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STABILITY ANALYSIS OF T-S FUZZY SYSTEMS WITH CONSTANT TIME DELAY BASED ON BESSEL-LEGENDRE INTEGRAL INEQUALITY

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Abstract. *This paper proposes a stability criterion for T-S fuzzy systems with constant time delay. Based on the Bessel-Legendre integral inequality, we propose a less conservative stability criterion than the existing ones for the T-S fuzzy systems with constant time delay. Finally, two classical examples are given to illustrate the effectiveness of the result.*

Keywords: Time delay, T-S fuzzy system, Bessel-Legendre integral inequality, Lyapunov-Krasovskii functional

1. **Introduction.** Fuzzy logic control is an efficient approach to model the plant without complete knowledge. Many applications of fuzzy control were reported in a lot of areas, e.g., industrial processes [2, 3], and mechanical process. Takagi-Sugeno (T-S) fuzzy model is the most popular in all of the fuzzy methods [4, 5, 6].

Time-delay is generally regarded as a main source of instability and poor performance, which arises in many processes such as manufacturing system, telecommunication, economic and chemical engineering system [9, 10]. So, many scholars devoted themselves to investigating the stability of fuzzy systems with time delay.

For example, by using LMI, [7, 8] presented controller design for a class of fuzzy dynamic systems with time delay in both continue and discrete case. In [7], the Lyapunov-Krasovskii was employed to analyze and synthesize the T-S fuzzy system with time-delay and a sufficient condition of delay-independent was developed. In [8], by using Lyapunov-Razumikhin functional approach, the similar work was done. While, as we know, the integral inequality plays an important role in reducing the conservatism of the result of time-delay systems. To some extent, we can say that the conservatism of integral inequality can decide the conservatism of result for time-delay systems. Recently, A. Seuret and F. Gouaisbaut [1] proposed an integral inequality called Bessel-Legendre integral inequality, which encloses the Jensen inequality and the improved Wirtinger-based inequality as special cases. There is some hope to prove that the conservatism of Bessel-Legendre integral inequality can be arbitrarily reduced.

Recently, many scholars focus their attention on the control problem of T-S fuzzy systems with time-delay. For instance, the problem of delay-dependent conditions for stability and stabilization of T-S fuzzy systems with time-delay were proposed in [11, 12, 13]. In [14], a delay partitioning approach was used for stability and stabilization of delayed T-S fuzzy systems. [15, 16] addressed the delay-dependent stability analysis and synthesis of uncertain T-S fuzzy systems with time-varying delay.

Motivated by the above work, the aim of this paper is to study the problem of stability analysis for T-S fuzzy systems with time delay by Bessel-Legendre integral inequality. In this paper, we proposed a sufficient condition for the stability of T-S fuzzy systems with constant delay. Based on Bessel-Legendre integral inequality, we give a less conservative stability criterion for the T-S fuzzy system with constant delay.

This paper is organized as follows. In Section 2, the T-S fuzzy model and Bessel-Legendre integral inequality are introduced. Section 3 presents the main results of the paper on the stability analysis of T-S fuzzy systems with constant time delay using classical Lyapunov-Krasovskii functional. Section 4 illustrates our results with two examples extracted from the literature. Section 5 gives the conclusions.

Notations: The notations used throughout this paper are fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with vector norm $|\cdot|$. The superscript "T" stands for matrix transpose, and the notation $P > 0$ ($P \geq 0$) means that matrix P is real symmetric and positive (or being positive semi-definite). I and 0 are used to denote appropriate dimensions identity matrix and zero matrix, respectively. The parameter $diag\{\cdots\}$ denotes a block-diagonal matrix. For given $t > 0$, $h > 0$ and continuous function $x(t)$ from $[-h, +\infty]$ to $\mathbb{R}^n$, set $x_t(s) = x(t+s)$ for all $s \in [-h, 0]$. Matrices, if not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. **Problem Statement and Preliminaries.** Consider a continuous-time T-S fuzzy system with constant time delay. The i-th rule of this T-S fuzzy model is of the following form:

Rule i: If $s_1(t)$ is F_{i1} and $s_2(t)$ is F_{i2} and $\ldots$ $s_p(t)$ is F_{ip} then,

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-h), & t \geq 0 \\ x(t) = \phi(t), & t \in [-h, 0] \end{cases} \tag{1}$$

where $s_1(t), s_2(t), \ldots, s_p(t)$ are the premise variables, and each F_{il} ($i = 1, 2, \ldots, r$; $l = 1, 2, \ldots, p$) is a fuzzy set. $x(t)$: $[0, \infty) \to R^n$ is the state vector, $\phi(t)$ is the initial condition, A_i and $A_{di} \in R^{n\times n}$ are constant matrices, and $h > 0$ is a constant time delay.

By a center-average defuzzier, product inference and singleton fuzzifier, the dynamic fuzzy model in [1] can be represented by

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_d(t)x(t-h), & t \geq 0 \\ x(t) = \phi(t), & t \in [-h, 0] \end{cases} \tag{2}$$

where

$$\begin{cases} A(t) = \displaystyle\sum_{i=1}^{r} h_i(s(t))A_i \\ A_d(t) = \displaystyle\sum_{i=1}^{r} h_i(s(t))A_{di} \\ h_i(s(t)) = \dfrac{\prod_{l=1}^{p} F_{il}(s_l(t))}{\sum_{i=1}^{r}\prod_{l=1}^{p} F_{il}(s_l(t))}, & i = 1, 2, \ldots, r \end{cases} \tag{3}$$

in which $F_{il}(s_l(t))$ is the grade of membership of $s_l(t)$ in F_{il} and $s(t) = (s_1(t), s_2(t), \ldots, s_r(t))$. By definition, the fuzzy weighting functions $\sum_{i=1}^{r} h_i(s(t)) = 1$ and $h_i(s(t)) \geq 0$. For simplicity, h_i is used to represent $h_i(s(t))$ in the following description.

In obtaining our main results of this paper, the following lemma plays an important role, and we show it as follows.

Lemma 2.1. [1] *For given symmetric positive definite matrix $R > 0$ and a differentiable function x: $[-h, 0] \to R^n$, the following inequality holds:*

$$\int_{-h}^{0} \dot{x}^T(u)R\dot{x}(u)du \geq \frac{1}{h}\varepsilon_N^T \left[\sum_{k=0}^{N}(2k+1)\Gamma_N(k)^T R \Gamma_N(k)\right]\varepsilon_N \tag{4}$$

holds, for all integer $N \in \mathbb{N}$, *where*

$$\varepsilon_N = \begin{cases} \left[x^T(0)\ x^T(-h)\right]^T, & \text{if } N = 0, \\ \left[x^T(0)\ x^T(-h)\ \frac{1}{h}\Omega_0^T\ \cdots\ \frac{1}{h}\Omega_{N-1}^T\right]^T, & \text{if } N > 0, \end{cases}$$

$$\Gamma_N(k) = \begin{cases} [I\ \ -I]^T, & \text{if } N = 0, \\ \left[I\ (-1)^{k+1}I\ \gamma_{Nk}^0 I\ \cdots\ \gamma_{Nk}^{N-1}I\right]^T, & \text{if } N > 0, \end{cases}$$

$$\gamma_{Nk}^i = \begin{cases} -(2i+1)\left(1-(-1)^{k+i}\right), & \text{if } N = 0, \\ 0, & \text{if } N > 0. \end{cases}$$

$\Omega_k = \int_{-h}^0 L_k(u)x(u)\mathrm{d}u$ *and* $L_k(u)$ *are the Legendre polynomials.*

3. **Main Results.**

Theorem 3.1. *For a given integer* N *and a constant delay* h, *assume that there exists a matrix* $P_N \in \mathbb{R}^{(N+1)n}$ *and two symmetric positive definite matrices* $S, R \in \mathbb{R}^n$ *such that the LMIs*

$$\Theta_N(h) = \begin{cases} P_N > 0, & \text{if } N = 0, \\ P_N + \frac{1}{h}diag\{0, S_{N-1}\} > 0, & \text{if } N > 0. \end{cases}$$

$$\Phi_N^i(h) = \Phi_{N0}^i(h) - \begin{bmatrix} \Gamma_N(0) \\ \cdots \\ \Gamma_N(N) \end{bmatrix}^T R_N \begin{bmatrix} \Gamma_N(0) \\ \cdots \\ \Gamma_N(N) \end{bmatrix} < 0$$

hold for $i = 1, 2, \ldots, r$, *then the system (2) is asymptotically stable for constant delay* h, *where,* $\Gamma_N(k)$ *for all* $k = 0, \ldots, N$, *are defined in Lemma 2.1 and*

$$\begin{aligned} \Phi_{N0}^i(h) &= G_N^T(h)P_N H_N + H_N^T P_N G_N(h) + \tilde{S}_N + h^2(F_N^i)^T R F_N^i, \\ \tilde{S}_N &= diag(S, -S, 0_{Nn}), \\ R_N &= diag\{R, 3R, \ldots, (2N+1)R\}, \\ S_N &= diag\{S, 3S, \ldots, (2N+1)S\}, \\ F_N^i &= [A_i, A_{di}, 0_{n,nN}], \quad i = 1, 2, \ldots, r, \\ G_N(h) &= \begin{bmatrix} I & 0_n & 0_{n,nN} \\ 0_{nN,n} & 0_{nN,n} & hI_{nN} \end{bmatrix}, \\ H_N &= \left[F_N^T, \Gamma_N^T(0), \Gamma_N^T(1), \ldots, \Gamma_N^T(N-1)\right]. \end{aligned}$$

Proof: Similar to [1], choose the augmented vector as

$$\tilde{x}_N(t) = \begin{bmatrix} x_t(0) \\ \int_{-h}^0 L_0(s)x_t(s)\mathrm{d}s \\ \vdots \\ \int_{-h}^0 L_{N-1}(s)x_t(s)\mathrm{d}s \end{bmatrix}$$

and

$$\varepsilon_N(t) = \begin{bmatrix} x_t(0) \\ x_t(-h) \\ \frac{1}{h}\int_{-h}^0 L_0(s)x_t(s)\mathrm{d}s \\ \vdots \\ \frac{1}{h}\int_{-h}^0 L_{N-1}(s)x_t(s)\mathrm{d}s \end{bmatrix}, \quad N \geq 1.$$

We choose the LKF as follows

$$V_N(x_t, \dot{x}_t) = \tilde{x}_N^T(t)P_N\tilde{x}_N(t) + \int_{t-h}^t x^T(s)Sx(s)\mathrm{d}s + h\int_{t-h}^t\int_\theta^t \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

Obviously, based on Lemma 3 in [1], we know that

$$V_N(x_t, \dot{x}_t) \geq \tilde{x}_N^T(t)\Theta_N \tilde{x}_N(t) + h\int_{t-h}^{t}\int_{\theta}^{t} \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

Then, we compute the derivative of V_N as

$$\begin{aligned}\dot{V}_N(x_t, \dot{x}_t) &= 2\tilde{x}_N^T(t)P_N\dot{\tilde{x}}_N(t) + x_t^T(0)Sx_t(0) - x_t^T(-h)Sx_t(-h) \\ &\quad + h^2\dot{x}_t^T(0)R\dot{x}_t(0) - h\int_{-h}^{0}\dot{x}^T(s)R\dot{x}(s)\mathrm{d}s.\end{aligned} \tag{5}$$

Noting that

$$\tilde{x}_N(t) = G_N(h)\varepsilon_N(t), \quad \dot{\tilde{x}}_N(t) = H_N\varepsilon_N(t), \quad \dot{x}_t(0) = \sum_{i=1}^{r} h_i(s(t))F_N^i\varepsilon_N(t).$$

we get that

$$\dot{V}_N(x_t, \dot{x}_t) = \sum_{i=1}^{r} h_i(s(t))\varepsilon_N^T(t)\Phi_{N0}^i(h)\varepsilon_N(t) - h\int_{-h}^{0}\dot{x}^T(s)R\dot{x}(s)\mathrm{d}s. \tag{6}$$

Applying Lemma 2.1 to (6), we can obtain that

$$\dot{V}_N(x_t, \dot{x}_t) \leq \sum_{i=1}^{r} h_i(s(t))\varepsilon_N^T(t)\Phi_N^i(h)\varepsilon_N(t).$$

Noting (3), we can get Theorem 3.1.

4. **Numerical Example.**

Example 4.1. *Let us consider the system (1) with*

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ -0.2 & -0.9 \end{bmatrix}, \ A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \ A_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}.$$

The largest allowable delay of h derived from [11, 12, 13, 14] and Theorem 3.1 is shown in Table 1. It can be concluded that the method proposed in this paper is less conservative than those in [11, 12, 13, 14].

Table 1. The largest allowable delay for Example 4.1

Methods	The largest h
[11]	0.65
[12]	3.37
[13]	3.85
[14, $m = 2$]	4.28
[Th.1, $N = 2$]	4.42

Example 4.2. *Consider the system (1) with*

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \ A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \ A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.$$

For this example, the largest allowable delay of h derived from [15, 16] and Theorem 3.1 is shown in Table 2. It can be concluded that the method proposed in this paper is better than those mentioned above.

Table 2. The largest allowable delay for Example 4.2

Methods	The largest h
[15]	1.5974
[16]	1.6341
[Th.1, $N = 2$]	2.04

5. **Conclusions.** The problem on the stability of T-S fuzzy systems with interval constant delay is addressed. Based on the Bessel-Legendre, a much less conservative stability criterion than some existing ones for the systems under consideration is presented. Finally, two numerical examples are given to illustrate the effectiveness of the result.

In future work, we will consider the H_∞ control of the T-S fuzzy systems with interval constant delay based on the result in this paper.

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