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LINEAR ESTIMATOR FOR RSS-BASED WIRELESS LOCALIZATION WITH UNKNOWN PATH LOSS EXPONENT

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Abstract. *Received signal strengthen (RSS) measurements are decayed with the increasing of propagation distance between the transmitter and the receiver. RSS measurements are affected by path loss exponent (PLE) which is related with environment factors. However, the obtaining of the PLE requires a lot of manual labors and material resources by experiment methods. Based on the log decay model with RSS measurements, a linear estimator for RSS-based source localization is proposed when the PLE is unavailable. By analyzing the impacts of PLE on the positioning, the estimation approach for coarse source position is put forward. Then the estimated position is refined iteratively by availing the reciprocal constraint relation. The simulations show that the designed iterative positioning algorithm is convergent and the positioning results maintain the stability basically when the iterative times is larger than two. The positioning accuracy of the proposed algorithm is very close to the Cramer-Rao low bound (CRLB) under the given noises conditions.*

Keywords: Wireless sensor networks, Localization, Received signal strength, Path loss exponent

1. **Introduction.** Localization techniques play a critical role in most of wireless sensor network (WSN) applications such as coverage calculation, event detection, object tracking, and location aware routing [1, 2, 3]. In such applications, sensor nodes are categorized into anchor nodes and source nodes. The main difference between them is that the anchor nodes know their locations, for instance with the help of GPS or labor survey, whereas they are unknown for the source nodes. A localization scheme tries to localize the source nodes using the information extracted from the signaling between the anchor nodes and source nodes. The information can manifest itself in the form of time of arrival (ToA) [4, 5], time differential of arrival (TDoA) [6] and received signal strength (RSS) [7]. Among them, RSS-based localization schemes are the most prevalent one due to easier implementation and less complexity. In this method, the distance between the anchor nodes and source nodes is estimated using a signal propagation model.

To estimate the source location, some algorithms including maximum likelihood (ML), semidefinite programming (SDP) method [8] and linear estimator are proposed for source localization [9, 10]. The ML estimator is always solved by the numerical method which requires initial solution to ensure the convergence. When the selected initial solution is far from the actual, it will be trapped in the local optimum. To overcome the shortcoming of the ML estimator, the SDP and linear estimator are proposed to obtain the robust source location estimates. By relaxing the nonconvex optimization into convex problem, the SDP method provides robust solution and improve the performance in the condition of larger noises. However, the complexity of SDP is high. The accuracy performance of SDP cannot achieve the optimal CRLB due to the relaxation. The linear estimator represents the source location estimates as closed-form solution by converting the nonlinear optimization function into linear model. The complexity of the linear estimator is much lower than that of SDP method.

RSS measurements decay with the increasing of the propagation distance and are related to the path loss exponent (PLE) when using the log decay model [11, 12]. Most researches focused on the localization model with known PLE. However, in the actual applications, the costs of the labor and material resource are immense for obtaining the PLE by experiments. Furthermore, the PLE will be fluctuated with the change of environment factors. In [13] an estimation method of source location is proposed by considering the unknown PLE. The proposed method derives the optimal source location with the equal step of PLE with iterative method, so the complexity of the algorithm is high. By considering the uncertainty of the PLE, in [14] a two-step weighted least squares estimator is proposed to avoid the search process.

Motivated by the above, we design a linear estimator specifically for the RSS-based localization problems. When considering the PLE as unavailable, we derive a nonconvex estimator that approximates the ML estimator but has no logarithm in the residual. Then, a linear estimator is proposed to obtain the initial coarse solution by using the Taylor approximation. To further improve the estimation performance, an iterative refinement technique is designed by using the linear method. The corresponding Cramér-Rao lower bounds (CRLBs) for this problem are derived as performance benchmarks. In this paper a linear estimator is proposed for RSS-based localization when the PLE is assumed to be unknown. By converting the nonlinear optimization problem into the linear equations by Taylor approximation, the linear estimator provides a closed-form solution and avoids the initialization of the ML estimator. To improve the accuracy performance of the proposed method, an iterative refinement technique is designed according to the weighted least square (WLS) method.

The rest of this paper is structured as follows. Section 2 presents the problem specification by considering PLE as unknown. Section 3 derives the CRLB of source location estimation by considering the unknown PLE. Section 4 in detail describes the proposed linear estimator and iterative refinement. Section 5 analyzes the simulation results. The conclusion is represented in Section 6. This paper contains a number of symbols. Following the convention, we represent the matrices as bold case letters. If we denote the matrix as $(*)$, $(*)^{-1}$ represents matrix inverse. $[\mathbf{A}]_{i,j}$ denotes the element at the ith row and the jth column of matrix $\mathbf{A}$.

2. **Problem Specification.** Assuming in two-dimensional plane there are anchor nodes with known positions which are denoted as $\mathbf{x}_i = [x_i \quad y_i]^T$, $i = 1, 2, \ldots, N$. The anchor nodes are used to derive the position of the source node which is denoted as $\mathbf{x} = [x \quad y]^T$. The RSS measurements received by anchor node i are denoted by p_i. Assuming that the RSS obeys the logarithmic decay model [12, 14], we can obtain that

$$p_i = p_0 - 10\beta \log_{10} d_i + \varepsilon_i \tag{1}$$

where $i = 1, 2, \ldots, N$, ε_i represents the noise which conforms to the Gaussian distribution with zero mean and variance δ_i^2. d_i is the measurement distance between the anchor node i and the source node. p_0 represents the transmit power of the source node at the 1 m reference point and can be obtained by the experiments or setting the transmit power of the source node. β is called as path loss exponent (PLE) and varied from 1 to 5. Typically β is equal to 2 in the free space circumstance. In most applications, PLE β can be estimated with the experiments in a prior. However, the PLE is fluctuated with the change of environment factors including temperature, humidity and propagation medium. The experiment to obtain the value of β requires a plenty of labor and material resource. So we consider the PLE as unavailable in the proposed RSS-based localization model. When the transmit power p_0 is assumed to be known and the PLE is unknown, the well known maximum likelihood (ML) estimator is written as

$$\min_{\mathbf{x},\beta} \sum_{i}^{N} \frac{1}{\delta_i^2} (p_i - p_0 + 10\beta \log_{10} d_i)^2 \tag{2}$$

where $d_i = \|\mathbf{x}_i - \mathbf{x}\|$, p_i is the observation value, and $i = 1, 2, \ldots, N$. The solution to ML estimator is always solved by the numerical calculation which requires an initial point. When the initial point is enough close to the actual solution, the positioning results will be trapped in the local optimum. So a linear estimator is proposed to avoid the shortcoming of numerical calculation.

To overcome the shortcoming of the ML estimator and fasten the iterative calculation, it is required that the initial point is close to the true solution as possible. Generally, the PLE β is varied from 1 to 5. It is observed that the impact of PLE on the position result is finite when the PLE β is slightly larger than the actual value. Therefore, we set the PLE as a given value which is larger than the true. Thus, an initial coarse solution for source location is derived with the linear estimator. The final solution to the source location is represented as closed-form with an iterative optimization.

3. **CRLB for Location Estimation.** The CRLB defines a lower bound on the variance of any unbiased estimator and is employed as a benchmark for evaluating the performance of estimators. As the PLE β is assumed to be unknown, an unknown vector is denoted as $\theta = [\mathbf{x}^T \quad \beta]^T$. The CRLBs of the unknown parameters are the diagonal elements of the inverse of the Fisher information matrix (FIM). Here when the PLE β is unknown, the FIM is denoted by $\mathbf{F}$, which is also written as

$$\mathbf{F} = -\frac{\partial^2 \ln P(\mathbf{p}|\mathbf{x})}{\partial \theta^T \partial \theta} \tag{3}$$

where

$$P(\mathbf{p}|\mathbf{x}) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\delta_i} \exp\left\{-\frac{(p_i - p_0 + 10\beta \log_{10} d_i)^2}{2\delta_i^2}\right\} \tag{4}$$

Substituting (4) in (3), then (3) is rewritten as

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_x & \mathbf{U} \\ \mathbf{U}^T & \mathbf{V} \end{bmatrix} \tag{5}$$

where

$$\mathbf{F}_x = \begin{bmatrix} \dfrac{\partial^2 \ln P(\mathbf{p}|\mathbf{x})}{\partial x^2} & \dfrac{\partial^2 \ln P(\mathbf{p}|\mathbf{x})}{\partial x \partial y} \\ \dfrac{\partial^2 \ln P(\mathbf{p}|\mathbf{x})}{\partial y \partial x} & \dfrac{\partial^2 \ln P(\mathbf{p}|\mathbf{x})}{\partial y^2} \end{bmatrix} \tag{6}$$

(6) is further rewritten as

$$\mathbf{F}_x = \begin{bmatrix} \sum\limits_{i=1}^{N} \dfrac{18.9\beta^2 (x - x_i)^2}{\delta_i^2 d_i^4} & \sum\limits_{i=1}^{N} \dfrac{18.9\beta^2 (x - x_i)(y - y_i)}{\delta_i^2 d_i^4} \\ \sum\limits_{i=1}^{N} \dfrac{18.9\beta^2 (x - x_i)(y - y_i)}{\delta_i^2 d_i^4} & \sum\limits_{i=1}^{N} \dfrac{18.9\beta^2 (y - y_i)^2}{\delta_i^2 d_i^4} \end{bmatrix} \tag{7}$$

$\mathbf{U}$ and $\mathbf{V}$ are given by

$$\mathbf{U} = \begin{bmatrix} \sum\limits_{i=1}^{N} \dfrac{43.4\beta (x - x_i) \log_{10} d_i}{\delta_i^2 d_i^2} & \sum\limits_{i=1}^{N} \dfrac{43.4\beta (y - y_i) \log_{10} d_i}{\delta_i^2 d_i^2} \end{bmatrix} \tag{8}$$

$$\mathbf{V} = \sum_{i=1}^{N} \frac{100 (\log_{10} d_i)^2}{\delta_i^2} \tag{9}$$

The CRLB of source location is denoted as $C([\theta]_r)$ which can be calculated by

$$C([\theta]_r) = \mathbf{F}^{-1}_{[r,r]} \tag{10}$$

where $C([\theta]_r)$ denotes the rth row element of the vector θ, $\mathbf{F}^{-1}_{[r,r]}$ denotes the rth row and the rth column element of inverse matrix $\mathbf{F}^{-1}$, and $r = 1, 2, 3$. (10) can be further rewritten as

$$C([\theta]_r) = \left[\left(\mathbf{F}_x - \mathbf{UVU}^T\right)^{-1}\right]_{r,r} \tag{11}$$

According to the definition of the vector θ, the CRLB for location estimation is written as

$$\mathrm{C}(\mathbf{x}) = \left[\mathbf{F}^{-1}\right]_{1,1} + \left[\mathbf{F}^{-1}\right]_{2,2} \tag{12}$$

4. **Linear Estimator.** In the following, we in detail describe the proposed linear estimator as two-step: initial coarse solution and iterative refinement for the source location.

4.1. **Initial coarse solution.** When the PLE β is selected as the value larger than the true, the positioning result is not sensible to the PLE β. Considering that the impact of the larger PLE β is less, we set the PLE β as a given value β_0. Here we firstly introduce the linear estimator with a given PLE. (1) is rewritten as

$$d_i^2 = 10^{\frac{p_0 - p_i + \varepsilon_i}{5\beta_0}} \tag{13}$$

where $i = 1, 2, \ldots, N$, ε_i is the noise which conforms to the Gaussian distribution with zero mean and variance δ_i^2. Expanding the right side of (13) with the Taylor series and neglecting the high order terms, (13) is rewritten as

$$d_i^2 = \lambda_i + \frac{\lambda_i \log_{10} 10}{5\beta} \varepsilon_i \tag{14}$$

where $\lambda_i = 10^{\frac{p_0 - p_i}{5\beta}}$, $i = 1, 2, \ldots, N$. (14) is also rewritten as

$$-2x_i x - 2y_i y + x^2 + y^2 = -x_i^2 - y_i^2 + \lambda_i + \frac{\lambda_i \log_{10} 10}{5\beta} \varepsilon_i \tag{15}$$

where the λ_i value is related to the given PLE β_0. Let $\mathbf{z} = [x \quad y \quad x^2 + y^2]$, and (14) is rewritten as a linear matrix form

$$\mathbf{Az} = \mathbf{b} + \alpha \tag{16}$$

where the row vector of $\mathbf{A}$ is equal to $[-2x_i \quad -2y_i \quad 1]$, and the elements of $\mathbf{b}$ and α are equal to $[-x_i^2 - y_i^2 + \lambda_i]$ and $\left[\frac{\lambda_i \log_{10} 10}{5\beta_0} \varepsilon_i\right]$. By using the weighting least square method, the estimate of the vector $\mathbf{z}$ is

$$\mathbf{z} = \left(\mathbf{A}^T \mathbf{\Sigma}_\alpha^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \Sigma_\alpha^{-1} \mathbf{b} \tag{17}$$

where $\mathbf{\Sigma}_\alpha = E\left(\alpha^T \alpha\right)$, and the elements of $\mathbf{\Sigma}_\alpha$ are written as

$$\mathbf{\Sigma}_\alpha[i,j] = \begin{cases} 0 & i \neq j \\ \dfrac{\lambda_i^2 \left(\log_{10} 10\right)^2}{25\beta_0^2} \delta_i^2 & i = j \end{cases} \tag{18}$$

where $i, j = 1, 2, \ldots, N$. Extracting from the vector $\mathbf{z}$, we derive the initial coarse solution for source location estimate $\mathbf{x}_0$ which is obtained with

$$\mathbf{x}_0 = \mathbf{z}(1:2) \tag{19}$$

Above solution for the source location is coarse due to the given inaccurate PLE β_0. To obtain more accurate estimate, we introduce the refinement technique to improve the solution.

4.2. **Iterative refinement.** Based on the initial estimates β_0 and $\mathbf{x}_0$, we further refine the source location in the following. The estimates in the kth iteration are denoted as $\mathbf{x}_k$ and β_k respectively. Considering the incremental equations

$$\left\{\begin{array}{l} \mathbf{x}_k = \mathbf{x}_{k-1} + \Delta\mathbf{x}_k \\ \beta_k = \beta_{k-1} + \Delta\beta_k \end{array}\right. \tag{20}$$

where $\Delta\mathbf{x}_k = [\Delta x_k \quad \Delta y_k]^T$ and $\Delta\beta_k$ are the corresponding optimized increment. Substituting (20) in (1), we obtain that

$$5\left(\beta_{k-1} + \Delta\beta_k\right)\log_{10}\left(2\mathbf{e}_{k-1}^T\Delta\mathbf{x}_k + d_{i,k-1}^2\right) = p_0 - p_i + \varepsilon_i \tag{21}$$

where $d_{i,k-1}^2 = (x_{k-1} - x_i)^2 + (y_{k-1} - y_i)^2$. Expanding the left side of (21) and neglecting the high order terms, (21) is rewritten as

$$\frac{10\beta}{d_{i,k-1}\ln 10}\mathbf{e}_{k-1}^T\Delta\mathbf{x}_k + 10\log_{10}d_{i,k-1}\Delta\beta_k = p_0 - p_i - 10\beta_{k-1}\log_{10}d_{i,k-1} + \varepsilon_i \tag{22}$$

Let $\theta_k = [\Delta\mathbf{x}_k \quad \Delta\beta_k]^T$, and (22) can be written as the matrix form

$$\mathbf{C}_{k-1}\Delta\theta_k = \mathbf{d}_{k-1} + \varepsilon \tag{23}$$

where the row vector of $\mathbf{C}_{k-1}$ is $\left[\frac{10\beta}{d_{i,k-1}\ln 10}(x_{k-1} - x_i) \quad \frac{10\beta}{d_{i,k-1}\ln 10}(y_{k-1} - y_i) \quad 10\log_{10}d_{i,k-1}\right]$, the row elements of $\mathbf{d}_{k-1}$ and ε are equal to $[p_0 - p_i - 10\beta_{k-1}\log_{10}d_{i,k-1}]$ and $[\varepsilon_i]$, and $i = 1, 2, \ldots, N$. So the WLS solution to (23) is

$$\Delta\theta_k = \left(\mathbf{C}_{k-1}^T\mathbf{\Sigma}_\varepsilon^{-1}\mathbf{C}_{k-1}\right)^{-1}\mathbf{C}_{k-1}^T\Sigma_\varepsilon^{-1}\mathbf{d}_{k-1} \tag{24}$$

where the elements of $\mathbf{\Sigma}_\varepsilon = E\left(\varepsilon^T\varepsilon\right)$ are

$$\mathbf{\Sigma}_\varepsilon[i,j] = \left\{\begin{array}{ll} 0 & i \neq j \\ \delta_i^2 & i = j \end{array}\right. \tag{25}$$

where $i, j = 1, 2, \ldots, N$. Then the refined estimates are obtained with

$$\Delta\theta_k = \Delta\theta_{k-1} + \Delta\theta_k \tag{26}$$

5. **Evaluation.** To test the performance of the proposed linear estimator and the iterative refinement method, the simulations are conducted in the MATLAB software. Six anchor nodes are set at the points $(70,10)$, $(40,150)$, $(150,50)$, $(10,80)$, $(190,110)$ and $(150,180)$ in a 200 m $\times$ 200 m square region. The location of the source node is set at $(100,100)$ in a prior. All noises are Gaussian with zero mean and variance δ^2. In the simulations, the transmit power p_0 and the true PLE β are set to -45 dB and 2, respectively. The accuracy performance is evaluated with mean square error (MSE) which is defined as

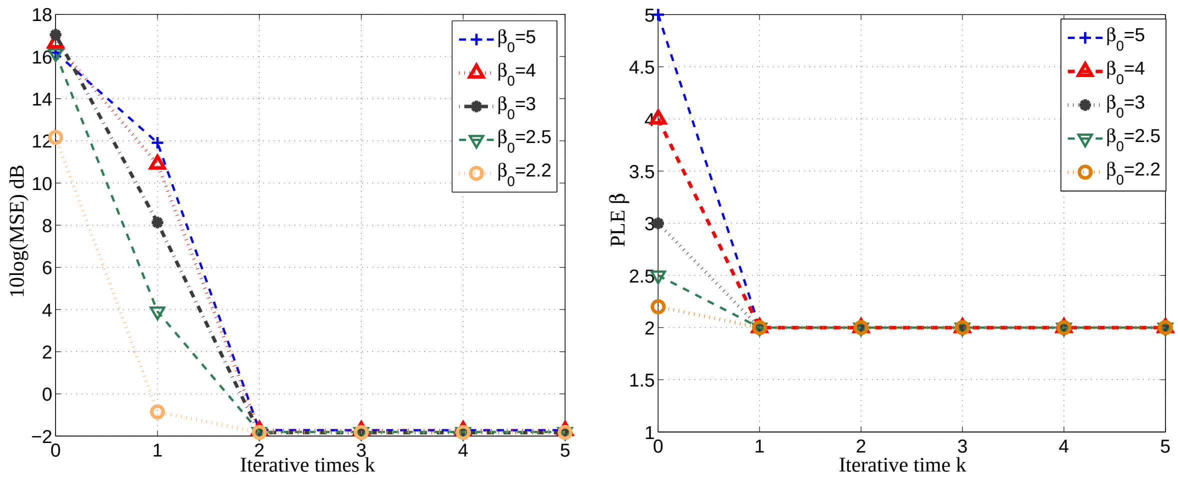
$$\mathrm{MSE} = \frac{1}{M_c}\sum_{i=1}^{M_c} \parallel \mathbf{x}_i - \mathbf{x}_i^o \parallel^2 \tag{27}$$

where M_c is called as the Monte Carlo times, and $\mathbf{x}_i$ and $\mathbf{x}_i^o$ denote the estimated and the true location of the source node in the ith Monte Carlo run, respectively. In our simulation, we use the average of 5000 Monte Carlo runs to evaluate the accuracy performance of the proposed algorithm. We firstly test the performance of the proposed algorithm with different β_0 and iterative times.

5.1. **Impact of iteration times.** To test the impact of the iteration times, δ^2 is set to 0.1^2. When the PLE β_0 is set to 2.2, 2.5, 3, 4 and 5, Figure 1(a) plots the MSE in log scale with different iteration times. It can be seen that the MSE performance is improved as the iteration times increases. When the iteration times is larger than 2, the MSE is convergent to -2 dB which provides the optimal performance. Apparently, when the larger β_0 is selected, the MSE performance is the worse. If β_0 is closer to the true PLE, the MSE in log scale is less. When β_0 is set to 2.2, the MSE in log scale of initial solution is 12.1 dB. However when β_0 is increased to 2.5, the MSE in log scale of initial solution achieves 16.2 dB.

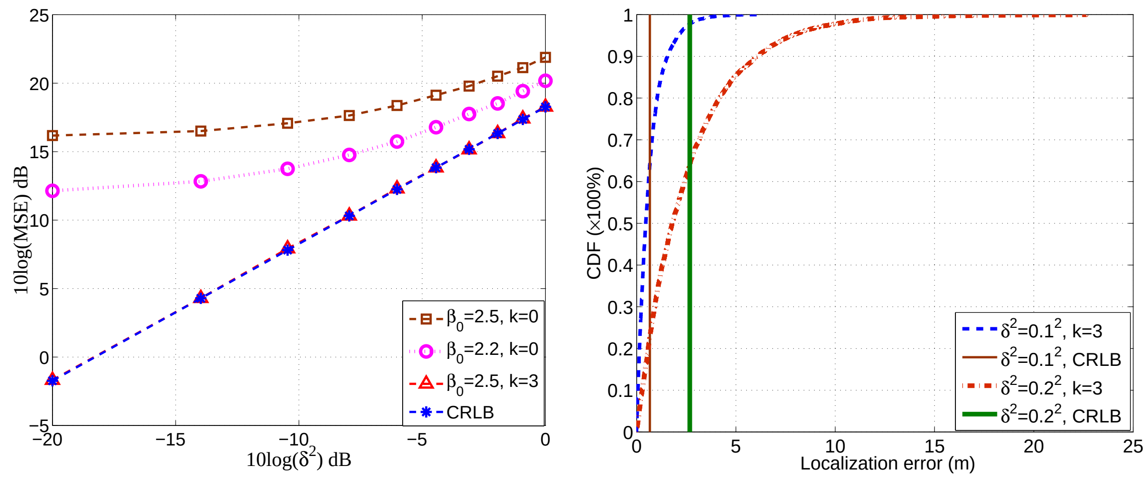
When the noise variance δ^2 is also set to 0.1^2, Figure 1(b) plots the estimated PLE with different iteration times. As can be seen that the estimated PLE is stable when the iterative time is larger than 1. The convergent speed of the estimated PLE is fast. In the second iteration, the estimated PLE is very close to the true result, so the accurate source location can also be obtained along with the estimated PLE.

5.2. **Impacts of noises.** To test the impacts of noises, we also perform Monte Carlo simulations with 5000 ensemble runs to evaluate the mean square error (MSE) of the



(a) MSE performance with different iteration times (b) Estimated PLE under different iteration times

FIGURE 1. Impacts of different iteration times



(a) MSE under different noises (b) CDF of localization error

FIGURE 2. Impacts of noises

source location estimation. When the noise variance δ^2 is varied from 0.1^2 to 1^2 (i.e., $10\log_{10}(\delta^2)$ is varied from -20 dB to 0 dB), the MSE in log scale of four different states is plotted in Figure 2(a). It can be seen from Figure 2(a) that the MSE performance in log scale is worse as the noise variance increases. When the iterative times is set to 3, the MSE performance is very close to the CRLB of source location estimation. When δ^2 is set to 0.1^2, $10\log_{10}(\text{MSE})$ is -1.7 dB in the third iteration. However, when δ^2 is increased to 1^2, $10\log_{10}(\text{MSE})$ achieves 18.2 dB in the third iteration. Similarly the performance order of the proposed algorithm in Figure 2(a) is the same as that of Figure 1(a) when β_0 is selected to be larger.

When the noise variance δ^2 is set to 0.1^2 and 0.2^2, Figure 2(b) plots the cumulative distribution function (CDF) of positioning error with 5000 MC runs. It can be seen that 90% of positioning error is less than 1.6 when δ^2 is set to 0.1^2. However, 90% of positioning error is less than 6.1 when δ^2 is set to 0.2^2. 65% of positioning error is less than the CRLB when δ^2 is set to 0.1^2 or 0.2^2.

6. **Conclusion.** When the PLE is unavailable, we introduce the linear estimator to obtain the source location. The linear estimator provides a closed-form solution to the source location and avoids the shortcoming of the ML estimator by setting a larger PLE in a prior. Then the iterative refinement technique is proposed to improve the initial coarse solution. The accuracy performance degrades as the noise increases for the proposed methods. When the iterative times is larger than two, the source location estimate is very close to the optimal CRLB performance. The estimated PLE is convergent as the iterative times increase.

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