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LAG SYNCHRONIZATION OF NEURAL NETWORKS WITH MIXED DELAYS VIA ADAPTIVE PERIODICALLY INTERMITTENT CONTROL

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Abstract. *The exponential lag synchronization for a class of neural networks with discrete delays and distributed delays (i.e., mixed delays) is studied via adaptive periodically intermittent control in this letter. Via the Lyapunov stability theory, combined with the method of the adaptive control and periodically intermittent control, the simple but robust adaptive periodically intermittent controllers are designed such that the response system can lag-synchronize with a drive system. The traditional assumptions on control width and time delays are removed in the results. So the derived results are less conservative. This leads to a larger application scope for our method. Lastly, numerical simulation is exploited to show the effectiveness of the results.*

Keywords: Neural networks, Mixed delays, Lag synchronization, Adaptive control, Periodically intermittent control

1. **Introduction.** In 1983, the competitive neural networks (CNNs) have been proposed by Cohen and Grossberg [1]. Afterward, the neural networks (NNs) have been got great development. Meyer-Bäse et al. proposed in [2-4] the so-called CNNs with different time scales, which can be seen as the extensions of Cohen and Grossberg's CNNs [1] and Amari's model [5] for primitive neuronal competition. Recently, the neural networks have drawn the attention of many researchers from different areas since they have been fruitfully applied in signal and image processing, associative memories, combinatorial optimization, automatic control [6-8].

Synchronization motion of its dynamical elements is one of the most significant dynamical behaviors and extensive research on analyzing neural networks [6-20]. Many synchronization phenomena are very useful for us. For example, synchronized oscillating neural activity has been shown to be critical in the advanced functions of the brain, such as information processing, cognitive, emotion and memory. The research of synchronization of coupled neural networks is an important step for understanding brain science and designing coupled neural networks for practical use. Many control techniques, such as linear feedback control [8,9], adaptive feedback control [10,11], impulsive control [12], pinning control [13] and intermittent control [14-20], have been developed to drive the synchronization of networks.

It is well known that there inevitably exists time delay when the signal travels through the complex neural network due to the finite speeds of spreading and transmission as well as traffic congestions. Time delay may cause undesirable dynamical behavior, for instance, oscillation and instability. So it is reasonable to require one neural network to synchronize the other neural network at a constant time lag. Lag synchronization may be a more appropriate technique to clearly indicate the fragile nature of neurons systems which compared with complete synchronization [10,17]. Hence, how to effectively lag synchronize two chaotic neural networks is an important problem for potentially practical application and theoretical research.

Based on the above arguments, this paper aims to handle the problem of exponential lag synchronization for the neural networks with mixed delays via adaptive periodically intermittent control. Based on the theory of Lyapunov stability combined with intermittent control techniques and adaptive control, improved lag synchronization criteria and the corresponding adaptive intermittent controllers which are more useful in practice and little costly are proposed. It should be pointed out that here two restrictive conditions that the control width should be larger than the time delay and the time delay should be smaller than the noncontrol width in [19,20] are not required. And the control gain which has obtained is smaller than before references. The adaptive intermittent controllers are proposed, which can be demonstrated to be less conservative comparing with linear intermittent control. Numerical example is presented to show the effectiveness of the proposed method.

The structure of the remaining paper is organized as follows. In Section 2, the model of neural networks with mixed delays and some preliminaries and lemmas are presented. In Section 3, exponential lag synchronization controllers of neural networks with mixed delays via adaptive periodically intermittent control are obtained. In Section 4, numerical example of neural networks with mixed delays is given to demonstrate the effectiveness of the proposed controllers. Conclusions are drawn in Section 5.

2. Model and Preliminaries.

We consider the neural networks with mixed delays as follows:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{N} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{N} b_{ij} f_j(x_j(t-\theta)) + \sum_{j=1}^{N} d_{ij} \int_{t-\eta}^{t} f_j(x_j(s))\mathrm{d}s, \quad (1)$$

where $i = 1, \dots, N$, $x_i(t)$ is the neuron current activity level, $f_j(x_j(t))$ is the output of neurons, $c_i > 0$ is the time constant of the neuron, a_{ij} represents the connection weight between the ith neuron and the jth neuron, b_{ij} and d_{ij} represent the synaptic weight of delayed feedback, scalars $\theta > 0$ and $\eta > 0$ are the discrete and the distributed time delay, respectively.

Let $x(t) = \left(x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), \dots, x_N^{\mathrm{T}}(t)\right)^{\mathrm{T}}$, $f(x(t)) = \left(f(x_1^{\mathrm{T}}(t)), f(x_2^{\mathrm{T}}(t)), \dots, f(x_N^{\mathrm{T}}(t))\right)^{\mathrm{T}}$, $C = diag(c_1, c_2, \dots, c_N)^{\mathrm{T}}$, $A = (a_{ij})_{N\times N}$, $B = (b_{ij})_{N\times N}$, $D = (d_{ij})_{N\times N}$. Then, (1) turns out to the following compact form:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\theta)) + D\int_{t-\eta}^{t} f(x(s))\mathrm{d}s, \quad (2)$$

The initial value of (2) is denoted by $x(t) = \phi^x(t) \in C([-\mu, 0], R^N)$, where $\mu = \max\{\theta, \eta\}$.

Based on the drive-response synchronization, we take (2) as the drive system. And we design the following response system:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t-\theta)) + D\int_{t-\eta}^{t} f(y(s))\mathrm{d}s + u(t), \quad (3)$$

where $u(t) = \left(u_1^{\mathrm{T}}(t), u_2^{\mathrm{T}}(t), \ldots, u_N^{\mathrm{T}}(t)\right)^{\mathrm{T}}$ is the controller to be designed, $u_i(t) \in R^n$ is the input vector of node i. The initial value of (3) is given as $y(t) = \varphi^y(t) \in C([-\mu, 0], R^N)$, where $\mu = \max\{\theta, \eta\}$. We assume that solutions of networks (2) and (3) are bounded and the output signals of the NNs (2) can be received by (3) with transmission delay $\tau \geq 0$.

Definition 2.1. *Drive-response systems (2) and (3) are said to be exponentially lag synchronized if there exist* $\alpha \geq 1$ *and* $\varepsilon > 0$ *such that*

$$\|y(t) - x(t-\tau)\| \leq \alpha e^{-\varepsilon(t-\tau)} \sup_{-\tau \leq \theta \leq 0} \|\varphi^y(t) - \phi^x(t-\tau)\|, \tag{4}$$

for any $t > \tau$. *Here,* ε *is called the degree of exponential lag synchronization.*

In order to study the exponential lag synchronization between (2) and (3) with the lag time $\tau \geq 0$, we define the error states $e(t) = y(t) - x(t-\tau)$. Subtracting (2) from (3) yields the following error system:

$$\dot{e}(t) = -Ce(t) + Ag(e(t)) + Bg(e(t-\theta)) + D\int_{t-\eta}^{t} g(e(s))\mathrm{d}s + u(t), \tag{5}$$

where $g(e(t)) = f(y(t)) - f(x(t-\tau))$, the initial condition of (5) is $e(t) = \varphi^y(t) - \phi^x(t-\tau)$.

To achieve synchronization of the objective (4), we need the following assumption and lemmas.

Assumption 2.1. *There exist positive constants* l_i, $(i = 0, 1, \ldots, N)$ *satisfying*

$$\|f_i(x)) - f_i(y)\| \leq l_i\|x - y\|, \quad \forall x, y \in R, x \neq y \tag{6}$$

Lemma 2.1. [1] *For any vectors* $x, y \in R^m$ *and positive definite matrix* $Q \in R^{m\times m}$, *the following matrix inequality holds:*

$$2x^{\mathrm{T}}y \leq x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y.$$

If not specified otherwise, inequality $Q > 0$ $(Q < 0,\ Q \geq 0,\ Q \leq 0)$ *means* Q *is a positive (or negative, or semi-positive, or semi-negative) definite matrix, where* Q *is a square matrix.*

Lemma 2.2. [21] *For any constant symmetric matrix* $M \in R^{n\times n}$, $M > 0$, *scalar* $h > 0$, *vector function* $\dot{x}(\cdot) \in C([-h, 0], R^n)$ *such that the integrations in the following are well defined, then:*

$$h\int_0^h x^{\mathrm{T}}(s)Mx(s)\mathrm{d}s \geq \left(\int_0^h x(s)\mathrm{d}s\right)^{\mathrm{T}} M \left(\int_0^h x(s)\mathrm{d}s\right).$$

3. Main Results.

In this section, we design the adaptive periodically intermittent control which is added to the neural networks with mixed delays (3) such that states of (3) can exponential lag-synchronize in mean square with (2). At the same time, the trivial solution of error system (5) is exponential stable in mean square. In order to realize lag synchronization of the neural networks with mixed delays by adaptive periodically intermittent control, the controllers are added to nodes of the network. In system (5), we define the adaptive intermittent feedback controllers as follows:

$$u_i(t) = \begin{cases} -k_i(t)e_i(t), & t \in [\kappa T, \kappa T + h), \\ 0, & t \in [\kappa T + h, (\kappa+1)T). \end{cases} \tag{7}$$

and the updating laws

$$\dot{k}_i(t) = \alpha_i \exp\{a_1 t\}\|e_i(t)\|^2. \tag{8}$$

where α_i $(i = 1, 2, \ldots, N)$ and a_1 are positive constants, $k_i(0) > 0$ $(i = 1, 2, \ldots, N)$ are initial value and $k_i((\kappa+1)T) = k_i(\kappa T + h)$. $T > 0$ denotes the control period, $0 < h < T$ and $\kappa = 0, 1, 2, \ldots$. Let $K(t) = diag(k_1(t), k_2(t), \ldots, k_N(t))$. Based on Assumption

2.1, the lag synchronization criterion under the adaptive periodically intermittent control scheme is deduced as follows.

Theorem 3.1. *Assume that Assumption 2.1 holds. If there exist positive constants* $a_1 > L$, a_2, ε *and* α_i $(i = 1, 2, \ldots, N)$, *such that*

$$\begin{aligned} &-2C + AA^{\mathrm{T}} + \left(L + \tfrac{L\eta^2}{a_1} - a_2\right) I_N + BB^{\mathrm{T}} + DD^{\mathrm{T}} \leq 0, \\ &\varepsilon = \lambda - a_2\left(1 - \tfrac{h}{T}\right) > 0, \end{aligned} \tag{9}$$

where $\lambda > 0$ *is the unique positive solution of the equation* $a_1 - \lambda - L\exp\{\lambda\theta\} = 0$. *Then the neural networks with mixed delays (2) and (3) globally exponentially lag synchronize under adaptive intermittent controllers (7) and the updating laws (8).*

Proof: Construct a Lyapunov-Krasovskii candidate function as follows

$$\begin{aligned} V(t) = &\frac{1}{2}\exp\{-a_1 t\} e^{\mathrm{T}}(t)e(t) + \frac{1}{2}\sum_{i=1}^{N}\exp\{-a_1 t\}\frac{(k_i(t) - k)^2}{\alpha_i} \\ &+ \frac{1}{2}\exp\{-a_1 t\}\int_{t-\eta}^{t}\int_{z}^{t} e^{\mathrm{T}}(s)Me(s)\mathrm{d}s\mathrm{d}z, \end{aligned} \tag{10}$$

where k is an undetermined sufficiently large positive constant. According to Assumption 2.1, let calculate the derivation of $V(t)$ with respect to t.

When $t \in [\kappa T, \kappa T + h)$, for $\kappa = 0, 1, 2, \ldots$

$$\begin{aligned} \dot{V}(t) = &\exp\{-a_1 t\} e^{\mathrm{T}}(t)\dot{e}(t) - \frac{a_1}{2}\exp\{-a_1 t\} e^{\mathrm{T}}(t)e(t) - \frac{a_1}{2}\sum_{i=1}^{N}\exp\{-a_1 t\}\frac{(k_i(t) - k)^2}{\alpha_i} \\ &+ \exp\{-a_1 t\}\sum_{i=1}^{N}(k_i(t) - k)e_i^{\mathrm{T}}(t)e_i(t) + \frac{a_1}{2}\exp\{-a_1 t\}e^{\mathrm{T}}(t)Me(t) \\ &- \frac{a_1}{2}\exp\{-a_1 t\}\int_{t-\eta}^{t} e^{\mathrm{T}}(s)Me(s)\mathrm{d}s - \frac{a_1}{2}\exp\{-a_1 t\}\int_{t-\eta}^{t}\int_{z}^{t} e^{\mathrm{T}}(s)Me(s)\mathrm{d}s\mathrm{d}z \\ = &\exp\{-a_1 t\}e^{\mathrm{T}}(t)\left[\left(\frac{1}{2}\eta M - C - K\right)e(t) + Ag(e(t)) + Bg(e(t-\theta)\right. \\ &\left. + D\int_{t-\eta}^{t} g(e(s))\mathrm{d}s\right] - a_1 V(t) - \frac{a_1}{2}\exp\{-a_1 t\}\int_{t-\eta}^{t} e^{\mathrm{T}}(s)Me(s)\mathrm{d}s. \end{aligned} \tag{11}$$

From Theorem of [1,21] and Assumption 2.1, we take $K = kI_N$ and $M = \frac{\eta L}{a_1}$. Because k is an undetermined sufficiently large positive constant, we can select k as we obtain

$$k > \frac{L}{2}\left(1 + \frac{\eta^2}{a_1}\right) + \lambda_{\max}\left(-C + \frac{1}{2}AA^{\mathrm{T}} + \frac{1}{2}BB^{\mathrm{T}} + \frac{1}{2}DD^{\mathrm{T}}\right). \tag{12}$$

So we have

$$\begin{aligned} \dot{V}(t) \leq &- a_1 V(t) + \frac{1}{2}\exp\{-a_1 t\}e^{\mathrm{T}}(t-\theta)L(e(t-\theta)) + \frac{L}{2}\sum_{i=1}^{N}\exp\{-a_1 t\} \\ &+ \frac{L}{2}\exp\{-a_1 t\}\int_{t-\theta-\eta}^{t-\theta}\int_{z-\theta}^{t-\theta} e^{\mathrm{T}}(s)Me(s)\mathrm{d}s\mathrm{d}z\frac{(k_i(t) - k)^2}{\alpha_i} \\ = &- a_1 V(t) + LV(t-\theta). \end{aligned} \tag{13}$$

Similarly, when $t \in [\kappa T + h, (\kappa+1)T)$, using condition in the first inequality of the (9), one has

$$\begin{aligned}\dot{V}(t) \leq{}& \exp\{-a_1 t\} e^{\mathrm{T}}(t)\left[-C + \frac{1}{2}AA^{\mathrm{T}} + \frac{1}{2}(L - a_2)I_N + \frac{1}{2}BB^{\mathrm{T}} + \frac{1}{2}DD^{\mathrm{T}} + \frac{1}{2}\eta M\right] e(t) \\ &+ \frac{1}{2}a_2 e^{\mathrm{T}}(t)e(t) + \frac{1}{2}\exp\{-a_1 t\} e^{\mathrm{T}}(t-\theta)L(e(t-\theta)) - a_1 V(t) \\ &+ \exp\{-a_1 t\}\int_{t-\eta}^{t} e^{\mathrm{T}}(s)\left(\frac{1}{2}\eta L\right)e(s)\mathrm{d}s - \frac{a_1}{2}\exp\{-a_1 t\}\int_{t-\eta}^{t} e^{\mathrm{T}}(s)Me(s)\mathrm{d}s \\ \leq{}& (a_2 - a_1)V(t) + LV(t-\theta).\end{aligned} \tag{14}$$

Since the first inequality of (9), the equation $a_1 - \lambda - L\exp\{\lambda\theta\} = 0$ has a unique positive solution $\lambda > 0$, obviously. Take $\bar{Q} = \sup\limits_{-\theta\leq s\leq 0} V(s)$ and $P(t) = \exp\{\lambda t\} \cdot V(t)$, where $t \geq 0$. Let $\Omega(t) = P(t) - \beta\bar{Q}$, where $\beta > 1$ is a constant. It is obvious that

$$\Omega(t) < 0, \quad \text{for all} \quad t \in [-\theta, 0]. \tag{15}$$

Then, we prove by contradiction that

$$\Omega(t) < 0, \quad \text{for all} \quad t \in [0, h]. \tag{16}$$

We adopt the reduction to absurdity. If (16) does hold, suppose to exist a $t_0 \in [0, h]$, such that

$$\Omega(t_0) = 0, \quad \dot{\Omega}(t_0) \geq 0. \tag{17}$$

$$\Omega(t) < 0, \quad -\tau \leq t \leq t_0. \tag{18}$$

Using (15), (17) and (18), we obtain

$$\begin{aligned}\dot{\Omega}(t_0) &= \lambda P(t_0) + \exp\{\lambda t_0\} \cdot \dot{V}(t_0) \\ &\leq \lambda P(t_0) - a_1\exp\{\lambda t_0\} \cdot V(t_0) + L\exp\{\lambda t_0\}V(t_0 - \theta) \\ &< (\lambda - a_1 + L\exp\{\lambda\theta\})\,\beta\bar{Q} = 0.\end{aligned} \tag{19}$$

This contradicts the second inequality in (17), so (16) holds.

Similar to the deduction above, when $\kappa T \leq t < \kappa T + h$ ($\kappa = 1, 2, \ldots$), we have

$$P(t) < \beta\bar{Q}\exp\{a_2\kappa(T-h)\} \leq \beta\bar{Q}\exp\left\{a_2\left(1 - \frac{h}{T}\right)t\right\}. \tag{20}$$

and for $\kappa T + h \leq t < (\kappa+1)T$, we have

$$P(t) < \beta\bar{Q}\exp\{a_2[t - (\kappa+1)h]\} \leq \beta\bar{Q}\exp\left\{a_2\left(1 - \frac{h}{T}\right)t\right\}. \tag{21}$$

Let $\beta \to 1$, from the definition of $P(t)$, one obtains

$$V(t) \leq \bar{Q}\exp\left\{-\left[\lambda - a_2\left(1 - \tfrac{h}{T}\right)t\right]\right\} = \sup_{-\theta\leq s\leq 0} V(s)\exp\{-\varepsilon t\}, \quad t \geq 0. \tag{22}$$

From condition in the second inequality of (9), the neural network (3) globally exponentially synchronizes to the system (2) under adaptive intermittent control. This completes the proof.

Remark 3.1. *Compared with [14,15], our results in this paper do not need some restrictions completely. We introduce the notation of periodically intermittent control in this letter, which reduces the traditional restriction in control period and control rate. In this respect, the conclusions generalize and enhance the previous results. In [16,17], the synchronization of complex network with delayed dynamical nodes has been studied via linear periodically intermittent control. The control gains are obtained to be larger than the needed values for practical problems. However, in our results, we adopt the adaptive* *control approach and give a rigorous proof for the synchronization scheme with adaptive controller. The adaptive controller has a certain regulating function. Thus, our results are less costly and more robust in practice than linear intermittent controller in [16,17].*

4. **Simulation Examples.** In this section, a numerical example is given to show the effectiveness of our results obtained in Section 3. Consider the following neural networks drive system with mixed delays described by

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{2} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{2} b_{ij} f_j(x_j(t-1)) + \sum_{j=1}^{2} d_{ij} \int_{t-1}^{t} f_j(x_j(s))\mathrm{d}s, \quad (23)$$

where $i = 1, 2$, $f_j(x) = \tanh(x)$, and $c_1 = c_2 = 1$, $a_{11} = 1.3$, $a_{12} = -0.1$, $a_{21} = -1.5$, $a_{22} = 0.2$, $b_{11} = -1.5$, $b_{12} = -0.4$, $b_{21} = 0.1$, $b_{22} = -2$, $d_{11} = -0.4$, $d_{12} = 0.1$, $d_{21} = -0.1$, $d_{22} = -0.6$.

Figure 1 is the numerical simulation of system (23), which shows that system (23) has a chaotic attractor. In the following, we consider the response system described by

$$\begin{aligned} \dot{y}_i(t) = &- c_i y_i(t) + \sum_{j=1}^{2} a_{ij} f_j(y_j(t)) + \sum_{j=1}^{2} b_{ij} f_j(y_j(t-1)) \\ &+ \sum_{j=1}^{2} d_{ij} \int_{t-1}^{t} f_j(y_j(s))\mathrm{d}s + u_i(t), \end{aligned} \quad (24)$$

for $t \geq \tau$, where $\tau = 5$, the parameters c_i, a_{ij}, b_{ij}, and d_{ij} are defined in system (23).

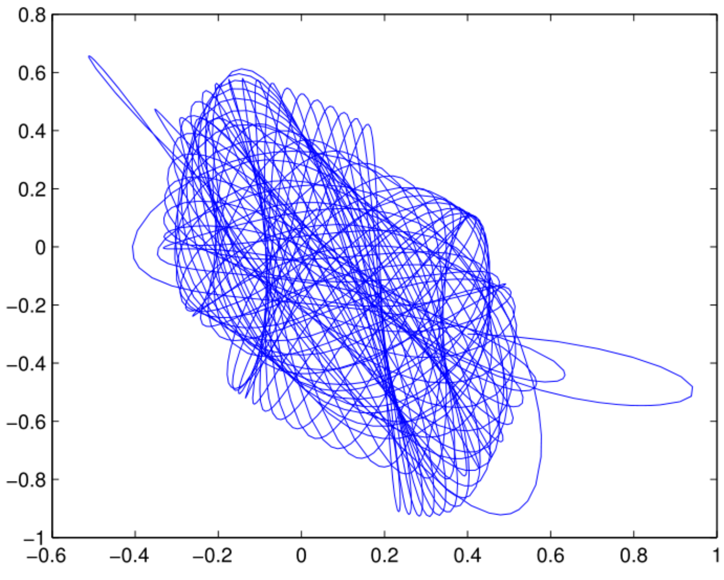
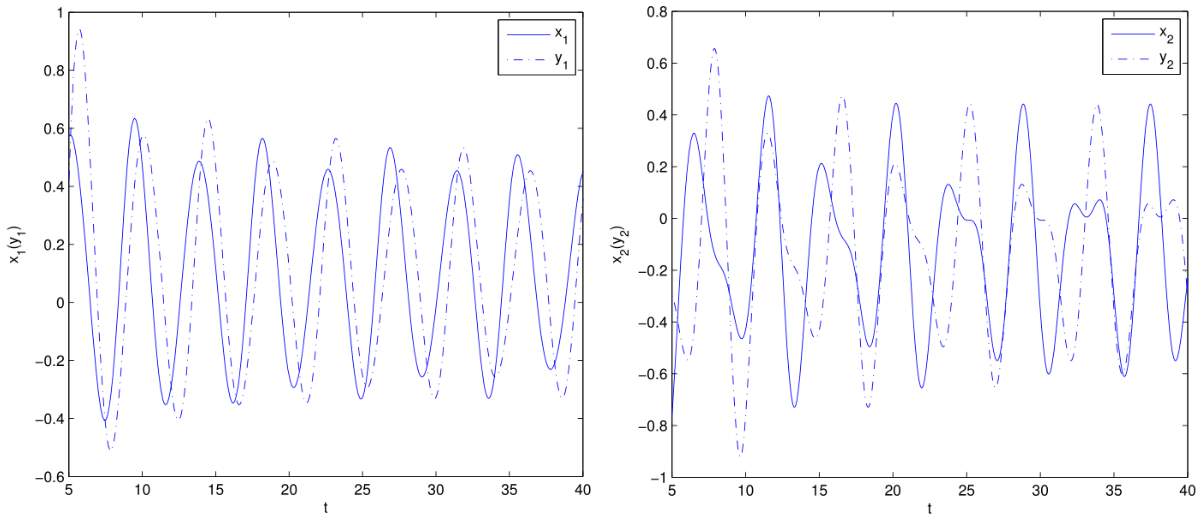


FIGURE 1. The chaotic attractor of system (23)

In this example, suppose that the desired exponential synchronization rate $\varepsilon = 1.1$. The values of the parameters for the controllers (7) and the updating laws (8) are taken as $T = 0.2$, $h = 0.15$, $\alpha_i = 5$ $(i = 1, 2)$. If we choose $a_1 = 30$ and $a_2 = 48$, it is easy to verify that (9) in Theorem 3.1 is satisfied. It is evident that all the conditions of Theorem 3.1 are satisfied in this case; hence, drive system (23) and response system (24) are exponentially lag synchronized under the controller (7) and updating laws (8). Take $\tau = 5$ and denote $e_i(t) = y_i(t) - x_i(t-5)$. In Figure 2, the solid line represents the x_1 and x_2 trajectory, and the dot dash line represents the y_1 and y_2 trajectory. Figure 2 shows the lag synchronization of neural networks (23) and (24) under the adaptive periodically intermittent controllers (7) and (8) when $\tau = 5$ is selected.



(a) (b)

Figure 2. Lag synchronization of x_i and y_i $(i = 1, 2)$ of neural network under the adaptive periodically intermittent controllers (7) and (8)

5. **Conclusions.** In this letter, adaptive periodically intermittent control technique is generalized to study lag synchronization of a class of neural networks with discrete delays and distributed delays. Based on the Lyapunov function theory combined with the technique of adaptive control, adaptive intermittent control method and some general criteria for ensuring neural networks with mixed delays synchronization have been derived. And the corresponding adaptive feedback synchronization controllers are designed. Moreover, our results are less conservative and more general. We introduce the notation of adaptive intermittent control, which reduces the traditional restriction in control period and control rate. And the conclusions in this paper enhance and generalize the previous results. Finally, numerical simulation has verified the effectiveness of the presented method.

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