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ROBUST FUZZY TRACKING CONTROL FOR UNCERTAIN FLEXIBLE AIR-BREATHING HYPERSONIC VEHICLES

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Abstract. *In this paper, robust fuzzy tracking control for flexible air-breathing hypersonic vehicles with parameter uncertainty is discussed. Considering the additive uncertainties of parameters, a linear parameter varying (LPV) Takagi-Sugeno (T-S) fuzzy model is firstly employed to approximate the uncertain hypersonic vehicles. Then, by introducing the integral of the tracking error, the corresponding augmented system is developed from the LPV T-S fuzzy model. Based on the approach of the linear matrix inequality (LMI), the robust tracking controller is designed and the stability of the closed-loop system is guaranteed. Finally, the designed controller is evaluated through the Monte Carlo simulation and the simulation results demonstrate excellent tracking performance with good robustness.*
Keywords: Flexible air-breathing hypersonic vehicle, Parameter uncertainty, Fuzzy control, Linear matrix inequality

1. **Introduction.** As a reliable and cost-efficient way for access to space, air-breathing hypersonic vehicles (AHVs) have been investigated by many researchers in recent decades [1]. The unique characteristics of the vehicles make the design of control systems for air-breathing hypersonic vehicles become a challenging task [2]. Hypersonic flight usually covers the large flight envelope which implies the huge variations in environmental and aerodynamic characteristics. The slender geometries and light structures required for these aircrafts result in significant uncertain flexible effects. In addition, the strong couplings also exist among propulsion, structure, aerodynamics, and control. Hence, comprehensive uncertainties resulting from aerodynamic parameter variations, environmental disturbances, flexibility and the strong couplings influence the flight of FAHVs all the time. For better description of the dynamic characteristics, a flexible AHV (FAHV) model which includes the flexible dynamics was introduced by Bolender et al. [3]. Based on this model, various robust control methods, such as linear and nonlinear control approaches [4-7], are intensively considered. Although a rapid progress of control methodology for FAHVs has been achieved, the controller design problem which should be robust to unknown environment still needs to be further investigated.

Since Takagi and Sugeno proposed T-S fuzzy model-based controller in 1985, T-S fuzzy control technique has become an effective control approach for nonlinear systems. To date, the T-S fuzzy control method has been extended to the control of hypersonic vehicles [8,9]. However, the model addressed in above literature mainly focused on hypersonic vehicles without considering model uncertainty. Therefore, it is worth further studying the fuzzy control problem of hypersonic vehicles with uncertainties. In [10], Wu et al. regard the model uncertainties as the external disturbance and a disturbance observer based fuzzy tracking controller is proposed to solve the tracking problem. However, the use of the disturbance observer makes tracking controller more complex and not easy to implement. Obviously, directly using uncertain system models to describe hypersonic vehicles for controller design is more accurate and convenient. Thus, in this paper, based on the result of analysis on the uncertainty [2], we develop a linear parameter varying (LPV) Takagi-Sugeno (T-S) fuzzy model for approximating the FAHVs. Then, based on the LMI method, the corresponding fuzzy controller is designed for tracking control. Finally, Monte Carlo simulation is conducted to validate the proposed controller.

2. **Problem Statement and Preliminaries.** The longitudinal dynamics of the FAHV model, derived from Lagrange's equations and including flexibility effects, are given as below [3]:

$$\dot{V} = (T\cos\alpha - D)/m - g\sin\gamma \tag{1}$$
$$\dot{h} = V\sin\gamma \tag{2}$$
$$\dot{\gamma} = (L + T\sin\alpha)/(mV) - g\cos\gamma/V \tag{3}$$
$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$
$$\dot{q} = M/I_{yy} \tag{5}$$
$$\ddot{\eta}_i = -2\xi_i\omega_i\dot{\eta}_i - \omega_i^2\eta_i + N_i, \quad i = 1,2,3 \tag{6}$$

Five rigid-body states V, h, γ, α, q, which represent the vehicle velocity, altitude, flight path angle, angle of attack (AOA) and pitch rate respectively, and six flexible states $\eta = [\eta_1, \dot{\eta}_1, \eta_2, \dot{\eta}_2, \eta_3, \dot{\eta}_3]$ for the flexible modes are contained in this model. The control inputs are the fuel equivalence ratio ϕ, canard deflection δ_c, and elevator deflection δ_e, which do not appear in (1)-(6) directly. Instead, they enter the aerodynamic forces and moment through the thrust T, drag D, lift L, pitch moment M, and generalized forces N_i. The forces and moments employed in the FAHVs are approximated as [3]:

$$T = \bar{q}S\left[C_{T,\phi}(\alpha)\phi + C_T(\alpha) + C_T^{\eta}\eta\right]$$
$$D = \bar{q}SC_D(\alpha, \delta_e, \delta_c, \eta)$$
$$L = \bar{q}SC_L(\alpha, \delta_e, \delta_c, \eta)$$
$$M = z_T T + \bar{q}S\bar{c}C_M(\alpha, \delta_e, \delta_c, \eta)$$
$$N_i = \bar{q}S\left(N_i^{\alpha^2}\alpha^2 + N_i^{\alpha}\alpha + N_i^{\delta_e}\delta_e + N_i^{\delta_c}\delta_c + N_i^0 + N_i^{\eta}\eta\right), \quad i = 1,2,3$$

where $\bar{q}$, S, $\bar{c}$ are the dynamic pressure, reference area, and mean aerodynamic chord, respectively. The corresponding coefficients in the thrust T, drag D, lift L, pitch moment M, and generalized forces N_i are obtained using curve-fitted approximations, which can be expressed as

$$\begin{aligned}
&C_{T,\phi}(\alpha) = C_T^{\phi\alpha^3}\alpha^3 + C_T^{\phi\alpha^2}\alpha^2 + C_T^{\phi\alpha}\alpha + C_T^{\phi}\\
&C_T(\alpha) = C_T^3\alpha^3 + C_T^2\alpha^2 + C_T^1\alpha + C_T^0\\
&C_M(\alpha, \delta_e, \delta_c, \eta) = C_M^{\alpha^2}\alpha^2 + C_M^{\alpha}\alpha + C_M^{\delta_e}\delta_e + C_M^{\delta_c}\delta_c + C_M^0 + C_M^{\eta}\eta\\
&C_L(\alpha, \delta_e, \delta_c, \eta) = C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^0 + C_L^{\eta}\eta\\
&C_D(\alpha, \delta_e, \delta_c, \eta) = C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e + C_D^{\delta_c^2}\delta_c^2 + C_D^{\delta_c}\delta_c + C_D^0 + C_D^{\eta}\eta\\
&C_j^{\eta} = \left[C_i^{\eta 1}, 0, C_i^{\eta 2}, 0, C_i^{\eta 3}, 0\right], \ i = T, M, L, D\\
&N_i^{\eta} = \left[N_i^{\eta 1}, 0, N_i^{\eta 2}, 0, N_i^{\eta 3}, 0\right], \ j = 1,2,3
\end{aligned}$$

The output to be controlled is selected as $y = [V, h]$. Define the velocity and altitude tracking reference trajectories as V_r and h_r, respectively. Then the control objective is to design a robust fuzzy controller so that velocity V and altitude h can track the reference trajectories V_r and h_r, respectively.

For LPV T-S modeling of the FAHV, some simplifications and processing of the model are adopted. Firstly, the effects of six flexible states are neglected during the controller design process. Then, since the variation of dynamic pressure $\bar{q}$ and the uncertainty of the pitch moment M_{yy} are the main uncertainties which influence the flight stability significantly [2], additive parameter uncertainties $\theta_{\bar{q}}\bar{q}$ and $\theta_M M_{yy}$ are added in the vehicle model, where $\theta_{\bar{q}}$ and θ_M are the unknown coefficients for representing the levels of different uncertainties. Through the above simplification and processing, the uncertain nonlinear vehicle model is rewritten as

$$\dot{x}_a(t) = f_a(x_a, \theta_{\bar{q}}, \theta_M) + g_a(x_a, \theta_{\bar{q}}, \theta_M)u_a \tag{7}$$

$$y_a = Cx_a \tag{8}$$

with $x_a = [V, h, \gamma, \alpha, \bar{q}]^T$, $u_a = [\phi, \delta_e]^T$, $y_a = [V, h]^T$,

$$f_a(x_a, \theta_{\bar{q}}, \theta_M) = \begin{bmatrix} f_{a,11} - g\sin\gamma \\ V\sin\gamma \\ f_{a,31} - g\cos\gamma/V \\ q - f_{a,31} + g\cos\gamma/V \\ f_{a,51} \end{bmatrix}, \; g_a(x_a, \theta_{\bar{q}}, \theta_M) = \begin{bmatrix} g_{a,11} & g_{a,12} \\ 0 & 0 \\ g_{a,31} & 0 \\ -g_{a,31} & 0 \\ g_{a,51} & g_{a,52} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

and

$$\begin{aligned}
f_{a,11} &= (1+\theta_q)\,\bar{q}S\left(C_T(\alpha)\cos\alpha - \left(C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^0\right)\right)\Big/m \\
f_{a,31} &= (1+\theta_{\bar{q}})\,\bar{q}S\left(C_L^{\alpha}\alpha + C_L^0 + C_T(\alpha)\sin\alpha\right)/(mV) \\
f_{a,51} &= (1+\theta_M)\,\bar{q}S\left(z_T C_T(\alpha) + \bar{c}\left(C_M^{\alpha^2}\alpha^2 + C_M^{\alpha}\alpha\right)\right)\Big/I_{yy} \\
g_{a,11} &= (1+\theta_{\bar{q}})\,\bar{q}SC_{T,\phi}(\alpha)/m \\
g_{a,12} &= -(1+\theta_{\bar{q}})\left(\left(C_D^{\delta_e^2} + k_{ec}^2 C_D^{\delta_c^2}\right)\delta_e + \left(C_D^{\delta_e} + k_{ec}C_D^{\delta_c}\right)\right) \\
g_{a,31} &= (1+\theta_{\bar{q}})\,\bar{q}SC_{T,\varphi}(\alpha)(\sin\alpha)/(mV) \\
g_{a,51} &= (1+\theta_M)\,\bar{q}Sz_T C_{T,\varphi}(\alpha)/I_{yy} \\
g_{a,52} &= (1+\theta_M)\,\bar{q}S\bar{c}\left(C_M^{\delta_e} + k_{ec}C_M^{\delta_c}\right)/I_{yy}
\end{aligned}$$

3. **LPV T-S Fuzzy Modeling.** Based on the nonlinear vehicle model with parameter uncertainties given above, the LPV T-S fuzzy model is built for the FAHV. First, motivated by [10], transforming the equilibrium condition into zero point, we obtain:

$$\dot{x}(t) = f(x, \theta_{\bar{q}}, \theta_M) + g(x, \theta_{\bar{q}}, \theta_M)u \tag{9}$$

$$y = Cx \tag{10}$$

where $x(t) = [x_1, x_2, x_3, x_4, x_5]^T \triangleq x_a - x_e$, $y \triangleq y_a - Cx_e$, $u(t) \triangleq u_a - u_e$, and (x_e, u_e) is the equilibrium point at a certain cruising flight condition. Next, based on the T-S fuzzy modeling method, the uncertain nonlinear model (9) and (10) can be described by an LPV T-S fuzzy model defined by the following fuzzy rules:

$$\begin{aligned}
&R_i : \textbf{IF } x_1 \text{ is } G_{k(i)} \text{ and } x_4 \text{ is } H_{l(i)} \\
&\textbf{THEN } \begin{cases} \dot{x} = A_i(\theta_{\bar{q}}, \theta_M)\,x + B_i(\theta_{\bar{q}}, \theta_M)\,u \\ y = Cx \end{cases} \\
&i = 1, \ldots, L, \quad k = 1, \ldots, m, \quad l = 1, \ldots, n
\end{aligned} \tag{11}$$

where R_i represents the ith fuzzy rule, and L is the total number of rules. $G_{k(i)}$ and $H_{l(i)}$ are the fuzzy sets corresponding to x_1, x_4 in the kth and lth fuzzy implication of the ith fuzzy rule, and m and n are the total number of fuzzy sets $G_{k(i)}$, $H_{l(i)}$, respectively. $A_i\left(\theta_{\bar{q}}, \theta_M\right)$ and $B_i\left(\theta_{\bar{q}}, \theta_M\right)$ are the LPV system matrices which contain the unknown coefficients $\theta_{\bar{q}}, \theta_M$. An optimum method in [11] is used to calculate $A_i\left(\theta_{\bar{q}}, \theta_M\right)$ and $B_i\left(\theta_{\bar{q}}, \theta_M\right)$. So the system matrices can be rewritten as $A_i\left(\theta_{\bar{q}}, \theta_M\right) = A_{i,0}+\theta_{\bar{q}}A_{i,1}+\theta_M A_{i,2}$, $B_i\left(\theta_{\bar{q}}, \theta_M\right) = B_{i,0} + \theta_{\bar{q}}B_{i,1} + \theta_M B_{i,2}$.

Let $G_{k(i)}(x_1)$, $H_{l(i)}(x_4)$ be the firing level of x_1, x_4 in the fuzzy set $G_{k(i)}$, $H_{l(i)}$, respectively. Then the LPV T-S fuzzy model of the system can be inferred as

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{L} h_i(t)\left[A_i\left(\theta_{\bar{q}}, \theta_M\right)x + B_i\left(\theta_{\bar{q}}, \theta_M\right)u\right] \\ y = Cx \end{cases} \tag{12}$$

where $h_i(t) = \bar{h}_i(t)\Big/\left(\sum_{i=1}^{L}\bar{h}_i(t)\right)$, $\bar{h}_i(t) = G_{k(i)}(x_1)H_{l(i)}(x_4)$.

x_1, x_4 are chosen as the premise variables because they are related to the flight states V and α which are sensitive to the flight dynamics. Here, we assume that $x_1 \in [-10^3, 10^3]$, $x_4 \in [-0.01, 0.01]$. Hence, Z-shaped and S-shaped functions are adopted in the fuzzy sets and the corresponding membership functions are selected in Figure 1.

Finally, by selecting the following four operating states as: $[-1000, 0, 0, -0.01, 0]^T$, $[-1000, 0, 0, 0.01, 0]^T$, $[1000, 0, 0, -0.01, 0]^T$, $[1000, 0, 0, 0.01, 0]^T$, the LPV T-S fuzzy model with four rules can be obtained by the above method.

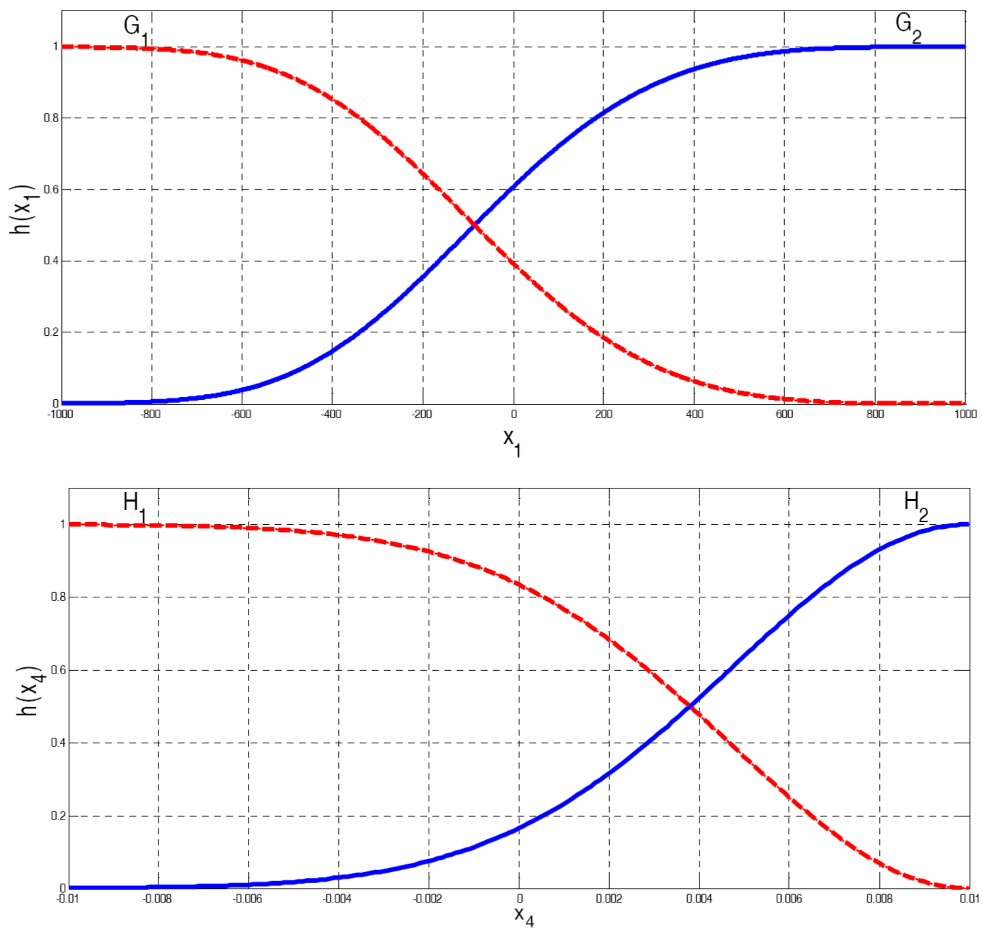

FIGURE 1. Definition of the membership functions of the fuzzy sets

4. **Robust Tracking Controller Design.** To track a reference command $y_{a,r} \triangleq [V_r(t), h_r(t)]^T$, it is desirable to introduce the integral action of the tracking error into the controller design. After augmentation with the integral error

$$\varsigma = \int_0^t (y_{a,r}(t) - y_a(t))dt = \int_0^t (y_r(t) - y(t))dt \tag{13}$$

where $y_r(t) \triangleq y_{a,r}(t) - Cx_e$, the LPV T-S fuzzy model is written in the form

$$\begin{cases} \dot{x}_{aug}(t) = \sum\limits_{i=1}^{L} h_i(t)\left[\bar{A}_i(\theta_{\bar{q}},\theta_M)x_{aug} + \bar{B}_i(\theta_{\bar{q}},\theta_M)u\right] + Gy_r \\ y = \bar{C}x_{aug} \end{cases} \tag{14}$$

where

$$x_{aug} \triangleq \left[x^T, \varsigma^T\right]^T,\ A_i\left(\theta_{\bar{q}},\theta_M\right) = \begin{bmatrix} A_i(\theta_{\bar{q}},\theta_M) & 0 \\ -C & 0 \end{bmatrix}$$

$$\bar{B}_i\left(\theta_{\bar{q}},\theta_M\right) = \begin{bmatrix} B_i(\theta_{\bar{q}},\theta_M) \\ 0 \end{bmatrix},\ \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix},\ G = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

Based on the parallel-distributed compensation (PDC) scheme [8], the robust fuzzy state-feedback controller for the augmented fuzzy model is constructed as

$$u(t) = \sum_{i=1}^{L} h_i(t)K_i x_{aug} \tag{15}$$

Substituting (15) into (14), the augmented closed-loop system can be written as

$$\begin{cases} \dot{x}_{aug}(t) = \sum\limits_{i=1}^{L}\sum\limits_{j=1}^{L} h_i(t)h_j(t)\left[\bar{A}_i(\theta_{\bar{q}},\theta_M) + \bar{B}_i(\theta_{\bar{q}},\theta_M)K_j\right]x_{aug} + Gy_r \\ y = \bar{C}x_{aug} \end{cases} \tag{16}$$

Thus, the output tracking controller design problem can be transformed into the stability problem of the closed-loop system (16) for all admissible $\theta_{\bar{q}}, \theta_M$.

Theorem 4.1. *For the LPV T-S fuzzy system (16), if there exists symmetric $X > 0$ matrices Y_j, and a scalar $s_1 < 0$ satisfying the following inequalities:*

$$\Gamma_{ii} - 2s_1 X < 0 \quad i = 1, 2, \ldots, L \tag{17}$$

$$\frac{1}{L-1}\Gamma_{ii} + \frac{1}{2}\left(\Gamma_{ij} + \Gamma_{ji}\right) < 0 \quad 1 \le i \ne j \le L \tag{18}$$

where $\Gamma_{ij} = \left(\bar{A}_{i,0} + \sum_{k=1}^{2}\delta_k\bar{A}_{i,k}\right)X + \left(\bar{B}_{i,0} + \sum_{k=1}^{2}\delta_k\bar{B}_{i,k}\right)Y_j + X\left(\bar{A}_{i,0} + \sum_{k=1}^{2}\delta_k\bar{A}_{i,k}\right)^T + Y_j^T\left(\bar{B}_{i,0} + \sum_{k=1}^{2}\delta_k\bar{B}_{i,k}\right)^T$, *and* $[\delta_1,\delta_2] \in \left\{[\delta_1,\delta_2]\middle|\bar{\delta}_1 = \bar{\theta}_{\bar{q}} \text{ or } \underline{\theta}_{\bar{q}}, \delta_2 = \bar{\theta}_M \text{ or } \underline{\theta}_M\right\}$, *then the fuzzy state-feedback controller* $u(t) = \sum_{i=1}^{L} h_i(t)K_i x_{aug}$, *where* $K_i = Y_i X^{-1}$ *keeps the system (16) stable for all admissible* $\theta_{\bar{q}}, \theta_M$.

Proof: Step 1: Choose $V(x,\theta) = x_{aug}^T P x_{aug}$ as a Lyapunov candidate function, where $P > 0$. Hence, system (16) is stable if inequality $\dot{V}(x_{aug}) < 0$, $\forall x \ne 0$ holds, i.e.,

$$\begin{aligned} &\sum_{i=1}^{L}\sum_{j=1}^{L} h_i(t)h_j(t)\Big[P\left(\bar{A}_i\left(\theta_{\bar{q}},\theta_M\right) + \bar{B}_i\left(\theta_{\bar{q}},\theta_M\right)K_j\right) \\ &+ \left(\bar{A}_i\left(\theta_{\bar{q}},\theta_M\right) + \bar{B}_i\left(\theta_{\bar{q}},\theta_M\right)K_j\right)^T P\Big] < 0 \end{aligned} \tag{19}$$

Pre- and post-multiply inequality (19) by $X = P^{-1}$ and its transpose. Then letting $Y_j = K_j X$ and applying Lemma 2 in [8], a sufficient condition is obtained for inequality (19) as follows:

$$\Phi_{ii} < 0 \quad i = 1, 2, \ldots, L \tag{20}$$

$$\frac{1}{L-1}\Phi_{ii} + \frac{1}{2}\left(\Phi_{ij} + \Phi_{ji}\right) < 0 \quad 1 \le i \ne j \le L \tag{21}$$

where $\Phi_{ij} = \left(\bar{A}_i\left(\theta_{\bar{q}}, \theta_M\right) X + \bar{B}_i\left(\theta_{\bar{q}}, \theta_M\right) Y_j\right) + \left(\bar{A}_i\left(\theta_{\bar{q}}, \theta_M\right) X + \bar{B}_i\left(\theta_{\bar{q}}, \theta_M\right) Y_j\right)^T$.

Step 2: For meeting some desired control performances, the closed-loop poles should be located in a prescribed sub-region in the complex left half plane. Thus, motivated by Lemma 3 in [4], we obtain that if there exists a positive symmetric matrix P satisfying

$$P\left(\bar{A}_i\left(\theta_{\bar{q}}, \theta_M\right) + \bar{B}_i\left(\theta_{\bar{q}}, \theta_M\right) K_i - s_1 I\right) + \left(\bar{A}_i\left(\theta_{\bar{q}}, \theta_M\right) + \bar{B}_i\left(\theta_{\bar{q}}, \theta_M\right) K_i - s_1 I\right)^T P < 0 \tag{22}$$

then all the eigenvalues of the closed-loop system $\bar{A}_i(\theta_{\bar{q}}, \theta_M) + \bar{B}_i(\theta_{\bar{q}}, \theta_M)K_i$ lie in the left side of s_1 in the complex left half plane. Next, by pre- and post-multiplying inequality (22) by $X = P^{-1}$ and its transpose, inequality (22) is equivalent to

$$\left(\bar{A}_i\left(\theta_{\bar{q}}, \theta_M\right) - s_1 I\right) X + \bar{B}_i\left(\theta_{\bar{q}}, \theta_M\right) Y_i + X\left(\bar{A}_i\left(\theta_{\bar{q}}, \theta_M\right) - s_1 I\right)^T + Y_i^T \bar{B}_i^T\left(\theta_{\bar{q}}, \theta_M\right) < 0 \tag{23}$$

where $Y_j = K_j X$.

Step 3: Note that the left sides of inequalities (20), (21) and (23) are convex functions with respect to $\theta_{\bar{q}}$, θ_M. Thus, by using the convex principle and combining (20), (21) and (23), we get the sufficient conditions (17) and (18) for stabilization of the system (16).

5. **Simulation Results.** In this section, the designed controller is evaluated under the Monte Carlo framework. Simulations are conducted on the FAHV model with flexible states. The tracking reference trajectories are set as a 1000 ft/s change and a 10000 ft change in velocity and altitude channel respectively. The initial trim condition of the FAHV is chosen as: $x_e = [7846.6, 85000, 0, 0.0219, 0]^T$, while the initial control input is $u_e = [0.12, 0.12]^T$. By using the LPV T-S fuzzy modeling method described in Section 3, the LPV T-S fuzzy tracking model of the FAHV can be established. Then the method proposed in Theorem 4.1 is used to design a robust tracking controller. Let $s_1 = -0.15$, $-0.2 \le \theta_{\bar{q}} \le 0.2$, $-0.2 \le \theta_M \le 0.2$. By Theorem 4.1, the fuzzy state-feedback gains can be obtained as (where only two control gain matrices are given for brevity).

$$K_1 = \begin{bmatrix} 1.94 & -69388.20 & -7.29 & -2927.60 & -294.23 & 1.08 & -1.87 \\ -0.08 & 532.69 & 0.03 & 42.82 & 6.69 & -0.04 & 0.005 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 1.03 & -36406.77 & -3.81 & -1555.07 & -156.65 & 0.57 & -0.98 \\ -0.05 & 353.63 & 0.024 & 27.78 & 4.13 & -0.026 & 0.004 \end{bmatrix}$$

To demonstrate the robustness of the designed controller, 200 tests are conducted with 20% random parameter uncertainties of L, T, D, M in (1)-(5). Thus, during each test, the parameter variations are randomly chosen within $|\Delta L| \le 20\%$, $|\Delta T| \le 20\%$, $|\Delta D| \le 20\%$, $|\Delta M| \le 20\%$, whereas the fuzzy controller (15) remains the same. The tracking results are shown in Figure 2 which is obtained by overlapping the simulation curves of 200 times corresponding to 200 sets of uncertain parameters. It exhibits no overshoot and no steady-state error, which suggests that the designed controller achieves good robustness. Under 200 sets of uncertain parameters, Figure 3 presents simulation results of the control inputs. Therefore, from the simulation results, it is concluded that the given uncertain FAHV is stable under the proposed fuzzy state-feedback controller.

6. **Conclusions.** In this paper, a robust tracking control strategy is proposed for the tracking problem of the longitudinal dynamics of the FAHV. Based on the description of the FAHV model with parameter uncertainty, an LPV T-S fuzzy model is established. Then, a robust fuzzy tracking controller with feedback of the state and tracking error integral has been designed by LMI approach. Simulations on the nonlinear FAHV longitudinal model demonstrate that the designed controller achieves excellent tracking performance with good robustness.

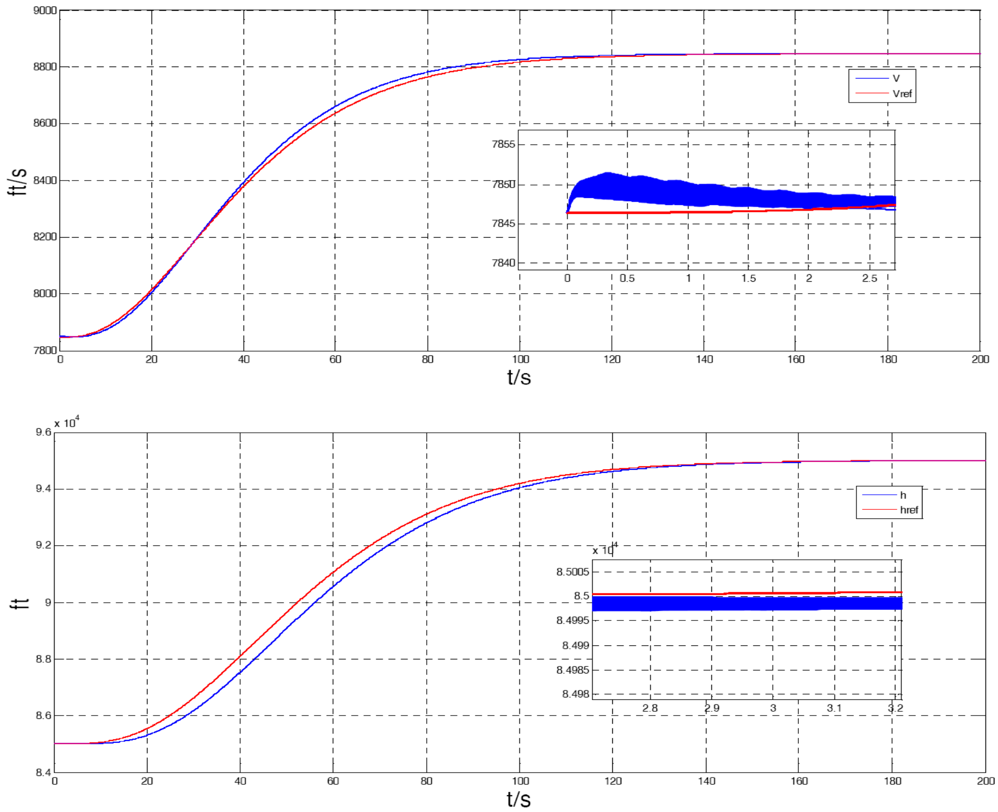

FIGURE 2. Velocity and altitude tracking response under 200 sets of uncertainty

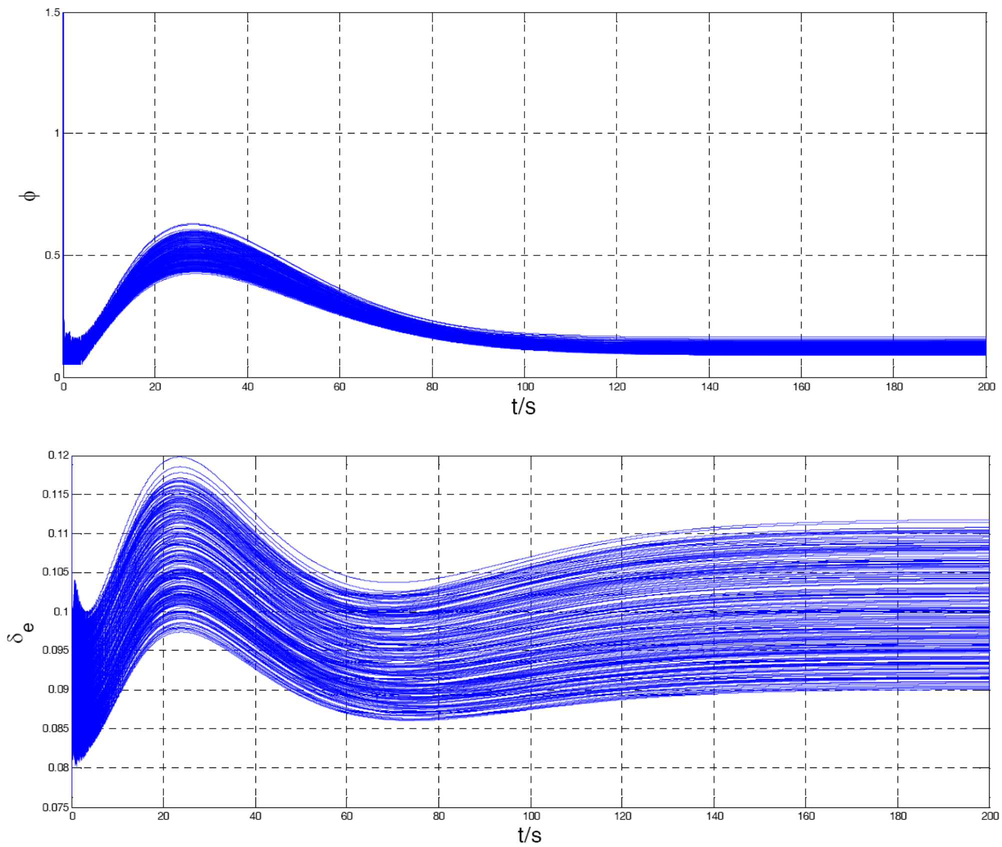

FIGURE 3. Control input signals under 200 sets of uncertainty

In this paper, we just take the uncertainties into consideration for robust fuzzy controller design. In the future research, we will further explore the robust fuzzy control scheme against measurement noise.

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