ICIC Express Letters
Volume **11**, Number **3**, March **2017**

ICIC International ©2017 ISSN 1881-803X
pp. 711–718

A CONSISTENCY DEGREE-BASED ATTRIBUTE REDUCTION METHOD OF INCONSISTENT DECISION SYSTEMS

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Received August 2016; accepted November 2016

Abstract. *In this paper, we define the concept of consistency degree of inconsistent decision system, and discuss the properties. Then we state the method for improving the consistency degree through a theorem. Finally, we design an algorithm that computes the reduct of inconsistent decision systems by two usual reduction properties. Several experiments are performed to demonstrate that our methods are effective to obtain high quality reduct in inconsistent decision table.*

Keywords: Attribute reduction, Inconsistent decision table, Consistency degree, Rough set

1. **Introduction.** Attribute reduction is a powerful data processing tool. It plays an essential role in numerous domains such as pattern recognition, data mining, machine learning and decision analysis [1-4]. Rough set can be a useful tool aiming to perform attribute reduction. The goal of attribute reduction is to find some particular subsets of the condition attributes. The redundant attributes can be removed while preserving even improving the classification quality. The minimum subsets of the condition attributes are called a reduct of a decision information system, which provides the same descriptive or classification ability as the entire set of attributes [5]. Numerous studies have been done for finding the set of all reducts or a single reduct [5-7]. In general, these work can be categorized as the following three aspects. 1) Attribute reduction based on partition. In [8], Miao et al. put all the definitions of relative reduct based on three different classification properties in the Pawlak rough set model into a unified framework, which will enhance the theoretical and logical understanding of the concept of relative reducts and lay the foundation for designing heuristics algorithms for reduction. Yang et al. [9] defined a relative discernibility relation of a condition attribute to characterize minimal elements in the discernibility matrix, and further developed two algorithms to find all reducts and one reduct in variable precision rough sets. Luan et al. [10] proposed an attribute reduction algorithm based on artificial fish swarm algorithm and rough set, which overcomes the difficulty that the calculation of equivalence classes is the most time-consuming. Shu and Qian [11] proposed an incremental attribute reduction algorithm for the incomplete decision systems that some of the attributes values for an object are incomplete (missing). Meng and Shi [12] proposed adivision algorithm for computing equivalence class for decision-theoretic rough set models, and further constructed a heuristic function for attribute reduction algorithms by extracting "monotonic ingredient" form decision systems. For large scale datasets, due to the existing algorithm's deficiency of computationally time-consuming, Liang et al. [13] developed an accelerator for attribute reduction, which simultaneously

reduced the size of the universe and the number of attributes at each iteration of the process of reduction. 2) Attribute reduction based on covering. Chen et al. [14] proposed the concept of induced cover and defined consistent and inconsistent covering decision systems, and then designed the algorithms using discernibility matrix to compute all the reducts. Thereafter, Wang et al. [15] improved their work in [14], and then developed a heuristic algorithm to find a subset of attributes that approximate an optimal reduction. James et al. [16] converted reduct problem into a set covering problem according to the positive regions in the variable precision rough set model, and then gave a set-covering heuristic function algorithm to compute the reduct, which can keep the positive regions consistent after the reduction. Moreover, there still have some work based on general relation decision systems. Chen et al. [17] used belief and plausibility functions to measure lower and upper approximations in neighborhood-covering rough sets, and then characterized the attribute reductions of covering information systems and decision systems by the respective functions. Further the authors gave the concepts of the significance and the relative significance of coverings to design algorithms for finding reducts. 3) Attribute reduction based on general relation. Liu et al. [18] gave the concept of general relation decision system which does not require a decision attribute set consisting of equivalence relations. Then the attributes reduction algorithms for both consistent and inconsistent relation decision systems have been proposed. Based on general binary relations, Wang et al. [19] defined relation information systems, consistent relation decision systems and relation decision systems, and developed the theorems necessary for computation of all the reducts. Chen et al. [20] introduced three kinds of dependency measure and proved the monotonicity, and then three types of heuristic algorithms are developed to obtain decision region preservation reducts. Zhang et al. [21] introduced an α-dominance relation based on inclusion measures, proposed a variable-precision-dominance-based rough set model, and then established an attribute reduction approach for interval-valued decision system.

The above only list partial achievements on attribute reduction. There are still much relative work. These work are categorized by the reduction criteria without considering the characteristics of decision systems. In fact, according to the characteristics of decision systems, they can be further divided into two kinds: one is for consistent decision systems, and the other is for inconsistent one. For the latter work, although there are many useful results, most of which only consider how to design the algorithms to improve the performance of reduction. Up to now, there are still few authors considering the decision system itself. In this paper, the main contribution is to propose the concept of consistency degree of inconsistent decision systems, and then perform attribute reduction with a given consistency degree. By using the consistency degree, we can remove some objects of original decision systems so as to obtain a new decision system with higher consistency, which can assure the reduction results more reliable. Therefore, our ultimate purpose is to perform attribute reduction, and consistency degree is only a strategy to improve the reduction quality. Under the guidance of theory frame, the reduction not only conforms to the preference of decision-makers, but also makes the results more reliable. Our work can enrich the existing theories and applications of rough sets to a certain degree. It can be used to construct the attribute importance meeting monotonicity requirements. Therefore, they will be widely applied in many fields such as performance evaluation, and engineering management.

The rest of the paper is structured as follows. Section 2 reviews some concepts of rough sets and inconsistent decision system. Section 3 gives the definition of consistency degree of inconsistent decision systems, and discusses some relevant properties, and further presents a method of improving the consistency degree of inconsistent decision systems through a theorem. Moreover, Section 4 presents attributes reduction algorithms, and analyzes the characteristics. Section 5 conducts the experiments by combining with some UCI data. Some concluding remarks are drawn in Section 6.

2. **Preliminaries.** First, we recall some basic concepts related to inconsistent decision systems in rough set theory, which can be found in [5,22].

Let $U = \{u_1, u_2, \ldots, u_n\}$ be a finite set of objects called the universe or a sample space. $P(U)$ is the power set of U.

A decision system (decision table) can be represented as $S = \left(U, C \bigcup D\right)$. $C = \{c_1, c_2, \ldots, c_m\}$ is called condition attributes and $D = \{d_1, d_2, \ldots, d_t\}$ is called a decision attribute. Each condition attribute c_j has a domain of values $V_j = \{c_j(u_i)\}_{i=1}^{n}$ and the decision attribute has a domain of values $V_d = \{d(u_i)\}_{i=1}^{n}$, where $c_j(u_i)$ and $d(u_i)$ are the values that attributes c_j and d take on the object u_i, respectively.

Suppose $R \subseteq U \times U$ is an equivalence relation on U, that is, R satisfies reflexivity, symmetric and transitivity. For any $x \in U$, the equivalence class of x with respect to R is defined by $[x]_R = \{y \in U \mid xRy\}$. The family of all equivalence class of R is called the quotient set induced by R, denoted by $U/R = \{[x]_R \mid x \in U\}$. For a subset $A \subseteq U$, the lower and upper approximations, the position and negative regions of A are defined as follows.

$\underline{apr}(A) = \left\{x \in U \mid [x]_R \sqsubseteq A\right\}$, $\overline{apr}(A) = \left\{x \in U \mid [x]_R \bigcap A \neq \phi\right\}$, $POS(A) = \underline{apr}(A)$, $NEG(A) = (\overline{apr}(A))^c$.

Definition 2.1. *Let U be a finite universe, and $S = \left(U, C \bigcup D\right)$ be a decision table. If $R_c = \bigcap_{i=1}^{m} c_i \subseteq R_D = \bigcap_{i=1}^{t} d_t$, where the sign $\bigcap$ is the intersection operation of a family of equivalence relations. Then $S = \left(U, C \bigcup D\right)$ is called consistent; otherwise, $S = \left(U, C \bigcup D\right)$ is called inconsistent.*

3. **Consistency Degree of Inconsistent Decision Systems.**

3.1. **Consistency degree and its properties.**

Definition 3.1. *For $x \in U$, let*

$$\eta_{\max}(x) = \max\left(\frac{|[x]_{R_A} \bigcap D_1|}{|[x]_{R_A}|}, \frac{|[x]_{R_A} \bigcap D_2|}{|[x]_{R_A}|}, \ldots, \frac{|[x]_{R_A} \bigcap D_m|}{|[x]_{R_A}|}\right), \tag{1}$$

$$CD(S) = \Sigma\eta_{\max}(x)/|U|. \tag{2}$$

Then we call $CD(S)$ the consistency degree of inconsistent decision systems.

$CD(S)$ reflects the consistency degree of decision systems from the whole. The higher $CD(S)$ is, the higher the reliability of the obtained knowledge from decision systems is with and vice versa.

Example 3.1. *Given a decision table $S = \left(U, C \bigcup D\right)$, $U/R_C = \{X_1, X_2, X_3, X_4\}$, $U/R_D = \{D_1, D_2, D_3\}$, where $X_1 = \{1, 2, 19, 20, 21\}$, $X_2 = \{3\}$, $X_3 = \{$4-13$\}$, $X_4 = \{$4-18$\}$, $D_1 = \{$1-12$\}$, $D_2 = \{$13-17$\}$, $D_3 = \{$18-21$\}$.*

It is easy to get $CD(S) = \frac{17}{21}$; $\underline{apr}D_1 = \{3\}$, $\overline{apr}D_1 = \{$1-13$, 19, 20, 21\}$; $\underline{apr}D_2 = \phi$, $\overline{apr}D_2 = \{$4-18$\}$; $\underline{apr}D_3 = \phi$, $\overline{apr}D_2 = \{1, 2,$ 14-21$\}$.

Definition 3.2. *If $CD(S) = 1$, then $S = \left(U, C \bigcup D\right)$ is consistent, or else it is inconsistent.*

Lemma 3.1. *$S = \left(U, C \bigcup D\right)$ is consistent if and only if $CD(S) = 1$.*

Definition 3.3. *A decision system $S = \left(U, C \bigcup D\right)$ is absolutely inconsistent iff $CD(S) =$ 0.5.*

Definition 3.3 shows it is the most difficult for knowledge acquisition when $CD(S) = 0.5$.

Definition 3.4. *Given a threshold β, if $CD(S) \geq \beta$, then $S = \big(U, C \bigcup D\big)$ is called β-consistent, or else it is called β-inconsistent.*

Proposition 3.1. *If $S = \big(U, C \bigcup D\big)$ is β-consistent, then $S = \big(U, C \bigcup D\big)$ is also consistent at any level $\beta_1 \leq \beta$.*

From Definition 3.4, we can improve the consistency degree of $S = \big(U, C \bigcup D\big)$ by some methods, i.e., removing the objects which can induce the inconsistency degree lower. Generally, we can take the two methods: the first is to remove the objects in an equivalence class with the obviously smaller inclination to a decision class; the second is to remove all the objects in an equivalence class with the same inclination to a decision class.

Example 3.2. *Given a decision table $S = \big(U, C \bigcup D\big)$, $U/R_C = \{X_1, X_2, X_3\}$, $U/R_D = \{D_1, D_2\}$, where $X_1 = \{1, 2\}$, $X_2 = \{3, 4, 5\}$, $X_3 = \{6\}$, $D_1 = \{1, 2, 3, 4\}$, $D_2 = \{5, 6\}$. Obviously, $CD(S) = \frac{5}{6}$, and it is an inconsistent decision table. Therefore, we can remove object 5 (with smaller inclination to D_2) to improve the consistency degree, and we can get a new decision table $S' = \big(U, C \bigcup D\big)$, $U/R_C = \{X_1, X_2, X_3\}$, $U/R_D = \{D_1, D_2\}$, where $X_1 = \{1, 2\}$, $X_2 = \{3, 4\}$, $X_3 = \{6\}$, $D_1 = \{1, 2, 3, 4\}$, $D_2 = \{6\}$, and we can get $CD(S') = 1$.*

Example 3.3. *Given a decision table $S = \big(U, C \bigcup D\big)$, $U/R_C = \{X_1, X_2, X_3\}$, $U/R_D = \{D_1, D_2\}$, where $X_1 = \{1, 2\}$, $X_2 = \{3, 4\}$, $X_3 = \{5\}$, $D_1 = \{1, 2, 3\}$, $D_2 = \{4, 5\}$. Obviously, $CD(S) = \frac{4}{5}$, and it is an inconsistent decision table. Therefore, we can remove object 3 and 4 (with the same inclination to D_1 and D_2) to improve the consistency degree, and we can get a new decision table $S' = \big(U, C \bigcup D\big)$, $U/R_C = \{X_1, X_2\}$, $U/R_D = \{D_1, D_2\}$, where $X_1 = \{1, 2\}$, $X_2 = \{5\}$, $D_1 = \{1, 2\}$, $D_2 = \{5\}$, and we can get $CD(S') = 1$.*

3.2. Improving $\boldsymbol{CD(S)}$ of a decision table.

Theorem 3.1. *For a decision table $S = \big(U, C \bigcup D\big)$, if we remove the objects which can induce the inconsistency degree lower according to the above mentioned methods, then $CD(S)$ can be improved.*

Proof: Let $U = \{x_1, x_2, \ldots, x_n\}$, $U/R_C = \{X_1, X_2, \ldots, X_m\}$ $(m \leq n)$, $U/R_D = \{D_1, D_2, \ldots, D_t\}$. Without loss of generality, suppose

$$x_{1i_1}, x_{2i_1}, \ldots, x_{ti_1} \in X_1,\ x_{1i_2}, x_{2i_2}, \ldots, x_{ti_2} \in X_2,\ \ldots,\ x_{1i_m}, x_{2i_m}, \ldots, x_{ti_m} \in X_m.$$

According to Equation (1), all $\eta_{\max}(x_i)$ are the same for any $x_i \in X_i$; let them be $\eta_1, \eta_2, \ldots, \eta_m$ respectively and suppose $\eta_1 \leq \eta_2 \leq \cdots \leq \eta_m$. Therefore, we firstly remove the objects in X_1. In the following, we will discuss it through two aspects.

1) Removing the objects in an equivalence class with the obviously smaller inclination to a decision class. For the original decision table S, we can get $\eta_1 = \frac{q_1}{t}$, $CD(S) = \frac{q_1 + s\eta_2 + \cdots + k\eta_m}{n}$, and here, q_1 is the number of the objects with a bigger inclination, i.e., the maximum number included in a D_i. Suppose to remove m_1 objects in X_1, and we can get $\eta_1' = \frac{q_1}{t - m_1}$, $CD(S') = \frac{q_1 + s\eta_2 + \cdots + k\eta_m}{n - m_1}$ for the new decision table S'. So we can get $CD(S) < CD(S')$. Similarly, we can get the same conclusion when removing the objects in other equivalence class with a smaller inclination.

2) Removing all the objects in an equivalence class with the same inclination to a decision class. Here we suppose $\frac{0}{0} = 0$. We can first remove the objects in an equivalence class belonging to a decision class, and then remove the remained objects belonging to another decision class. According to the idea of 1), we can get $CD(S) < CD(s')$ easily.

Corollary 3.1. *If $S = \big(U, C \bigcup D\big)$ is inconsistent, $\underline{apr}(D_i)$ is monotone increasing with the increasing of $CD(S)$, while $\overline{apr}(D_i)$ is monotone decreasing with the increasing of $CD(s)$.*

The CD value plays a vital role in feature selection and attributes reduction based on rough sets. In the following, we will explain how to improve CD. An algorithm can be realized. Several steps are designed as follows.

Algorithm 1. Improve CD of a decision table S.

Input: A decision table $S = \left(U, C \bigcup D\right)$ and given β.

Output: A decision table $S' = \left(U', C \bigcup D\right)$ with $CD(S') \geq \beta$.

Step 1: Compute the $CD(S)$, and if $CD(S) \geq \beta$, then end; else return to Step 2;

Step 2: For any $x_i \in U$, let $\xi = \min(\eta_{\max}(x_i))$,

for $i = 1, 2, \ldots, n$ $(|U| = n)$

if $\eta_{\max}(x_i) = \xi$, then $U_1 = [x_i]_{R_A}$, end;

end;

$D = \phi$;

Step 3: for $i = 1, 2, \ldots, n_1$ $(|U_1| = n_1)$

$D = [D, d(x_i)]$, end;

Step 4: Split $D \rightarrow [d_1, d_2, \ldots, d_t]$;

Let $m_1 = |d_1|, m_2 = |d_2|, \ldots, m_t = |d_t|$,

Ranking m_i, $i = 1, 2, \ldots, t$ according to decreasing sequence, suppose $m_1 > m_2 > \cdots > m_t$, take out m_t;

Step 5: find out x_i included in d_t and remove x_i,

Step 6: end; get a new $S' = \left(U', C \bigcup D\right)$.

Through Algorithm 1, the consistency degree of $S = \left(U, C \bigcup D\right)$ can be improved according to the decision preference.

Example 3.1 (continued) *If we remove the second sample, then* $\underline{apr}D_1 = \{3\}$, $\overline{apr}D_1 = \{1, 3\text{-}13, 19, 20, 21\}$, $CD(S) = \frac{17}{20}$; *if removing the first and second samples, then* $\underline{apr}D_3 = \{19, 20, 21\}$, $\overline{apr}D_3 = \{14\text{-}21\}$, $CD(S) = \frac{17}{19}$. *Through Example 3.1, we can verify Theorem 3.1 easily.*

4. A $CD(S)$-Based Attribute Reduction for Inconsistent Decision Systems.

Attribute reduction is a vital application of rough set theory. It is the minimal set of condition attributes that keep some properties of a decision system unchanged. The concept of reduct plays an important role in the analysis of a decision system. There are many different definitions of relative reducts for decision systems. Some are applicable to consistent decision systems, and others are applicable to both consistent and inconsistent decision systems. In [8], the authors have already given three definitions of relative reducts and they have proven that the three definitions are equivalent in consistent decision systems. However, in real applications, a large number of decision systems are inconsistent. In the following, we first introduce two definitions of [8].

4.1. Related work.

Definition 4.1. [8] *Given a decision table* $S = \left(U, C \bigcup D\right)$, *a condition attribute set* $A \subseteq C$, *the generalized decision of an object* $x \in U$ *is denoted as* $\delta(x/A) = \{V_d(y) \mid y \in [x]_A\}$. *The set of generalized decisions of all objects in* U *is denoted as the general decision* $\triangle(A)$, *that is,* $\triangle(A) = (\delta(x_1/A), \delta(x_2/A), \ldots, \delta(x_n/A))$, *where* $|U| = n$.

1) Region preservation reduct

An attribute set $A \subseteq C$ is a region preservation reduct of C with respect to D, if it satisfies the two conditions: a) $POS_A(D) = POS_C(D)$; b) for any $A' \subseteq A$, $POS_{A'}(D) \neq POS_C(D)$.

2) Decision preservation reduct

An attribute set $A \subseteq C$ is a decision preservation reduct of C with respect to D, if it satisfies the two conditions: a) $\triangle(A) = \triangle(C)$; b) for any $A' \subseteq A$, $\triangle(A') \neq \triangle(C)$.

4.2. **Algorithm design for attribute reduction in inconsistent decision systems.** Attribute reduction is the minimal set of condition attributes that keep some properties of a decision system unchanged. According to the definition of inconsistent decision systems, we know that some rules extracted from them may not be consistent. In order to improve the consistency degree of decision rules, we have discussed the method of revising the original decision system. In the following, we give Algorithm 2.

Algorithm 2. $CD(S)$-based attribute reduction algorithm.
Input: A decision table $S = \left(U, C \bigcup D\right)$ and a given β.
Output: A reduct of S.
Step 1: Compute $CD(S)$
Step 2: for $i = 1, 2, \dots, n$,
If $CD(S) < \beta$, use Algorithm 1, and obtain S',
Else $CD(S) \geq \beta$
End; return to Step 3,
Step 3: Perform reduction according to definition of region preservation reduct or decision preservation reduct;
Step 4: output a reduct of S.

5. Experimental Analysis.

Example 5.1. *Let $(U, C \bigcup D)$ be an inconsistent decision table as shown in Table 1, $U = \{x_1, x_2, \dots, x_{10}\}$, $C = \{C_1, C_2, C_3, C_4\}$, and $D = \{d\}$.*

Table 1. An inconsistent decision table

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
C_1	1	1	2	1	3	1	1	1	1	1
C_2	1	1	1	1	1	2	2	2	2	3
C_3	1	1	1	2	2	1	2	1	2	1
C_4	1	1	2	2	2	3	4	3	4	3
d	1	1	1	2	2	1	2	3	3	3

In the following, we will perform reduction through four methods: 1) Region preservation reduct (RPR); 2) Decision preservation reduct (DPR); 3) $CD(S)$-based RPR ($CD(S)$-RPR); 4) $CD(S)$-based DPR ($CD(S)$-DPR). The results are listed as Table 2.

Table 2. Reduction results for Example 3.3

CD	RPR	DPR	$CD(S)$-RPR	$CD(S)$-DPR
0.8	1,2,4	1,2,4	1,2,4	1,2,4
0.9			1,4	1,4
1			1,4	1,4

To further show our proposed algorithm's efficiency, we employ five datasets from UCI Machine Learning Repository to verify the performance of our method. The datasets are described in Table 3. In the following experiments, we still use RPR, DPR, $CD(S)$-RPR and $CD(S)$-DPR. The results are listed as Table 4.

From Tables 2 and 4, we find that all the above methods can effectively reduce the datasets attributes. The reduction will vary with the consistency degree of decision table. When $CD(S) = 0.8, 0.9$, the reduction of $CD(S)$-RPR and $CD(S)$-DPR keep the same with that of RPR and DPR, which is because the original dataset keeps unchangeable (i.e., the samples cannot be removed with $CD(S) = 0.8, 0.9$). Therefore, our method can effectively merge decision preference into the reduction process, which is more suitable

Table 3. Description of data sets

No.	Data sets	Samples	Attributes	Class
1	Contraceptive	1473	9	3
2	Energy efficiency	768	8	6
3	Fertilit	100	9	2
4	Haberman's Survival	306	3	2
5	Sampbase	4601	58	2

Table 4. Comparison of reduction results for four methods

	RPR	DPR	$CD(S)$-RPR			$CD(S)$-DPR		
			$CD = 0.8$	$CD = 0.9$	$CD = 1$	$CD = 0.8$	$CD = 0.9$	$CD = 1$
Contraceptive	1,2,3 4,5,6 7,8,9	1,2,3 4,5,6 7,8,9	1,2,3 4,5,6 7,8,9	1,2,3 4,5,6 7,8,9	1,2,3 4,5,6 7,8,9	1,2,3 4,5,6 7,8,9	1,2,3 4,5,6 7,8,9	1,2,3 4,5,6 7,8,9
Energy efficiency	6,8	6,8	6,8	6,8	6,8	6,8	6,8	6,8
Fertilit	1,2,5 9	1,2,5 9	1,2,5 9	1,2,5 9	1,2,4 6,7,9	1,2,5 9	1,2,5 9	1,2,4 6,7,9
Haberman's Survival	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
Sampbase	1,10 19,25 27,42, 45,46 50,52 55,56 57	1,10 19,25 27,42, 45,46 50,52 55,56 57	1,10 19,25 27,42, 45,46 50,52 55,56 57	1,10 19,25 27,42, 45,46 50,52 55,56 57	1,10 19,21 25,26 27,31 35,42 44,45 46,48 50,52 55,56 57	1,10 19,25 27,42, 45,46 50,52 55,56 57	1,10 19,25 27,42, 45,46 50,52 55,56 57	1,10 19,21 25,26 27,31 35,42 44,45 46,48 50,52 55,56 57

for practical problems. Moreover, with the improvement of $CD(S)$, our method can be widely applied to designing the uncertainty measure meeting monotonicity requirements for heuristic algorithms.

6. **Conclusions.** In this paper, by defining the concept of consistency degree of inconsistent decision system, we generalize the definition of consistent (inconsistent) decision system. We also give some properties of consistency degree, and further propose a method of improving the consistency degree of inconsistent decision system. Then we develop an algorithm to perform attribute reduction with the above studies. We set up the theoretical foundation for reduction of inconsistent decision systems, which should be associated with real application and data. In the future, constructing uncertainty measure meeting monotonicity requirements for heuristic algorithms design will be important and interesting work for different application fields.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (71540001, 71371064) and the Natural Science Foundation of Hebei Province (F2015208099, F2015208100).

REFERENCES

[1] K. Kaneiwa, A rough set approach to multiple dataset analysis, *Applied Soft Computing*, vol.11, pp.2538-2547, 2011.

[2] J. Su, B. Wang, C. Y. Hsiao and V. S. Tseng, Personalized rough-set-based recommendation by integrating multiple contents and collaborative information, *Information Sciences*, vol.180, pp.113-131, 2010.

[3] Z. Lu, Z. Qin, J. Zhang and J. Fang, A fast feature selection approach based on rough set boundary regions, *Pattern Recognition Letters*, vol.36, pp.81-88, 2014.

[4] Y. Liu, W. Huang, Y. Jiang and Z. Zeng, Quick attribute reduct algorithm for neighborhood rough set model, *Information Sciences*, vol.271, pp.65-81, 2014.

[5] Z. Pawlak, Rough sets, *International Journal of Computer and Information Sciences*, vol.111, pp.341-356, 1982.

[6] X. H. Hu and N. Cercone, Learning in relational databases: A rough set approach, *Computational Intelligence*, vol.11, pp.325-338, 1995.

[7] Q. H. Hu, Z. X. Xie and D. R. Yu, Hybrid attribute reduction based on a novel fuzzy-rough model and information granulation, *Pattern Recognition Letters*, vol.40, pp.3509-3527, 2007.

[8] D. Q. Miao, Y. Zhao, Y. Y. Yao, H. X. Li and F. F. Xu, Relative reducts in consistent and inconsistent decision tables of the Pawlak rough set model, *Information Sciences*, vol.179, pp.4140-4150, 2009.

[9] Y. Y. Yang, D. G. Chen and Z. Dong, Novel algorithms of attribute reduction with variable precision rough set model, *Neurocomputing*, vol.139, pp.336-344, 2014.

[10] X. Y. Luan, Z. P. Li and T. Z. Liu, A novel attribute reduction algorithm based on rough set and improved artificial swarm algorithm, *Neurocomputing*, vol.174, pp.522-529, 2016.

[11] W. H. Shu and W. B. Qian, An incremental approach to attribute reduction from dynamic incomplete decision systems in rough set theory, *Data Knowledge Engineering*, vol.100, pp.116-132, 2015.

[12] Z. Q. Meng and Z. Z. Shi, On quick attribute reduction in decision-theoretic rough set models, *Information Sciences*, vol.330, pp.226-244, 2016.

[13] J. Y. Liang, J. R. Mi, W. Wei and F. Wang, An accelerator for attribute reduction based on perspective of objects and attributes, *Knowledge-Based Systems*, vol.44, pp.90-100, 2013.

[14] D. G. Chen, C. Z. Wang and Q. H. Hu, A new approach to attribute reduction of consistent and inconsistent covering decision systems with covering rough sets, *Information Sciences*, vol.177, pp.3500-3518, 2007.

[15] C. Z. Wang, Q. He, D. G. Chen and Q. H. Hu, A novel method for attribute reduction of covering decision systems, *Information Sciences*, vol.254, pp.181-196, 2014.

[16] J. N. K. Liu, Y. X. Hu and Y. L. He, A set covering based approach to find the reduct of variable precision rough set, *Information Sciences*, vol.275, pp.83-100, 2014.

[17] D. G. Chen, W. L. Liu, X. Zhang and S. Kwong, Evidence-theory-based numerical algorithms of attribute reduction with neighborhood-covering rough sets, *International Journal of Approximation Reasoning*, vol.55, pp.908-923, 2014.

[18] G. L. Liu, L. Li, J. T. Yang, Y. B. Feng and K. Zhu, Attribute reduction approaches for general relation decision systems, *Pattern Recognition Letters*, vol.65, pp.81-87, 2015.

[19] C. Z. Wang, C. Wu and D. G. Chen, A systematic study on attribute reduction with rough sets based on general binary relations, *Information Sciences*, vol.178, pp.2237-2261, 2008.

[20] Y. M. Chen, Z. Q. Zeng, Q. X. Zhu and C. H. Tang, Three-way decision reduction in neighborhood systems, *Applied Soft Computing*, vol.38, pp.942-954, 2016.

[21] H. Y. Zhang, Y. Leung and L. Zhou, Variable-precision-dominance-based rough set approach to interval-valued information systems, *Information Sciences*, vol.244, pp.75-91, 2013.

[22] A. Skowron and C. Rauszer, The discernibility matrices and functions in information systems, in *Intelligent Decision Support, Handbook of Applications and Advances of the Rough Sets Theory*, R. Slowinsk (ed.), Kluwer Academic Publishers, 1992.