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ADAPTIVE FINITE-TIME SUPPRESSION CONTROL FOR A CLASS OF UNCERTAIN CHAOTIC SYSTEMS

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Abstract. *This paper considers the problem of stabilization control of a class of chaotic systems with fully unknown parameters in a finite time. By combining the finite-time stability theory and adaptive control technique, an adaptive controller is developed to realize finite-time chaos stabilization of chaotic systems. Finally, some simulation results show that our control methods work very well in stabilizing a class of chaotic systems in a finite time.*

Keywords: Adaptive control, Finite-time control, Chaotic system

1. Introduction.

Chaotic dynamic system is a class of complex nonlinear systems, which shows irregular and unpredictable phenomena and commonly undesirable in practice. Thus, the consideration on chaotic suppression and control is an interesting topic attracted by many researchers, and some remarkable control approaches have been developed, for example, adaptive control [1, 2, 3], backstepping control [4, 5, 6] and fuzzy control [7]. In [1], a complete adaptive control approach based on the rigorous Lyapunov stability theorem is proposed for chaotic control and synchronization. Then, Zeng and Singh [2] consider the problem of adaptive control of chaos in Lorenz system; Tian and Gao [3] propose an adaptive control approach for chaotic continuous-time systems with delay. During the past decades, adaptive backstepping control has become one of the most popular design methods for nonlinear systems in triangular structure. By using backstepping technique, some adaptive backstepping control schemes are proposed for uncertain chaotic system, such as Lorenz system [4], Liu system [5] and unified chaotic system [6]. To reduce the number of adaptive laws, Chen et al. [7, 8] presented several adaptive fuzzy control schemes to control uncertain chaotic systems. However, all the control approaches aforementioned are applied to controlling the uncertain chaotic systems asymptotically. From a practical point of view, it is more useful to stabilize a control system in a given time. To obtain a faster convergence for control systems, an effective method is using finite time control methods.

Motivated by the above observations, in this paper, we consider the problem of adaptive finite-time control for a class of uncertain chaotic systems with unknown parameters. By combining the finite-time stability theory and adaptive control technique, an adaptive controller is developed to realize finite-time chaos stabilization of uncertain chaotic systems. Finally, some simulation results show that our control methods work very well in stabilizing a class of chaotic systems in a finite time.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Section 2. An adaptive finite-time control scheme is presented in Section 3. The simulation results are given in Section 4, and followed by Section 5 which concludes the work.

2. Problem Statement and Preliminaries.

In this paper, the controlled uncertain chaotic system is described by [9]

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 + u_1, \\ \dot{x}_2 = -b x_1 x_3 + c x_1 + u_2, \\ \dot{x}_3 = d x_1 x_2 - \varepsilon x_3 + u_3, \end{cases} \tag{1}$$

where $x = [x_1, x_2, x_3]^T$ is the state vector, and u_1, u_2 and u_3 are the control inputs.

The system (1) is chaotic when the parameter values are taken as

$$a = 20, \quad b = 5, \quad c = 40, \quad d = 4, \quad \varepsilon = 3. \tag{2}$$

The 3-D phase portrait of the system (1) is shown in Figure 1, when the parameter values are chosen as in (2) and the initial conditions are taken as follows: $x_1(0) = x_2(0) = x_3(0) = 10$.

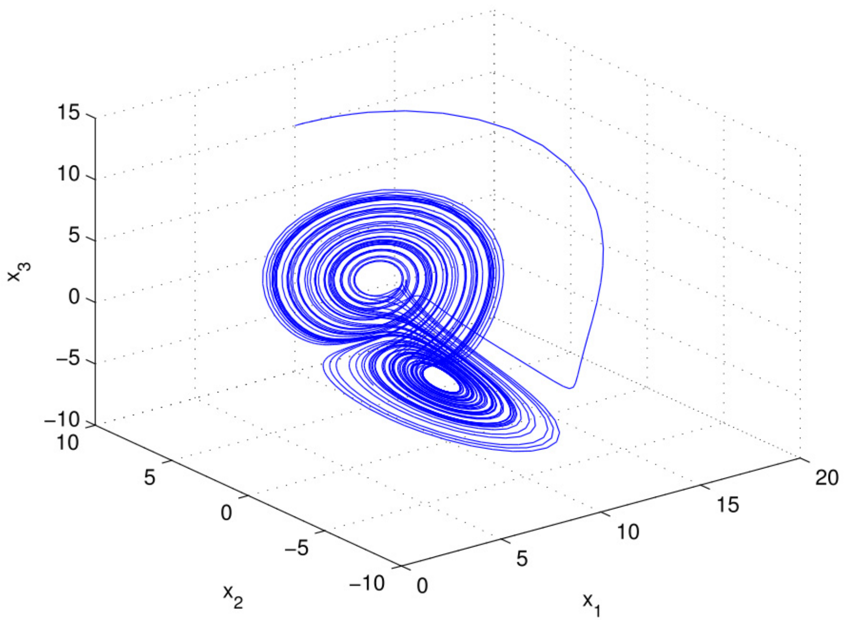

FIGURE 1. The 3-D phase portrait of the system (1)

The main control objective is to design an adaptive control scheme u_i $(i = 1, 2, 3)$ for the chaotic system (1) with unknown parameters such that the chaos of the system is suppressed in finite time.

Suppose that the parameters a, b, c, d and ε are unknown in advance and define $\phi = [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5]^T = [a, b, c, d, \varepsilon]^T$ as the vectors of the unknown parameters in system (1). Next, the following assumption is needed.

Assumption 2.1. *There exists positive constant* Φ *such that*

$$\|\phi\| \leq \Phi, \tag{3}$$

where $\|\cdot\|$ *is the Euclidean norm in* $\mathbf{R}^n$.

Definition 2.1. *Consider the autonomous chaotic system described by (1). If there exists a constant $T = T(x(0)) > 0$, such that*

$$\lim_{t\to T} \|x(t)\| = 0 \tag{4}$$

and $\|x(t)\| \equiv 0$ if $t \geq T$, then the chaos suppression of the autonomous chaotic system (1) is achieved in a finite time.

Lemma 2.1. *Assume that a continuous, positive definite function $V(t)$ satisfies the following differential inequality:*

$$\dot{V}(t) \leq -c_0 V^{\xi}(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0, \tag{5}$$

where $c_0 > 0$, $0 < \xi \leq 1$ are two constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\xi}(t) \leq V^{1-\xi}(t_0) - c_0(1-\xi)(t-t_0), \quad t_0 \leq t \leq t_1, \tag{6}$$

and $V(t) \equiv 0$, $\forall t \geq t_1$ with t_1 given by

$$t_1 = t_0 + V^{1-\xi}(t)/c_0(1-\xi). \tag{7}$$

Lemma 2.2. *For $a_1, a_2, \ldots, a_n \in \mathbf{R}$, the following inequality holds:*

$$|a_1| + |a_2| + \cdots + |a_n| \geq \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}\,. \tag{8}$$

3. **Adaptive Finite-Time Control Design.** In this section, an adaptive control scheme will be proposed such that the chaos of the system is suppressed in finite time, and the finite-time convergence and stability of the proposed control scheme will be proved via the Lyapunov stability theory.

For clarity, we first give the adaptive controller which will be constructed in the following form:

$$\begin{aligned} u_1 &= -x_2x_3 - \hat{\phi}_1(x_2 - x_1) - \mu\left(\Phi + \left\|\hat{\phi}\right\|\right)\frac{x_1}{\|x\|^2} - \eta_1 \mathrm{sgn}(x_1), \\ u_2 &= \hat{\phi}_2 x_1 x_3 - \hat{\phi}_3 x_1 - \mu\left(\Phi + \left\|\hat{\phi}\right\|\right)\frac{x_2}{\|x\|^2} - \eta_2 \mathrm{sgn}(x_2), \\ u_3 &= \hat{\phi}_4 x_1 x_2 - \hat{\phi}_5 x_3 - \mu\left(\Phi + \left\|\hat{\phi}\right\|\right)\frac{x_3}{\|x\|^2} - \eta_3 \mathrm{sgn}(x_3), \end{aligned} \tag{9}$$

where $\hat{\phi} = \left[\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3, \hat{\phi}_4, \hat{\phi}_5\right]^T$ are estimations for unknown parameters ϕ, $\mu = \min\{\eta_i\}$, $i = 1, 2, 3$, $\eta_i > 0$ is a constant gain, sgn(.) is the sign function, and if $x(t) = 0$, then $\frac{x_1}{\|x\|^2} = \frac{x_2}{\|x\|^2} = \frac{x_3}{\|x\|^2} = 0$.

Then, the parameter adaptive laws are designed as

$$\dot{\hat{\phi}}_1 = x_2 - x_1, \quad \dot{\hat{\phi}}_2 = -x_1x_3, \quad \dot{\hat{\phi}}_3 = x_1, \quad \dot{\hat{\phi}}_4 = x_1x_2, \quad \dot{\hat{\phi}}_5 = x_3. \tag{10}$$

Further, based on the control scheme aforementioned, the main result is summarized by the following theorem.

Theorem 3.1. *If the autonomous chaotic system (1) with fully unknown parameters is controlled by the control laws (9) with the adaptation laws (10), then the system trajectories will converge to zero in finite time and the chaotic behavior of the system will be suppressed.*

Proof: Choose a Lyapunov function candidate as

$$V(t) = \frac{1}{2}\left(\|x\|^2 + \left\|\hat{\phi} - \phi\right\|^2\right). \tag{11}$$

Then, the time derivative of $V(t)$ along the solution of (1) satisfies

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{3} x_i\dot{x}_i + \left(\hat{\phi}-\phi\right)^T \dot{\hat{\phi}} \\ &\leq x_1\left(\phi_1(x_2-x_1)+x_2x_3+u_1\right)+x_2\left(-\phi_2x_1x_3+\phi_3x_1+u_2\right) \\ &\quad +x_3\left(\phi_4x_1x_2-\phi_5x_3+u_3\right)+\left(\hat{\phi}-\phi\right)^T\dot{\hat{\phi}}. \end{aligned} \tag{12}$$

Next, by constructing adaptive control inputs u_1, u_2 and u_3 in (9), then we can rewrite (12) as

$$\begin{aligned}\dot{V}(t) &\leq -\tilde{\phi}_1(x_2-x_1)+\tilde{\phi}_2x_1x_3-\tilde{\phi}_3x_1-\tilde{\phi}_4x_1x_2-\tilde{\phi}_5x_3+\left(\hat{\phi}-\phi\right)^T\dot{\hat{\phi}} \\ &\quad -\mu\left(\Phi+\left\|\hat{\phi}\right\|\right)\left(\frac{x_1^2+x_2^2+x_3^2}{\|x\|^2}\right)-\eta_1|x_1|-\eta_2|x_2|+\eta_2|x_2|. \end{aligned} \tag{13}$$

Furthermore, with the help of adaptive laws given in (10), one has

$$\dot{V}(t) \leq -\eta_1|x_1|+\eta_2|x_2|-\eta_2|x_2|-\mu\left(\Phi+\left\|\hat{\phi}\right\|\right) \tag{14}$$

where the fact of $\frac{x_1^2+x_2^2+x_3^2}{\|x\|^2}=1$ and $x\text{sgn}(x)=|x|$ has been used in the above equation.

By using Assumption 2.1, we have

$$\left\|\hat{\phi}-\phi\right\| \leq \left\|\hat{\phi}\right\|+\|\phi\| \leq \left\|\hat{\phi}\right\|+\Phi. \tag{15}$$

Substituting (15) into (14) and using Lemma 2.2 result in

$$\begin{aligned}\dot{V}(t) &\leq -\mu\left(\|x\|+\left\|\hat{\phi}-\phi\right\|\right) \\ &\leq -\sqrt{2}\mu\left(\frac{1}{2}\left(\|x\|^2+\left\|\hat{\phi}-\phi\right\|^2\right)\right)^{1/2} \\ &= -\sqrt{2}\mu V^{1/2}(t) \end{aligned} \tag{16}$$

Therefore, based on Lemma 2.1, the system trajectories $x(t)$ will converge to zero in the finite time $T=\frac{\sqrt{2}}{\mu}\left(\frac{1}{2}\left(\|x(0)\|^2+\left\|\hat{\phi}(0)-\phi\right\|^2\right)\right)^{1/2}$. So the chaotic behavior of the system (1) is suppressed in finite time. The proof is thus complete.

4. **Simulation Results.** To illustrate the effectiveness of the proposed adaptive finite-time control scheme for chaotic suppression of the uncertain chaotic system (1), some simulation results are presented. In the simulation, the system parameters $a=20$, $b=5$, $c=40$, $d=4$ and $\varepsilon=3$ are chosen to guarantee the existence of chaos behavior for (1). In this way, the parameters' bound $\Phi=46$. The simulation is carried out for $[x_1(0),x_2(0),x_3(0)]^T=[10,10,10]^T$ and $\hat{\phi}_1(0)=12$, $\hat{\phi}_2(0)=36$, $\hat{\phi}_3(0)=1$, $\hat{\phi}_4(0)=41$, $\hat{\phi}_5(0)=5$ and $\eta_1=\eta_2=\eta_3=1$.

The simulation results are shown in Figures 2-4. Figure 2 shows the system variables x_1, x_2 and x_3. From Figure 2, we can see that the trajectories converge to zero fast which implies the system (1) is stabilized in a finite time and is not chaotic anymore. The trajectories of control inputs and the adaptive laws are shown in Figures 3 and 4.

5. **Conclusion.** In this paper, the problem of finite-time chaos suppression of a class of chaotic systems with uncertain parameters is considered. Based on finite time control techniques together with adaptive control approach, an adaptive state-feedback control laws are designed which can guarantee that all state variables converge to zero in a finite time. Simulation results further illustrate the effectiveness of our results. Our future work

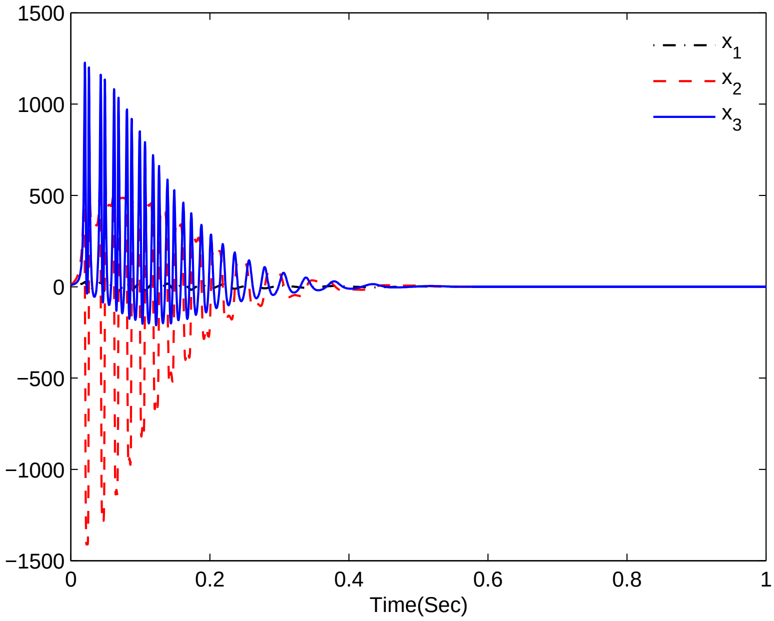

Figure 2. State variables x_1, x_2 and x_3

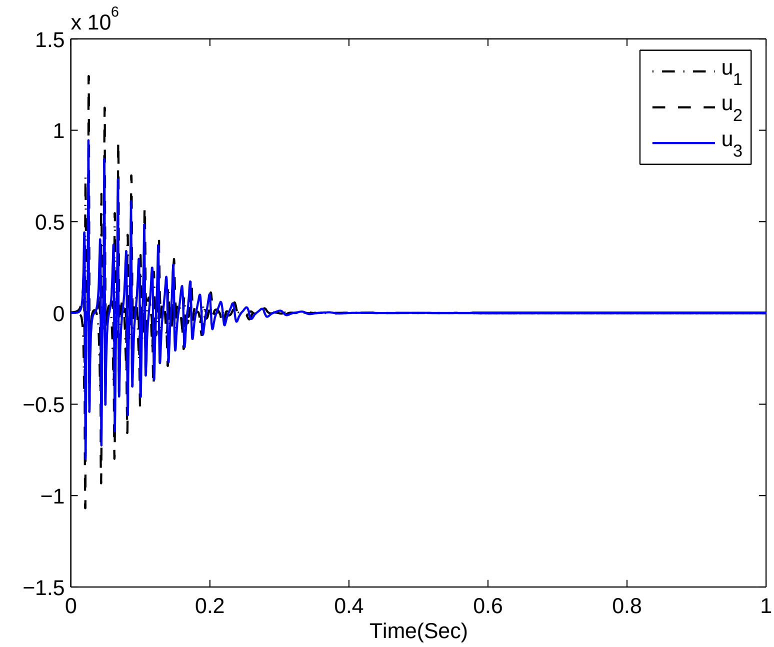

Figure 3. The control variables u_1, u_2 and u_3

will focus on designing output-feedback adaptive finite-time controller for the original system (1).

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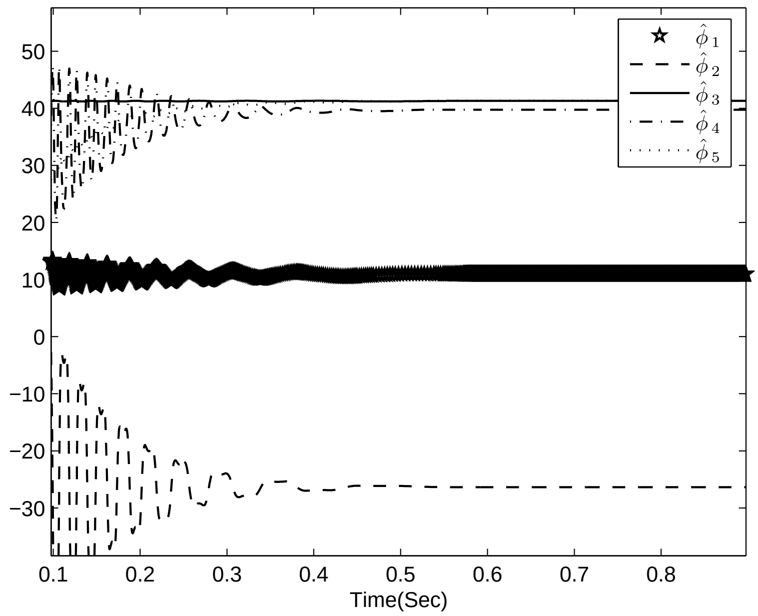

FIGURE 4. Adaptive parameters $\hat{\phi}_1$, $\hat{\phi}_2$, $\hat{\phi}_3$, $\hat{\phi}_4$ and $\hat{\phi}_5$

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