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ADAPTIVE PROBABILITY HYPOTHESIS DENSITY FILTER FOR TARGET TRACKING UNDER UNCERTAIN CONDITIONS

Bo Li*, Shuo Wang and Xu Jia

School of Electronics and Information Engineering
Liaoning University of Technology
No. 169, Shiying Street, Guta District, Jinzhou 121001, P. R. China
*Corresponding author: leeboo@yeah.net

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Abstract. *Target tracking is widely used in various surveillance systems. In order to deal with the difficulty of target tracking under uncertain conditions, an adaptive probability hypothesis density (PHD) filter is presented in this paper. Employing the beta distribution and augmented parameter in extended state space, we propose the novel PHD filter and its particle implementation on the basis of adaptive wave gate. Simulation results confirm the efficiency of the adaptive PHD filter.*

Keywords: Target tracking, PHD filter, Cardinality, Uncertain conditions

1. **Introduction.** The purpose of target tracking is to estimate cardinality of targets and related motion states from the current measurements [1]. With the rapid development of the random finite set (RFS) theorem, the probability hypothesis density (PHD) filter has been used in various surveillance fields [2].

Note that the probability of detection varies over time owing to the physical characteristics of passive sensors, and the clutters randomly occur in surveillance region during the tracking process. The uncertain detection probability and the uncertain clutter rate, as two uncertain conditions, are seriously restricted applications of the standard PHD filter. How to track targets under uncertain environment has become vital in practice. Firstly, the uncertain probability of detection would lead to unstable cardinality estimates. The work regarding target tracking based on the uncertain detection probability was proposed in [3]. According to a Markov transition, the linear Gaussian trajectory of target was observed, and the simulation results indicated that the proposed filter reasonably performed. In [4], the PHD recursions without the knowledge of detection probability were derived, and the closed form solutions were computed under the framework of the Gaussian sum. With respect to uncertain clutter rate, it blends target dynamic characteristics with clutter statistics, thus making it more difficult to extract actual targets from clutters. In [5], a new PHD filter in augmented state space was proposed when the clutter model and the prior knowledge were mismatching. Applying the Gaussian mixtures (GM) method, a novel algorithm for multiple targets tracking in the case of unknown clutter density was proposed in [6], and the simulation results validated the filter better than the conventional PHD filter in uncertain clutter environment. Recently, in order to deal with the uncertain detection profile, the robust cardinalized PHD (CPHD) filter was proposed to learn non-uniform detection profile and clutter background, and the experimental results showed that it corrected for discrepancies in detection parameters [7]. However, the implementations of the mentioned works are mostly dependent on the GM method that can be only applicable to the linear Gaussian dynamics. With regard to the nonlinear and non-Gaussian system, we have to apply the sequential Monte Carlo (SMC) implementation to estimate target dynamics. As known, the SMC methods are often a set of genetic-type particle Monte Carlo methodologies to solve the filtering problem based on the statistical and probabilistic point of view. Therefore, an adaptive PHD filter and its SMC implementation for target tracking under uncertain conditions are presented in this paper. The innovations of this work can be summarized as: first, the beta distribution and the augmented parameter in extended state space are employed to solve the difficulty of target tracking under uncertain conditions; besides, the adaptive detection gate has been also applied in the proposed filter to distinguishing the actual targets from enormous clutters.

The remainder of this note is organized as follows. In Section 2, the preliminaries of the standard PHD filter are briefly formulated. Section 3 proposes the adaptive PHD filter and its SMC implementation in detail. In Section 4, the simulation results evaluate the tracking performance of the adaptive PHD filter. We summarize this paper by providing the future work in the last section.

2. **Preliminaries.** Suppose that the target state vector $\boldsymbol{X}_k = \{\boldsymbol{x}_{1,k}, \ldots, \boldsymbol{x}_{n_k,k}\}$ is in the state space $\mathcal{X} \subseteq \mathbb{R}_{n_k}$ and the measurement vector $\boldsymbol{Z}_k = \{\boldsymbol{z}_{1,k}, \ldots, \boldsymbol{z}_{m_k,k}\}$ is in the measurement space $\mathcal{Z} \subseteq \mathbb{R}_{m_k}$, and then the stochastic dynamic system at time k is modeled as [1,2]:

$$\boldsymbol{x}_k = \boldsymbol{F}_{k|k-1}\boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1} \tag{1}$$

$$\boldsymbol{z}_k = h_k(\boldsymbol{x}_k) + \boldsymbol{u}_k \tag{2}$$

where $\boldsymbol{F}_{k|k-1}$ is the state transition matrix, $\boldsymbol{v}_{k-1}$ is the process noise vector at time $k-1$, $h_k(\cdot)$ denotes a deterministic mapping from $\boldsymbol{x}_k$ to $\boldsymbol{z}_k$, and $\boldsymbol{u}_k$ is the measurement noise vector at time k.

As we know, the PHD filter can propagate the posterior PHD, where the inner product of the PHD in the given state space is considered as the estimated cardinality and the associate peaks within the region can be regarded as the estimated positions. During the filtering process, the detection profile should be given before computing the PHD propagation. However, both detection probability and clutter rate have notable uncertainty in practice, even the prior knowledge is not completely available. As a result, the given modeling parameters are unrealistic, which cannot satisfy these uncertain conditions.

3. **The Adaptive PHD Filter.** To accommodate jointly uncertain clutter rate and uncertain detection probability, we outline a new PHD filter based on the adaptive scheme, where clutters or false alarms are modeled by an unknown and time varying number of clutters. Both actual targets and clutters are represented using the augmented variable, in addition to their kinematical states, to describe the unknown and time varying probability of detection.

3.1. **Filtering principle.** Assume $\mathcal{X}^{\mathrm{P}} = [0, 1]$ is the state space of uncertain probability of detection and $\mathcal{X}^{\ell} = \{0, 1\}$ is the discrete space of label ℓ for actual targets ($\ell = 1$) and clutters ($\ell = 0$), and then the extended state space $\mathcal{X}^{\dagger}$ can be defined as:

$$\mathcal{X}^{\dagger} = \mathcal{X} \cup \mathcal{X}^{\mathrm{P}} \cup \mathcal{X}^{\ell} \tag{3}$$

Let $f_\ell(x, \rho)$ denote the transition density function based on the augmented value $\rho \in \mathcal{X}^{\mathrm{P}}$, and then the integration in $\mathcal{X}^{\dagger}$ can be written as:

$$\int\limits_{\mathcal{X}^{\dagger}} f_\ell(x, \rho)\mathrm{d}x\mathrm{d}\rho = \int\limits_{\mathcal{X}\cup\mathcal{X}^{\mathrm{P}}} f_0(x, \rho)\mathrm{d}x\mathrm{d}\rho + \int\limits_{\mathcal{X}\cup\mathcal{X}^{\mathrm{P}}} f_1(x, \rho)\mathrm{d}x\mathrm{d}\rho \tag{4}$$

Time update: Considering the uncertain probability of detection $\rho_{1,k}$ and the state $(x_k, \rho_{1,k}) \in \mathcal{X} \cup \mathcal{X}^{\mathrm{P}}$ for actual targets at time k, we define the probability of survival

$p_{S,1,k}(x_k)$, the transition density $f_{1,k|k-1}(x_k|x_{k-1}) f_{\mathrm{P},k|k-1}(\rho_{1,k}|\rho_{1,k-1})$, and the birth intensity $\gamma_{1,k}(x_k,\rho_{1,k})$ respectively. Then, the predicted PHD is given by:

$$D_{1,k|k-1}(x_k,\rho_{1,k}) = \gamma_{1,k}(x_k,\rho_{1,k}) + \int_0^1 \int \substack{p_{S,1,k}(x_{k-1}) f_{1,k|k-1}(x_k|x_{k-1}) \\ \times f_{\mathrm{P},k|k-1}\left(\rho_{1,k}|\rho_{1,k-1}\right) D_{1,k-1}\left(\rho_{1,k-1},x_{k-1}\right)} \mathrm{d}x_{k-1}\mathrm{d}\rho_{1,k-1} \tag{5}$$

where $D_{1,k-1}(\rho_{1,k-1},x_{k-1})$ is the posterior PHD for actual targets at time $k-1$.

On the other hand, we have in hand the uncertain probability of detection $\rho_{0,k}$ and the state $(x'_k,\rho_{0,k}) \in \mathcal{X} \cup \mathcal{X}^{\mathrm{P}}$ for clutters ($\ell = 0$) at time k. Similarly, the probability of survival $p_{S,0,k}$, the transition density $f_{0,k|k-1}\left(x'_k|x'_{k-1}\right)$, and the birth intensity $\gamma_{0,k}(x'_k,\rho_{0,k})$ can be defined respectively. Thus, we have the predicted PHD:

$$D_{0,k|k-1}(\rho_{0,k}) = \gamma_{0,k}(\rho_{0,k}) + p_{S,0,k} D_{0,k-1}(\rho_{0,k-1}) \tag{6}$$

Note that $D_{0,k|k-1}(\rho_{0,k})$ is mainly characterized by the single dependent augment because the false alarms have nothing with the value of clutter state x'_k.

Measurement update: To make sure the available measurements more efficient, we introduce an adaptive round wave gate with the radius of $r = \varepsilon \dot{x}_{\max} T$, where $\varepsilon \geq 1$ is the adaptive control parameter, $\dot{x}_{\max}$ is the maximal velocity of the actual targets, and T is the sampling period. When $Z_{1,k} = \left\{ z_k \middle| \left| z_k - h_k\left(\hat{x}_k^{(m)}\right) \right| \leq r \right\}$, $Z_{1,k}$ can be regarded as the target-generated measurement set. Given that the available measurement set at time k is Z_k, then the measurement set for clutters can be written as $Z_{0,k} = Z_k - Z_{1,k}$. Let $g_k(z_k|x_k)$ and $\kappa_k(z_k)$ be likelihood functions for single target measurement and clutter respectively, and then we have:

$$\begin{aligned} &D_{1,k}(x_k,\rho_{1,k}) \\ &= D_{1,k|k-1}(x_k,\rho_{1,k}) \left(1 - \rho_{1,k} + \sum_{z_k \in Z_{1,k}} \frac{\rho_{1,k} g_k(z_k|x_k)}{\left\langle D_{0,k|k-1}, \rho_{0,k}\kappa_k \right\rangle + \left\langle D_{1,k|k-1}, \rho_{1,k} g_k(z_k|x_k) \right\rangle} \right) \end{aligned} \tag{7}$$

$$\begin{aligned} &D_{0,k}(\rho_{0,k}) \\ &= D_{0,k|k-1}(\rho_{0,k}) \left(1 - \rho_{0,k} + \sum_{z_k \in Z_{0,k}} \frac{\rho_{0,k}\kappa_k(z_k)}{\left\langle D_{0,k|k-1}, \rho_{0,k}\kappa_k(z_k) \right\rangle + \left\langle D_{1,k|k-1}, \rho_{1,k} g_k(z_k|x_k) \right\rangle} \right) \end{aligned} \tag{8}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product operation.

State estimation: The estimated cardinality of actual targets and clutters can be written as:

$$\hat{N}_{\ell,k} = \langle D_{\ell,k}(x_k,\rho_{\ell,k}), 1 \rangle \tag{9}$$

Further, the mean number of clutters is estimated as:

$$\hat{\lambda}_k = \langle D_{0,k}(\rho_{0,k}), \rho_{0,k} \rangle \tag{10}$$

Finally, the target state estimates are achieved using the k-means clustering method which iteratively attempts to find $\hat{N}_{1,k}$ clusters. Suppose that $\hat{x}_k^{(m)}$ is the center of each cluster, and then the target states at time k can be estimated by $\left\{ \hat{x}_k^{(m)} \right\}_{m=1}^{\hat{N}_{1,k}}$.

3.2. **Particle implementation.** According to the mentioned filtering frame of the adaptive PHD filter, the related SMC implementation is derived in this subsection.

Prediction: At time $k-1$, the posterior PHDs of actual targets and clutters can be approximated with the weighted particle set $\left\{x_{k-1}^{(i)}, w_{\ell,k-1}^{(i)}\right\}_{i=1}^{L_{\ell,k-1}}$:

$$D_{1,k-1}(x_{k-1}, \rho_{1,k-1}) = \sum_{i=1}^{L_{1,k-1}} w_{1,k-1}^{(i)} \beta_{u_{1,k-1},v_{1,k-1}}(\rho_{1,k-1}) \delta\left(x_{k-1} - x_{k-1}^{(i)}\right) \tag{11}$$

$$D_{0,k-1}(\rho_{0,k-1}) = \sum_{i=1}^{L_{0,k-1}} w_{0,k-1}^{(i)} \beta_{u_{0,k-1},v_{0,k-1}}(\rho_{0,k-1}) \tag{12}$$

where $L_{\ell,k-1}$ is the required number of particles, $\beta_{u_{\ell,k-1},v_{\ell,k-1}}(\rho_{\ell,k-1})$ is the beta distribution on the variable $\rho_{\ell,k-1}$ at time $k-1$, and the distribution's shape parameters meet $u_{\ell,k-1}, v_{\ell,k-1} > 0$ [3,7].

Time update: At time k, two time-predicted PHDs can be given by:

$$D_{1,k|k-1}(x_k, \rho_{1,k}) = \sum_{i=1}^{L_{1,k|k-1}} w_{1,k|k-1}^{(i)} \beta_{u_{1,k|k-1},v_{1,k|k-1}}(\rho_{1,k}) \delta\left(x_k - x_{k|k-1}^{(i)}\right) \tag{13}$$

$$D_{0,k|k-1}(\rho_{0,k}) = \sum_{i=1}^{L_{0,k|k-1}} w_{0,k|k-1}^{(i)} \beta_{u_{0,k|k-1},v_{0,k|k-1}}(\rho_{0,k}) \tag{14}$$

where the beta distribution can be written as:

$$\beta_{u_{\ell,k|k-1},v_{\ell,k|k-1}}(\rho_{\ell,k-1}) = \int_0^1 f_{\mathrm{P},k|k-1}(\rho_{\ell,k}|\rho_{\ell,k-1}) \beta_{u_{\ell,k-1},v_{\ell,k-1}}(\rho_{\ell,k-1})\, \mathrm{d}\rho_{\ell,k-1} \tag{15}$$

Especially, the shape parameters and the associate variance are defined as:

$$u_{\ell,k|k-1}^{(i)} = \frac{u_{\ell,k-1}^{(i)}}{u_{\ell,k-1}^{(i)} + v_{\ell,k-1}^{(i)}} \left(\frac{u_{\ell,k-1}^{(i)} v_{\ell,k-1}^{(i)}}{\left(\sigma_{\ell,k|k-1}^{(i)}\right)^2 \left(u_{\ell,k-1}^{(i)} + v_{\ell,k-1}^{(i)}\right)^2} - 1 \right) \tag{16}$$

$$v_{\ell,k|k-1}^{(i)} = \frac{v_{\ell,k-1}^{(i)}}{u_{\ell,k-1}^{(i)} + v_{\ell,k-1}^{(i)}} \left(\frac{u_{\ell,k-1}^{(i)} v_{\ell,k-1}^{(i)}}{\left(\sigma_{\ell,k|k-1}^{(i)}\right)^2 \left(u_{\ell,k-1}^{(i)} + v_{\ell,k-1}^{(i)}\right)^2} - 1 \right) \tag{17}$$

$$\left(\sigma_{\ell,k|k-1}^{(i)}\right)^2 = \left(\frac{1}{u_{\ell,k-1}^{(i)} + v_{\ell,k-1}^{(i)}} + \varepsilon \right) \frac{u_{\ell,k-1}^{(i)} v_{\ell,k-1}^{(i)}}{\left(u_{\ell,k-1}^{(i)} + v_{\ell,k-1}^{(i)}\right)\left(u_{\ell,k-1}^{(i)} + v_{\ell,k-1}^{(i)} + 1\right)} \tag{18}$$

where the threshold $\varepsilon \in [0,1]$ can be utilized to adjust the value of $\left(\sigma_{\ell,k|k-1}^{(i)}\right)^2$.

Measurement update: At time k, the measurement-updated PHDs can be computed based on the adaptive measurement set $Z_{\ell,k}$:

$$\begin{aligned} D_{1,k}(x_k, \rho_{1,k}) = \sum_{i=1}^{L_{1,k|k-1}} \Bigg(& w_{1,U,k}^{(i)} \beta_{u_{1,k|k-1},v_{1,k|k-1}}(\rho_{1,k}) \delta\left(x_k - x_{k|k-1}^{(i)}\right) \\ & + \sum_{z_k \in Z_{1,k}} w_{1,D,k}^{(i)}(z_k) \beta_{u_{1,k|k-1},v_{1,k|k-1}}(\rho_{1,k}) \delta\left(x_k - x_{k|k-1}^{(i)}\right) \Bigg) \end{aligned} \tag{19}$$

$$D_{0,k}(\rho_{0,k}) = \sum_{i=1}^{L_{0,k|k-1}} \left(w_{0,U,k}^{(i)} \beta_{u_{0,k|k-1},v_{0,k|k-1}}(\rho_{0,k}) + \sum_{z_k \in Z_{0,k}} w_{0,D,k}^{(i)}(z_k) \beta_{u_{0,k|k-1},v_{0,k|k-1}}(\rho_{0,k}) \right) \tag{20}$$

Considering the probability generating function $B(\cdot,\cdot)$ of the beta distribution, we can obtain the measurement-updated weights for undetected and detected components:

$$w_{\ell,U,k}^{(i)} = \frac{B\left(u_{\ell,k|k-1}^{(i)}, v_{\ell,k|k-1}^{(i)}+1\right)}{B\left(u_{\ell,k|k-1}^{(i)}, v_{\ell,k|k-1}^{(i)}\right)} \frac{w_{\ell,k|k-1}^{(i)}}{\sum_{\ell=0}^{1}\sum_{i=1}^{L_{\ell,k|k-1}} w_{\ell,k|k-1}^{(i)}} \tag{21}$$

$$w_{\ell,D,k}^{(i)}(z_k) = \frac{\underbrace{\frac{B\left(u_{\ell,k|k-1}^{(i)}+1, v_{\ell,k|k-1}^{(i)}\right)}{B\left(u_{\ell,k|k-1}^{(i)}, v_{\ell,k|k-1}^{(i)}\right)}\kappa_k(z_k)}_{\ell=0} + \underbrace{\frac{B\left(u_{\ell,k|k-1}^{(i)}+1, v_{\ell,k|k-1}^{(i)}\right)}{B\left(u_{\ell,k|k-1}^{(i)}, v_{\ell,k|k-1}^{(i)}\right)} g_k(z_k|x_k)}_{\ell=1}}{\sum_{\ell=0,i=1}^{L_{\ell,k|k-1}} \frac{u_{\ell,k|k-1}^{(i)}}{u_{\ell,k|k-1}^{(i)}+v_{\ell,k|k-1}^{(i)}}\kappa_k(z_k)\, w_{\ell,k|k-1}^{(i)} + \sum_{\ell=1,i=1}^{L_{\ell,k|k-1}} \frac{u_{\ell,k|k-1}^{(i)}}{u_{\ell,k|k-1}^{(i)}+v_{\ell,k|k-1}^{(i)}} g_k(z_k|x_k)\, w_{\ell,k|k-1}^{(i)}} w_{\ell,k|k-1}^{(i)} \tag{22}$$

State estimation: To solve the problems of particle degeneracy, we resample and get a new weighted particle set $\left\{x_k^{(i)}, w_{\ell,k}^{(i)}\right\}_{i=1}^{L_{\ell,k}}$. Then, the estimated cardinality of actual targets and clutters can be written as:

$$\hat{N}_{\ell,k} = \left[\sum_{i=1}^{L_{\ell,k}} w_{\ell,k}^{(i)}\right] \tag{23}$$

where $[\cdot]$ denotes the integer operation. Further, the mean number of clutters is estimated as:

$$\hat{\lambda}_k = \sum_{\ell=0,i=1}^{L_{\ell,k}} \frac{u_{\ell,k|k-1}^{(i)}}{u_{\ell,k|k-1}^{(i)}+v_{\ell,k|k-1}^{(i)}} w_{\ell,k}^{(i)} \tag{24}$$

Finally, the estimated state of actual targets $\left\{\hat{x}_k^{(m)}\right\}_{m=1}^{\hat{N}_{1,k}}$ can be obtained.

4. **Simulation Results and Analyses.** We use a typical simulation to compare the adaptive PHD filter with the standard PHD filter under 500 Monte Carlo runs. Our experimental environment was: Intel$^{\text{TM}}$ Core$^{\text{TM}}$ i5, RAM 4 GB, Windows$^{\text{TM}}$ 7, and MATLAB$^{\text{TM}}$ V8.0. In the simulation experiment, four targets (T1$\sim$T4) randomly move with the constant velocity (CV) motion in the half-disc surveillance region $[-2000, 2000] \times [0, 2000]$ m^2, where the initial positions are $(1000, 1500)$ m, $(250, 750)$ m, $(-1500, 250)$ m, and $(-250, 1000)$ m respectively. Their velocities are $(-20, -15)$ m/s, $(40, -25)$ m/s, $(45, 45)$ m/s, and $(40, 10)$ m/s. Especially, the surveillance time is 50 s and the sampling period is 1 s. The optimal sub-pattern assignment (OSPA) distance is used to evaluate tracking performance over the standard PHD filter with the given clutter number of 10 and detection probability of 95%.

Figure 1 shows the true target tracks and measurements. Note that four targets move in cluttered region during their motion cycles marked in this figure. As expected, the trajectories are straight lines that represent the CV motion state. It is also obvious that the measurements become more concentrated around the passive sensor located on the original point $(0, 0)$ m.

Figures 2 and 3 plot the x and y positions of true tracks, measurements and two filter estimates versus time, where "$\circ$" and "$\bullet$"denote the position estimates from the adaptive PHD filter and the standard PHD filter respectively, "$-$" and "$\times$" denote the true tracks and measurements. As seen, the position estimates of two filters make a steady approach

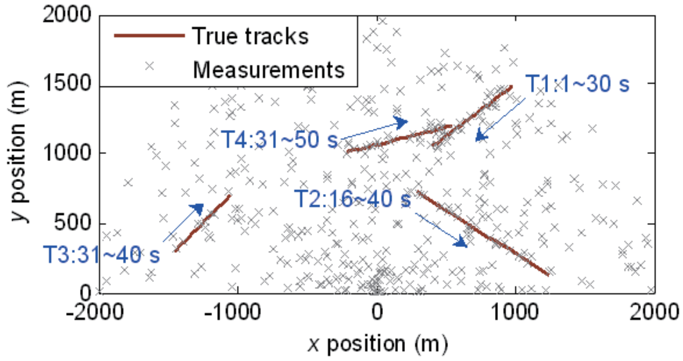

FIGURE 1. Target tracks and measurements

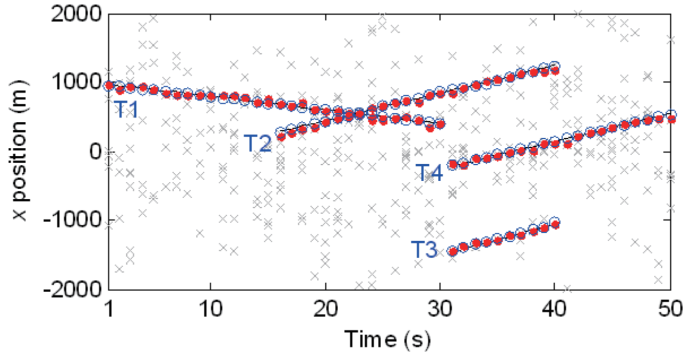

FIGURE 2. Position estimates in x position

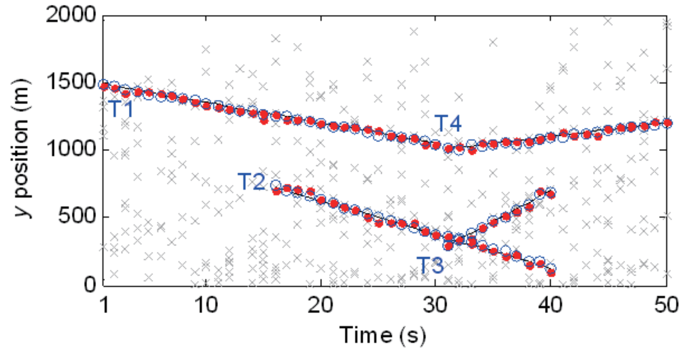

FIGURE 3. Position estimates in y position

to the true target positions. However, the standard PHD filter erroneously estimates the positions of clutter-generated measurements. Especially, it gives the unstable position estimates on the 15^{th} s, whereas the adaptive PHD filter boosts estimation accuracy.

Figure 4 shows the cardinality estimates of two filters. It can be seen the standard PHD filter exaggerates the cardinality estimates during the surveillance time owing to the inherent defects. It mistakes a clutter for actual target on the 15^{th} s. As a result, the cardinality of targets is inevitably under-estimated. For comparison, the adaptive PHD filter adheres to its confidence in the cardinality estimates with the adaptive scheme. Moreover, setting the adaptive wave gate, we distinguish the actual targets from enormous clutters.

Figure 5 demonstrates the OSPA distance of two filters under consideration. We can see that at the time of intensity peaks in this figure, the adaptive PHD filter achieves the

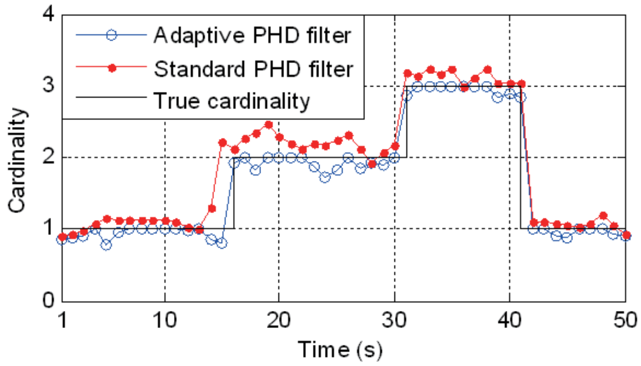

Figure 4. Cardinality estimates

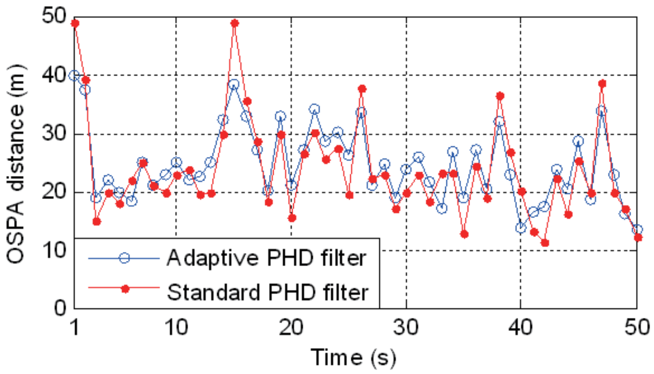

Figure 5. OSPA distance

lower distance error as a direct result of always approaching the true cardinality. Further analyses indicate although the total OSPA distances of two filters are approximately equivalent, the adaptive PHD filter has more advantages on cardinality component.

Finally, we evaluate the overall performance of the adaptive PHD filter, and then the estimated clutter rate is 9.72 with the detection probability of 94.15%. Compared with the existing PHD filters, the proposed filter can adaptively complete the task of multi-target tracking with promising performance in the environment of both uncertain detection probability and uncertain clutter rate, which is more applicable for practical applications.

5. **Conclusions.** This paper discusses an adaptive PHD filter under the uncertain conditions. Employing the beta distribution and augmented parameter in extended state space, we propose the filtering principle and the SMC implementation. According to the adaptive wave gate, the available measurements are identified for the actual targets and clutters. Simulation results further confirm the tracking performance of the adaptive PHD filter. In the future, we plan to reduce the computational complexity of this filter.

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