldsymbol{x}) - \boldsymbol{R}_d(\boldsymbol{x})] \frac{\partial \boldsymbol{H}_d}{\partial \boldsymbol{x}}(\boldsymbol{x}) \\ &= -\left[\frac{\partial \boldsymbol{H}_d(\boldsymbol{x})}{\partial \boldsymbol{x}}\right]^T \boldsymbol{R}_d(\boldsymbol{x}) \frac{\partial \boldsymbol{H}_d(\boldsymbol{x})}{\partial \boldsymbol{x}} \leq 0\end{aligned} \tag{12}$$

According to La Salle's invariance principle, if the closed loop system's maximal invariant set is $\{\boldsymbol{x}_0\}$, including the maximal invariant sets

$$\left\{x \in R^n | [\partial \boldsymbol{H}_d(\boldsymbol{x})/\partial \boldsymbol{x}]^T \boldsymbol{R}_d(\boldsymbol{x}) [\partial \boldsymbol{H}_d(\boldsymbol{x})/\partial \boldsymbol{x}] = 0\right\},$$

the Hamilton controller of inverter system is asymptotically stable.

4. **Simulation Results.** In order to validate the proposed control algorithm, the virtual circuit is built in MATLAB/Simulink. The inverter system parameters are as follows: the peak of the AC voltage $U_m = 311$ V, the frequency of grid $f = 50$ Hz, the inductance of AC side $L = 10$ mH, the resistance of AC side $R = 0.1\ \Omega$, the input voltage $E = 200$ V, the inductance of boost circuit $L_1 = 9$ mH, and the capacitance of DC side $C = 50$ μF. The i_{d0} and i_{q0} are obtained according to the output power of photovoltaic cells. The desired equilibrium point is $i_{d0} = 10$ A, $i_{q0} = 0$ A, and $V_{dc} = 700$ V. The Hamilton controller parameters are: $j_{12} = 10$, $r_1 = 200$, $r_2 = 200$, $r_4 = 250$.

Figure 3 shows that the curves of voltage and current keep the same phase after a very short period of time. Figure 4 shows that the power factor is one when the inverter system reaches the steady state. Figure 5 represents the curves of i_d and i_q. The current achieves the desired current value in about 0.02 s and the start-up speed is very fast. According to the conclusion of the second section, when the output power of the solar cell changes, the reference current of the d axis changes. The i_d alters from 10 A to 20 A at 0.2 second. Figure 3 shows that the waveform of grid-connected current is not distorted. In addition, it shows that the current reaches the new set value after a very short dynamic process in Figure 5. The voltage of capacitance is shown in Figure 6. It is shown that the voltage not only has good dynamic response, but also reaches the desired value and remains unchanged. The experimental results show that the Hamiltonian control of the inverter system is feasible and has good control performance.

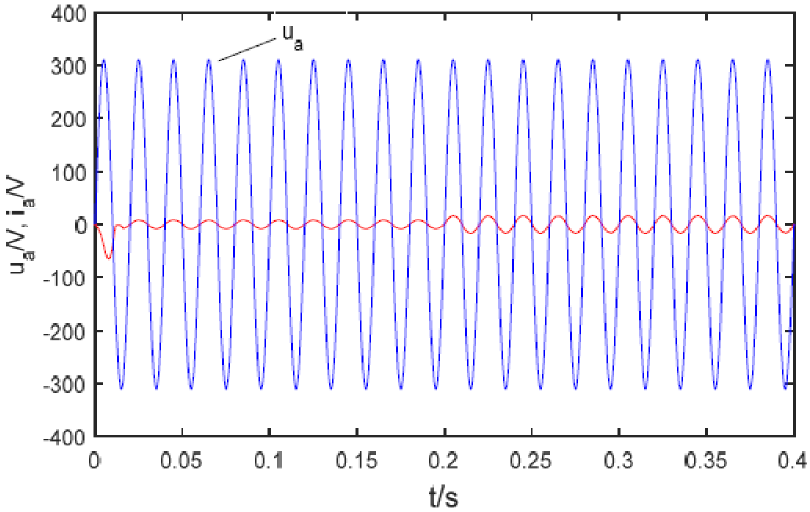

Figure 3. The curves of A phase voltage and current

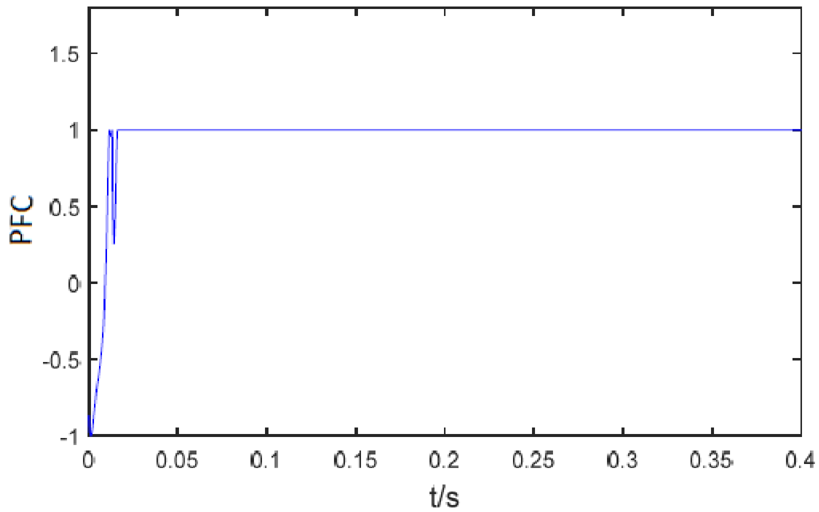

FIGURE 4. The curve of PFC

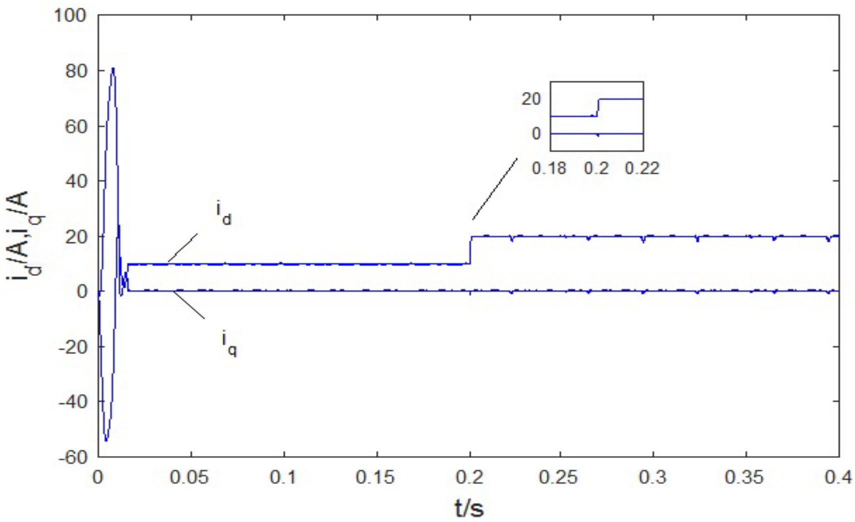

FIGURE 5. The curves of i_d and i_q

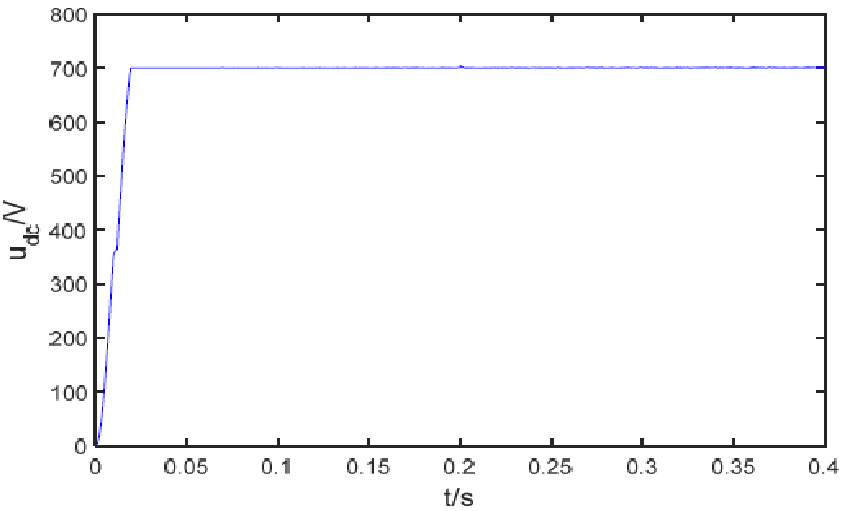

FIGURE 6. The curve of u_{dc}

5. **Conclusions.** In this paper, the boost circuit and the inverter circuit are analyzed as a whole inverter system to eliminate the error caused by separate modeling. The model of the inverter system is established based on a novel PCH and ES control principle. The Hamiltonian controller of the system is designed by interconnection assignment and damping injection. The designed controller is simple and can be implemented easily. Simulation studies have been performed to validate the proposed controller. The experimental results show that the PCH controller of the grid-connected inverter system has good dynamic and steady-state performance. Moreover, the proposed control method is also applicable to other converters.

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