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RESEARCH ON TWO-STAGE FUZZY PROGRAMMING MODEL BASED ON COMPREHENSIVE SATISFACTION DEGREE

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Abstract. *Fuzzy programming has been a widespread problem in resource allocation and optimization decision. It is also a hot research in today's academic circles and application fields. Starting from the general model of fuzzy problem, we analyze its essential features and discuss the shortcomings of existing methods. Then aiming at the fuzzy programming with concrete multi-constraints, we put forward a model of comprehensive satisfaction degree for processing the constraints. Further, we establish the second model to make final decision. That is the two-stage fuzzy programming model based on comprehensive satisfaction degree (denoted as BCSD-TFPM, for short). Theoretical analysis and calculation results show that the two models established in this paper not only can merge decision consciousness into solving process effectively, but also have good structure characteristic and strong interpretability. Therefore, this research has certain theoretical significance and application prospect.*

Keywords: Fuzzy programming, Multi-constraints, Comprehensive satisfaction degree, Two-stage fuzzy programming

1. **Introduction.** Because of the complexity of environment increasing, fuzziness has been a widespread phenomenon in the real world and it is unavoidable in many practical decision problems. In order to make decision better, many scholars usually use fuzzy programming as an effective tool in dealing with fuzzy decision. In 1965, Zadeh [1] first proposed the fuzzy set theory, which has become an effective tool to deal with subjective uncertainty, especially in the fields of fuzzy control and fuzzy optimization. Along with the rapid development of fuzzy set theory and the application universality of fuzzy programming, there are many beneficial researches on how to build different models and solving methods of fuzzy programming under different backgrounds, for example, Bellman and Zadeh [2] proposed the basic model of the fuzzy multi-objective decision. By applying a convex programming technique, and considering the criteria of probability maximization and fractile optimization simultaneously, Yano [3] proposed a fuzzy decision method for multi-objective stochastic linear programming problems with variance covariance matrices. Taeyong et al. [4] proposed a method based on nonlinear fuzzy membership function, which enriched the expression and the processing method of fuzzy number in fuzzy linear programming. By some criterion function to regulate and integrate different goals, Tsai and Lin [5] presented a method to solve multi-objective fuzzy programming problems. Li and Jin [6] introduced fuzzy inequity degree for processing fuzzy constraints, and then gave a solution of the numerical fuzzy programming combined with genetic algorithm.

Through treating fuzzy constraint satisfaction as a constraint processing strategy, Li et al. [7] put forward a fuzzy programming theory which was based on comprehensive effect, and gave its concrete application. For the max-min operator method of Werners, Lian and Qin [8] proposed an optimization model with weight coefficient by means of the weighted summation of the objective function and constraints' membership degree. Aiming at the comprehensive problem of the optimal decision under different threshold values, Li and Liu [9] gave the concept of level utility function which could reflect the threshold's credibility, and also set up the measurement mode of fuzzy optimal value based on level utility model. Combined with deviation degree measures and weighted max-min method, Cheng et al. [10] proposed a new method for solving fuzzy multi-objective linear programming problems where the coefficients are triangular fuzzy numbers. Based on the nearest interval approximation operator, Luhandjula and Rangoaga [11] presented a new approach for solving a fuzzy multi-objective programming problem. As the promotion of the quadratic programming problem with fuzzy relations, Yang et al. [12] mainly discussed the feasible set structure, and then gave a global optimal solution of this problem. In view of the whole fuzzy linear fractional programming problem, Chinnadurai and Muthukumar [13] applied a numerical method of fuzzy number and proposed a method for computing an optimal value based on (α, γ). For fuzzy linear programming problems with fuzzy constraints and coefficients, Kuar and Kumar [14] presented a sequence structure method based on L-R fuzzy numbers.

Despite that the references mentioned above have achieved successful application in actual problems, there are still some limitations: 1) the existing abstract models cannot describe the fuzzy states of practical problems comprehensively. Because in practical decision, with the increase of the problems' complexity, there may be multiple decision variables in constraints, and the constraints which describe the relationship among decision variables are different; 2) based on different constraints, the consideration of the importance of constraints may be different from different decision makers. However, the existing methods often ignore the influence of constraints' importance for final decision. Since these limitations must be faced in fuzzy decision, we mainly do the following works: 1) we analyze the essential characteristics of the general model in fuzzy programming, and then put forward its basic methods and existing shortcomings; 2) after making abstract fuzzy constraints into concrete ones, we can give different weights (which is used to describe the importance of constraints) of each constraint depending on the preference of decision makers. Based on that, we propose the comprehensive satisfaction degree calculation model. And then we do the second programming which uses the state of comprehensive satisfaction degree as a flexible constraint. That is the two-stage fuzzy programming model based on comprehensive satisfaction degree (BCSD-TFPM); 3) in case analysis, we denote the form of fuzzy sets on discrete universe to discuss the effectiveness of the proposed models.

The rest of this paper is organized as follows. Section 2 introduces the formal representation of fuzzy programming, and then discusses two methods which are commonly used in fuzzy programming. In Section 3, we propose a new method to solve the fuzzy programming problem based on comprehensive satisfaction degree. Here, we give the characteristics of the two models and some remarks that we should pay attention to. After that, we give the solving steps for this method. In Section 4, a simple example with one situation of A_i is used to illustrate the proposed models. And then we analyze the results. In Section 5, the conclusion is derived.

2. **The Essential Characteristics of Fuzzy Programming Problem.** The essence of programming problem is to seek optimal decision scheme under some constraints, and its general form is as follows:

$$\begin{cases}\max f(x),\\ \text{s.t. } x \in A.\end{cases} \tag{1}$$

Here, A denotes a collection of universe U (called as feasible region), and $f(x)$ denotes the function of a certain number of features on U (called as objective function), which is to measure the decision scheme x good or bad. When $f(x)$ has fuzziness on U or A is a fuzzy set on U, we call (1) fuzzy programming problem.

Although model (1) can well reflect the essence of fuzzy programming problem, it cannot reflect the decision consciousness. So this model is just a formal description. Coupled with the complexity of the decision environment increasing, only using this abstract model is often unable to carry out specific operations. Therefore, in order to integrate different decision consciousness into the model better, many scholars have done a lot of beneficial researches under different emphases. Below, we give the characteristics and limitations of two common methods, which is based on the fuzzy programming problem with crisp target (that is, the objective function is a real function) and fuzzy constraints (that is, the feasible domain is fuzzy).

Method 1: The method based on synthesizing effect function. This method considers the satisfaction of targets and constraints at the same time, and its general description [15] is as follows:

$$\begin{cases}\max S(f(x), A(x)),\\ \text{s.t. } x \in U.\end{cases} \tag{2}$$

Here, $S(u, v)$ is known as synthesizing effect function, if it satisfies the two conditions: 1) $S(u, v)$ is monotone non-decreasing on u and v; 2) $S(u, v)$ is monotone increasing on u. If remember u and v respectively denote the objective function $f(x)$ and the constraint satisfaction $A(x)$ in model (1), $S(f(x), A(x))$ is the synthesizing effect function of comprehensively describing the property of x. Thus, model (2) is the promotion of model (1).

The analysis above shows that synthesizing effect function in model (2) can bring the processing concept of fuzzy information into the quantitative operation. It is an effective tool for treating the decision consciousness. And also, model (2) simultaneously considers the target value and constraint satisfaction degree of the alternatives. Compared with model (1), model (2) can better reflect the basic characteristic of fuzzy decision. However, due to the fact that the constraints in actual problems are always various, and the restriction relationship of targets and constraint satisfactions is complex, we often fail to score it through simple function relationship. Therefore, this method is just a model in an abstract sense. It lacks adequate generality and operability.

Method 2: The method based on level cut sets. This method aims at multiple fuzzy constraints which are in the form of $x \in A^{(i)}$ (here, $A^{(i)}$ are fuzzy sets, $i = 1, 2, \ldots, n$). It uses some level cut sets to describe the fuzzy sets in constraints. And the basic model [16] is as follows:

$$\begin{cases}\max f(x),\\ \text{s.t. } x \in \bigcap_{i=1}^{n} A^{(i)}_{\lambda_i}.\end{cases} \tag{3}$$

In this method, we should determine the processing strategy of the level cut sets firstly. Then the fuzzy sets $A^{(i)}$ in constraints can be converted into a group of crisp sets $A^{(i)}_{\lambda_i} = \left\{x | A^{(i)}(x) \geq \lambda_i\right\}$ (that is a relatively crisp description of the fuzzy sets $A^{(i)}$ on the threshold level λ_i, here, $\lambda_i \in [0, 1]$). After that, we can turn the fuzzy programming

(1) into a crisp one with an objective function $f(x)$ and a set of feasible regions $A_{\lambda_i}^{(i)}$. And the decision result is different along with the difference of λ_i.

Obviously, compared with model (1), model (3) can enrich the existing fuzzy set theory, and also can reflect the consciousness of different decision-makers to a certain extent. That provides a good theoretical basis to solve the decision method based on constraint satisfaction degree. However, the choice of λ_i is difficult. And what model (3) describes is the optimal solution under different thresholds, not the whole optimal value of model (1). It is also lack of universality (for example, when $\bigcap_{i=1}^{n} A_{\lambda_i}^{(i)} = \emptyset$ holds, $x \in \bigcap_{i=1}^{n} A_{\lambda_i}^{(i)}$ is of meaningless, and then we cannot make decision), and it ignores the objective function values and fuzzy constraints of the different emphasis. So model (3) cannot fully describe the practical decision.

From what has been discussed above, to some extent, the two methods mentioned in this paper can enrich the existing fuzzy set theory. However, because model (1) is an abstract description of the fuzzy programming problem, model (2) and (3) are abstract in a sense. Especially when the decision environment in actual problems becomes complex, there are multiple decision variables and fuzzy constraints; the forms of fuzzy constraints are various; it has different function relationships among variables in different constraints. And also different decision variable values may correspond to different constraint satisfaction degrees; and different decision makers may have different views of the constraints' importance. All of those can lead to the different final decision result. In that case, if we continue to use the two methods to solve fuzzy programming problems, we cannot achieve a better implementation effect, and it may have no solution. Below, we will combine the basic feature of fuzzy decision to discuss a kind of programming with reified multiple fuzzy constraints. This not only makes up for the inadequacy of the existing fuzzy decision methods, but has important practical value.

3. **The Two-Stage Fuzzy Programming Model Based on Comprehensive Satisfaction Degree (BCSD-TFPM).** In practical problems, the multi-objective programming problems can be turned into single objective ones through some strategies, so we mainly discuss a solving method of the fuzzy programming problem with one objective. Considering the complexity of fuzzy environment, there is not just one decision variable in constraints, and usually the function relationships among decision variables are different under different fuzzy constraints. Thus we can propose a model of this problem as follows:

$$\begin{cases} \max f(x), \\ \text{s.t. } g_i(x) \in A_i, \quad i = 1, 2, \ldots, n. \end{cases} \tag{4}$$

Here, $x = (x_1,\ x_2,\ \ldots,\ x_n) \in R^n$ denotes the decision vector. $f(x)$ and $g_i(x)$ are both real function on the universe U. A_i is some kind of fuzzy set on the domain U (when A_i is a crisp set on U, model (4) can be converted into a general crisp programming model).

In decision activities, the solving method has a strong dependence on the objectives and constraints. That is to say a kind of method can give reasonable results only in the corresponding decision-making environment. So selecting a method to adapt to the decision environment is a key factor to reasonable decision. In fact, the choice of decision environment is a kind of embodiment of different processing consciousness for fuzziness from decision-makers. Although different processing consciousness corresponds to different decision environment, we still need to follow some principle. That is, constraints are often reflected by the form of goals, at this point, the realization of constraints is to achieve a certain state of people's willing, and thus the solution should satisfy the constraints to a large extent. So how to judge the satisfaction degree of constraints becomes a key to realizing fuzzy optimization. However, because of different decision variable values corresponding to different constraint satisfaction degrees, and along with too many constraints, considering each satisfaction degree alone is difficult to operate. And the integration of different decision consciousness may have inconsistent understanding of constraint's importance degree. Therefore, in order to better reflect the consciousness, before establishing a general solution method of fuzzy programming with multiple constraints, we should comprehensively consider the satisfaction degree of different constraints by some strategies (we call it comprehensive satisfaction degree). The model is as follows:

$$\begin{cases} \max \sum_{i=1}^{m} w_i A_i(g_i(x)), \\ \text{s.t. } x \in U. \end{cases} \tag{5}$$

Here, $A_i(g_i(x))$ denotes each constraint satisfaction degree. w_i $(i = 1, 2, \ldots, m)$ denotes different importance of constraints (that is the weight, let $w_i \in [0, 1]$, $\sum_{i=1}^{m} w_i = 1$). The higher importance the constraint has, the larger the value of w_i will be.

In practical decision, combined with different decision backgrounds, the decision makers have different understandings of the importance of various constraints. For example, a company plans to manufacture a product, for the production department, the higher the production is and the lower the cost is, the better the scheme is; for the environment department, it is better of lower pollution. Because the final purpose of producers is to pursue profits, through weighing the relationship between various constraints, here we can divide the constraints into different categories. Suppose the influence of constraints to the final decision can be divided into three categories, including pretty good, good, not bad. Then we can give their weight numbers 1, 0.8, 0.6, so the weights corresponding to each category are $1/(1+0.8+0.6) = 0.417$, $0.8/(1+0.8+0.6) = 0.333$, $0.6/(1+0.8+0.6) = 0.25$.

Obviously, model (5) uses satisfaction degree to describe the implementation of constraints, and considers the maximal value of comprehensive satisfaction degree as principle and the weight as a description mechanism of constraints' importance (here, the weight describes the preference relationship among fuzzy constraints from different decision makers). It not only can well reflect the influence of different importance on the decision result, but also avoids multiple consideration of different satisfaction degrees to a certain extent and greatly reduces the complexity of the calculation. Then we can build a concrete fuzzy programming model based on constraint satisfaction degree more conveniently. If remember $\alpha = \max\{\sum_{i=1}^{m} w_i A_i(g_i(x)) \mid x \in U\}$, we have $\alpha \in (0, 1]$ easily.

Remark 3.1. *When there are some absolute constraints, the constraint $x \in U$ in model (5) should be changed as x belongs to the absolute constraints.*

Remark 3.2. *When $\alpha = 0$ holds, it indicates the satisfaction degree $A_i(g_i(x)) = 0$. That is to say, the constraints cannot be realized, and then we cannot make decision. To avoid this situation happening, in model (5), we set at least a constraint satisfaction degree $A_i(g_i(x))$ to make up for $A_i(g_i(x)) > 0$.*

Remark 3.3. *Different existence forms of fuzzy constraints may have different descriptions of A_i, so the evaluation method of $A_i(g_i(x))$ based on A_i may be different. Especially when A_i is triangular fuzzy number or trapezoidal fuzzy number, we can use the membership degree to describe the satisfaction degree $A_i(g_i(x))$.*

When we get the maximal comprehensive satisfaction degree, we can make a further programming to the final decision. However, there may be no solution under the condition of meeting the maximal satisfaction degree absolutely. Thus, in a certain deviation permit, we give the comprehensive satisfaction degree a certain elasticity. From that, we can propose the second programming model:

$$\begin{cases} \max f(x), \\ \text{s.t. } \sum_{i=1}^{m} w_i A_i(g_i(x)) \geq \beta. \end{cases} \tag{6}$$

Easily to know, model (6) regards the comprehensive satisfaction degree as a basis of the decision. Then the fuzzy programming with complex multi-constraints can be transformed into the programming problem with a single constraint (i.e., the comprehensive satisfaction degree is no less than a certain value). Obviously, if $\beta > \alpha$ holds, there is no x satisfying $\sum_{i=1}^{m} w_i A_i(g_i(x)) \geq \beta$. Then we could not make the final decision. And whether the final decision under the maximal comprehensive satisfaction degree α is the optimal one is uncertain. Thus we can use β to denote some thresholds which make the constraint in model (6) be a flexible constraint. And then the value of β is the minimum requirement for the given fuzzy constraints. After that, we can get $\beta \in (0, \alpha]$.

Remark 3.4. *From model (6), we know the comprehensive satisfaction degree describes the reliability of the decision. So the value of β cannot be too small (in this paper, we can restrict the value not to be less than 0.8α); otherwise, the final decision will have lower reliability.*

Remark 3.5. *When all the constraint satisfaction degrees $A_i(g_i(x)) = 1$, the comprehensive satisfaction degree $\alpha = 1$. Then model (4) can be converted into a general crisp programming model. Thus, to a certain extent, the discussion in this article is a promotion of crisp programming method.*

The established two models provide a specific method to determine the optimal solution of fuzzy programming with multiple fuzzy constraints, and the implementation steps are as follows.

Step 1. According to different understandings of the constraint's importance, we can give the weights w_i $(i = 1, 2, \ldots, m)$ based on the method in Section 3, and determine the constraint satisfaction degree $A_i(g_i(x))$ based on the different descriptions of A_i.

Step 2. For w_i and $A_i(g_i(x))$ under different situations, according to model (5), determine the maximum comprehensive satisfaction degree α.

Step 3. According to actual problem, give a certain elastic range to β, and then according to (6) to solve the optimal solution.

In fact, for dealing with fuzzy programming problems with complex constraint, when different decision makers give a set of different constraint satisfaction degrees, we can get the corresponding optimal scheme. Therefore, this method based on comprehensive satisfaction degree will greatly decrease the complexity of the calculation. And model (5) that we have established in this paper as the first programming not only can make up for the limitations above, but also provides a reliable decision environment for the second programming of model (6) which describes the reliability of the final decision. Particularly, this method can make up for the limitations of the two methods in Section 2, and avoid the condition of being unable to solve.

4. **The Case Analysis.** In this section we further expound the application of the BCSD-TFPM in decision problems combined with a concrete case.

Case description: To obtain more profit, an enterprise decided to expand the scope of business. Now they drew up investing two new products A, B for production and marketing. Through in-depth investigation, we found that the production raw materials and the equipment holding time are two key factors of the production quantities of A, B. However, due to the influence of processing another original products, as well as the total amount of investment being restricted, the respective production x_A, x_B of A, B could not have an unlimited number. Producing per kg A, B, we need use raw material a_1 kg, a_2 kg, and take up the equipment for b_1 h, b_2 h. After analysis, there are 10 alternatives $\{(x_{A_1}, x_{B_1}), (x_{A_2}, x_{B_2}), (x_{A_3}, x_{B_3}), (x_{A_4}, x_{B_4}), (x_{A_5}, x_{B_5}), (x_{A_6}, x_{B_6}), (x_{A_7}, x_{B_7}), (x_{A_8}, x_{B_8}), (x_{A_9}, x_{B_9}), (x_{A_{10}}, x_{B_{10}})\}$ of the production of A, B (here, $X_j = (x_{A_j}, x_{B_j})$, $j = 1, 2, \ldots, 10$). The support rate of each alternative respectively on the whole raw materials and equipment holding time is in Table 1.

If we consider A_1, A_2 respectively denotes the whole raw materials and equipment holding time, A_1, A_2 can be regarded as two fuzzy sets on U. Then the support rate of alternatives X_j can be regarded as the constraint satisfaction degree.

Table 1. The net profit rate and the constraint satisfaction degree of each alternative project

Alternative project X_j	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
$A_1(g_1(X_j))$	0.8	0.65	0.45	0.87	0.7	0.64	1	0.9	0.5	0.9
$A_2(g_2(X_j))$	0.9	0.5	1	0.8	0.76	0.72	0.7	0.82	0.5	0.4
Net profit rate $p(X_j)$	0.27	0.23	0.19	0.3	0.25	0.24	0.22	0.28	0.17	0.32

Obviously, if we regard the net profit rate $p(X)$ as a measure of the ultimate profit, this problem can be represented as the following fuzzy programming model:

$$\begin{cases} \max p(X), \\ \text{s.t. } a_1 x_A + a_2 x_B \in A_1, \\ \qquad b_1 x_A + b_2 x_B \in A_2. \end{cases} \tag{7}$$

Because there are multiple alternatives, and the support rates are different, considering each constraint satisfaction degree separately to make decision cannot fully describe the reliability of the decision scheme. And also, different decision makers have different importance of constraints (i.e., the weight), which has a certain influence on the choice of decision results. Below we combine the third part, on the basis of model (5), to analyze the effect of different importance on the maximal comprehensive satisfaction degree. The results are shown in Table 2.

Table 2. The maximal comprehensive satisfaction degree under some different weights

w_i	The comprehensive satisfaction degree of each alternative										α
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
$w_1 = 0.7$, $w_2 = 0.3$	0.83	0.605	0.615	0.894	0.718	0.664	0.91	0.876	0.5	0.75	0.91
$w_1 = 0.55$, $w_2 = 0.45$	0.845	0.5825	0.6975	0.8385	0.727	0.676	0.865	0.864	0.5	0.675	0.865
$w_1 = 0.5$, $w_2 = 0.5$	0.85	0.575	0.725	0.835	0.73	0.68	0.85	0.86	0.5	0.65	0.86
$w_1 = 0.4$, $w_2 = 0.6$	0.86	0.56	0.78	0.828	0.736	0.688	0.82	0.852	0.5	0.6	0.86

From Table 2, we know that: 1) when the constraint satisfaction degrees are certain, the greater the weight with high satisfaction degree is (i.e., the importance of the constraint is greater), the higher comprehensive satisfaction degree is; 2) when the weights are certain, if the satisfaction degree is higher corresponding to high weight, comprehensive satisfaction degree is higher (of course, there exists the situation of comprehensive satisfaction degree equaling).

When we get the maximal comprehensive satisfaction degree, we can depend on the actual situation to select the value of β for the second stage programming using model (6). Below we analyze the influence of different comprehensive satisfaction degrees on the final decision results, which is shown in Table 3. In this table, $*$ denotes there is no optimal value or no optimal decision under some circumstances.

Synthesizing the data in Table 3, we find the weights directly affect the size of the comprehensive satisfaction degree. Thus, when we make decision, we could determine

Table 3. The optimal decision under some different comprehensive satisfaction degrees

w_i	α	β	The satisfied alternatives	$\max p(X)$	The optimal decision
$w_1 = 0.7$, $w_2 = 0.3$	0.91	0.91	$\{X_7\}$	0.22	X_7
		0.86	$\{X_4, X_7, X_8\}$	0.3	X_4
		0.84	$\{X_4, X_7, X_8\}$	0.3	X_4
		0.728	$\{X_1, X_4, X_7, X_8, X_{10}\}$	0.32	X_{10}
$w_1 = 0.55$, $w_2 = 0.45$	0.865	0.91	$\emptyset$	$*$	$*$
		0.86	$\{X_7, X_8\}$	0.28	X_8
		0.84	$\{X_1, X_7, X_8\}$	0.28	X_8
		0.692	$\{X_1, X_3, X_4, X_5, X_7, X_8\}$	0.3	X_4
$w_1 = 0.5$, $w_2 = 0.5$	0.86	0.91	$\emptyset$	$*$	$*$
		0.86	$\{X_8\}$	0.28	X_8
		0.84	$\{X_1, X_7, X_8\}$	0.28	X_8
		0.688	$\{X_1, X_3, X_4, X_5, X_7, X_8\}$	0.3	X_4
$w_1 = 0.4$, $w_2 = 0.6$	0.86	0.91	$\emptyset$	$*$	$*$
		0.86	$\{X_1\}$	0.27	X_1
		0.84	$\{X_1, X_7, X_8\}$	0.28	X_8
		0.688	$\{X_1, X_3, X_4, X_5, X_6, X_7, X_8\}$	0.3	X_4

the weights of various constraints first, and then in the corresponding weights to analyze the influence of different comprehensive satisfaction degrees on decisions. However, due to the fact that constraint satisfaction degree describes the implementation condition of constraints, and comprehensive satisfaction degree reflects the reliability of decision results, we should choose the optimal decisions weighing the relationship between the optimal value or the size of the comprehensive satisfaction degree but not considering one of them. For example, when we choose the weights $w_1 = 0.7$, $w_2 = 0.3$, different comprehensive satisfaction degrees may correspond to different optimal decisions. Although the decision X_7 has the maximal comprehensive degree, compared with the decision X_4 or X_{10}, its net profit rate is lower. In addition, when the comprehensive satisfaction degree $\beta = \alpha$, although the reliability of the decision is higher, the target may be relatively lower. For example, when the final decision is X_7, relative to the alternative under another comprehensive satisfaction degree, its net profit net is low. When $\beta > \alpha$, there is no such decision variables to meet this condition, and thus we cannot make decision. So by giving an elasticity of comprehensive satisfaction degree β for the second programming has more practical significance, and to some extent, it can avoid the situation of being unable to solve the decision. However, for the final decision scheme, decision makers can choose the one with higher satisfaction degree according to the actual situation (that is, the final decision result has higher reliability), and the profits may be lower; or choose the higher profit one, which needs to assume a relatively larger risk.

In conclusion, compared with the original fuzzy programming with abstract constraint, the two models proposed in this paper describe the diversity of complex fuzzy constraints, and also reflect the reliability of the optimal value. Although along with different consciousness integrated in, the final selected decision will vary. This method can also be used as a reasonable reference of a kind of fuzzy programming problems. And it has more practical significance.

5. **Conclusion.** Aiming at the programming problem with fuzzy constraints, we analyze the basic features of the general form and the shortcomings of the existing two methods. Then through embodying the abstract fuzzy constraints, we discuss a solving method

of this kind of problem. In the process of decision, because of different consciousness and different constraints, if we consider each constraint satisfaction degree respectively, that will greatly increase the calculation. However, the comprehensive satisfaction degree measurement model we have proposed can make up for this shortcoming. And also this model can intuitively describe the overall implementation of constraints. After that, we consider the comprehensive satisfaction degree as a basis of the second programming to seek optimal value. Further, we analyze the characteristics of the two models through a concrete case. The results show that this method BCSD-TFPM has good structure characteristics. As a whole, it can reflect the comprehensive reliability of the decision result, and also lay a foundation for further establishing more perfect theory and method of fuzzy programming.

Obviously, the calculation model (5) is a theory expression in an abstract sense which describes the comprehensive satisfaction degree. In this paper, we just aim at a fuzzy programming problem with one situation of A_i to analyze the implementation of satisfaction degree. However, the real problems are often reflected by nonlinear programming and the forms of constraints are more complex. Therefore, based on models (5) and (6), our further work is to combine the fuzzy set theory to discuss the systemic and operable method under more complicated environment.

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