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PARAMETERS OPTIMIZATION AND INTERVAL CONCEPT LATTICE COMPRESSION WITH CHANGE OF PARAMETERS

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Abstract. *The interval parameters $[\alpha, \beta]$ in interval concept lattice affect the extension of each concept, structure of lattice and quantity and precision of extracted association rules. In order to obtain the optimal α and β with the biggest compression degree of interval concept lattice, firstly, the definitions of similarity degree of the binary relations pairs and covering-neighborhood-space are put forward, and then the similarity degree matrix and covering-neighborhood of binary relations pairs are obtained. Secondly, update algorithm of concept sets with the changing parameters is raised, where concept sets are got on the basis of the non-reconstruction. Combining with covering-neighborhood of binary relations pairs with the changing parameters, further the model of parameters optimization of interval concept lattice is built based on compression theory. According to the size of the compression degree and its change trends, the optimal values of interval parameters are found. Finally, the validity of the model is demonstrated by an example.*
Key words: Interval parameters, Relation similarity matrix, Compression degree, Covering-neighborhood space

1. **Introduction.** Concept lattice [1-3] is a kind of concept hierarchical structure based on the binary relation between objects and attributes from data sets, which is suitable as a basic data structure to mine rules. Rough concept lattice [4,5] uses the upper and lower approximation theory of rough set to describe the concepts of partial attributes. If formal context is too sparse, there might be a lot objects who only meet an attribute of intensions, and association rules extracted by it have low credibility and support. Interval concept lattice [6] is an expansion of classical concept lattice, whose concept extension is an object set meeting a certain-percentage (α and β) intension.

The problem of interval parameters is primary for building interval concept lattice [7] and mining association rules. In previous studies, the main evaluation standard of the model of parameters optimization [8] is that lattice structure tends to be stable under the optimal interval parameters, namely when interval parameters change into the optimal with equal steps, the update degree of lattice structure tends to 0. At the same time, based on it, the number of these association rules is moderate and the accuracy is high. Succeeding studies are based on three-way decision space [9]. The evaluation standard of the optimal parameters is to help users make reject or accept decision, not non-commitment. However, these optimization models have not considered the disadvantages about redundancy of concept nodes in lattice structure and data size booming. Therefore, reducing concept lattice and improving the efficiency of extracting rules have become another important research subject. [10,11] put forward methods of reduction by analyzing attributes and nodes, but they are all based on the existing lattice structure. [12,13] directly used the formal context to compress concept lattice structure, and proposed some definitions about concept similarity degree and attribute similarity degree.

By learning from compression theory, firstly the compression algorithm of interval concept lattice is raised. In the process of compressing, binary relation pairs are built based on formal context, and then a similarity matrix is formed by calculating the similarity degree of binary relation pairs. Use the coverage of binary relation pairs to obtain neighborhood space and solve the neighborhood of each object under various parameters. Secondly, parameters optimization algorithm of interval concept lattice based on compression degree is put forward and the compression of original concept lattice with different degrees is realized. The effect of interval parameters on compression degree of concept lattice has been discussed. Finally, use an example to verify the model and the relation of parameters and compression degree is obtained. When the compression degree tends to maximum, the interval parameters are the optimal.

2. **Preliminary Knowledge.**

2.1. **Interval concept lattice theory.**

Definition 2.1. [14] *(U, A, R) is a formal context. $U = \{x_1, x_2, \ldots, x_n\}$ is an object set. $A = \{a_1, a_2, \ldots, a_m\}$ is an attribute set. R is a binary relation between U and A. If $(x, a) \notin R$, call x has attribute a. Else, call x does not have attribute a. Use "1" to express $(x, a) \in R$, and "0" to express $(x, a) \notin R$. Formal context can be shown by the table including 0 and 1. For the formal context (U, A, R), define operation on object set $X \subseteq U$ and attribute set $B \subseteq A$:*

$$X^* = \{a|a \in A, \forall x \in X, (x, a) \in R\}; \quad B^* = \{x|x \in U, \forall a \in B, (x, a) \in R\}$$

Definition 2.2. [6] *The formal context (U, A, R) is given. $L(U, A, R)$ is a classic concept lattice based on it. (M, N, Y) is a rough concept lattice based on RL. Assume the interval $[\alpha, \beta]$, $0 \le \alpha < \beta \le 1$.*

α-upper extension M^α : $M^\alpha = \{x|x \in M, |f(x) \cap Y|/|Y| \ge \alpha, 0 \le \alpha \le 1\}$
β-lower extension M^β : $M^\beta = \{x|x \in M, |f(x) \cap Y|/|Y| \ge \beta, 0 \le \alpha < \beta \le 1\}$

Among them, Y is the intension of the concept. $|Y|$ is the number of elements contained by set Y. M^α expresses the objects covered by at least $\alpha \times |Y|$ attributes from Y.

Definition 2.3. [6] *The ternary ordered pairs (M^α, M^β, Y) is called interval concept, among which, Y is the intension; M^α is the α-upper extension; M^β is the β-lower extension.*

Definition 2.4. [6] *Using $L_\alpha^\beta(U, A, R)$ to express all interval concepts within $[\alpha, \beta]$, record: $\left(M_1^\alpha, M_1^\beta, Y_1\right) \le \left(M_2^\alpha, M_2^\beta, Y_2\right) \Leftrightarrow Y_1 \supseteq Y_2$, and "$\le$" is the partial ordering relation of $L_\alpha^\beta(U, A, R)$. If all concepts in $L_\alpha^\beta(U, A, R)$ meet the partial ordering relation "$\le$", call $L_\alpha^\beta(U, A, R)$ is the interval concept lattice on formal context (U, A, R).*

2.2. **Compression theory.** Based on formal context build binary relation pairs of attribute and object $t = (x, a)$, namely x has attribute a.

Definition 2.5. [13] *Binary relation pairs $t_1 = (x_1, a_1)$, $t_2 = (x_2, a_2)$ $(t_1, t_2 \in R)$, the similarity degree between t_1 and t_2 is* $r(t_1, t_2) = \frac{1}{2}\left(\frac{|f(x_1) \cap f(x_2)|}{|f(x_1) \cup f(x_2)|} + \frac{|g(a_1) \cap g(a_2)|}{|g(a_1) \cup g(a_2)|}\right)$

Among them, $\forall x \in U$, $f(x) = \{y|\forall y \in A, xRy\}$; $\forall y \in A$, $g(y) = \{x|\forall x \in U, xRy\}$.

Make $t_1 = (x_1, a_1)$, $[t_1] = \{t_2 = (x_2, a_2) \in R\,|r(t_1, t_2) \ge \gamma\}$ and $\gamma \in [0, 1]$. We call $[t_1]$ is γ-similarity class of t_1 and $\{[t_1]\,|t_1 \in R\}$ constitutes coverage on R.

Definition 2.6. [13] *Assume C_R is a coverage on R. $\forall t \in R$, $Md(t) = \{K \in C_R | t \in K \wedge (\forall S \in C_R \wedge t \in S \wedge S \subseteq K \Rightarrow K = S)\}$ is called minimum description of t. R is a domain, and C_R is a coverage on R. Call (R, C_R) a covering-neighborhood space.*

Definition 2.7. [13] *(R, C_R) is a covering-neighborhood space, $\forall t \in R$, call $\cap\{K|K \in C_R\}$ is neighborhood of t, record $N(t)$. Similarly, record neighborhood of x is $N(x)$.*

Definition 2.8. [15] *$\forall t(x, a) \in R$, and its neighborhood is $N(t)$. $N(t)^*$ expresses object sets of neighborhood of binary relation pairs including attribute a. Call $N(t)^*_\alpha = \left\{x_j | (x_j \in N(t)^*) \wedge \frac{|x_j.Y \cap f(x)|}{|f(x)|} \geq \alpha\right\}$ α is the interval parameters in interval concept lattice. $|.|$ expresses the number of elements in set.*

Definition 2.9. [15] *$\forall x \in U$, and the neighborhood of x is $N(x) = \cap\{N(t_j)^*_\alpha | t_j = (x, a_j)\}$; $N(x)'$ is attribute set that all objects in $N(x)$ share.*

The process of solving $N(x)$ is actually a clustering process where we can obtain sets of objects which are similar to x. According to $N(x)'$, we can delete some attributes of x. These attributes are of low association.

Definition 2.10. [15] *(U, A, R) is a formal context. Define operators as follows: α and β are interval parameters in interval concept lattice. $Y' = Y$;*

$$M^{\alpha'} = \{x | x \in M^\alpha \wedge |N(x)' \cap Y|/|Y| \geq \alpha\};\ M^{\beta'} = \{x | x \in M^\beta \wedge |N(x)' \cap Y|/|Y| \geq \beta\}$$

$\forall (M^\alpha, M^\beta, Y)$, if $M^{\alpha'} = M^\alpha$ and $M^{\beta'} = M^\beta$, call (M^α, M^β, Y) compressed interval concept.

Definition 2.11. [15] *(U, A, R) is a formal context. $L^\beta_\alpha(U, A, R)$: before compressed, $L^{\beta'}_\alpha(U, A, R)$: after compressed. Define compression degree:*

$$Rd = \left(\left|L^\beta_\alpha(U, A, R)\right| - \left|L^{\beta'}_\alpha(U, A, R)\right|\right) \Big/ \left|L^\beta_\alpha(U, A, R)\right|$$

2.3. Compression algorithm of $L^{\beta_0}_{\alpha_0}(U, A, R)$.

Input: Formal context M, interval parameters $[\alpha_0, \beta_0]$, threshold of similarity class $\gamma_0 \in [0, 1]$.

Output: Compression degree of concept lattice Rd.

Step 1. According to formal context M, extract binary relation pairs sets: $R = \{t_1, t_2, \ldots, t_n\}$. Based on Definition 2.6, calculate similarity degree and obtain similarity degree matrix of binary relation pairs.

Step 2. Set $\gamma_0 \in [0, 1]$. Based on Definition 2.6, calculate γ_0 similarity class of $t_1, t_2, \ldots, t_n$, and obtain the coverage C_R. From Definition 2.8, obtain the neighborhood of $t_1, t_2, \ldots, t_n$. From Definition 2.9 and $[\alpha_0, \beta_0]$, respectively calculate $N(t)^*$ and $N(t)^*_\alpha$ of each t_i.

Step 3. Based on Definition 2.10 and $N(t)^*_\alpha$, calculate $N(x)$ of each object and $N(x)'$.

Step 4. Obtain interval concepts sets [7] from given M, and record the number n_0 of concepts; then use Definition 2.11 to judge each concept in sets and realize the compression of interval concept lattice. Record the number n_1 of concepts. Finally, calculate $Rd = (n_0 - n_1)/n_0$ and output it.

3. Optimization Model of Interval Parameters Based on Compression Degree.

In order to obtain the best interval parameters, introduce compression degree to measure the parameters. By changing interval parameters, obtain compression degrees under different parameters. Determine the best parameters based on the value and trend of compression degree. Because compression degree is only related to the number of concepts in concept lattice, do not need to reconstruct lattice structure after changing parameters, but update concept sets.

3.1. **Update of concept sets based on changing parameters.** When α and β are changing, concepts in lattice will change. Based on the characteristic of changing concepts, some definitions are given in [8]. Compared with the algorithm (update algorithm of interval concept lattice based on parameters) of [8], the only difference is that the update of concept sets does not need to adjust the relationship of father-children.

3.2. **Parameters optimization algorithm of interval concept lattice based on compression degree.** The previous optimization models of interval parameters ignore the redundancy of concepts caused by weak association in formal context. Compressing concept lattice can solve these problems to a certain extent. This algorithm is proposed to find the best parameters according to compression degree.

Algorithm: Parameters optimization algorithm of interval concept lattice based on compression degree.

Input: Formal context M, threshold of similarity class $\gamma_0 \in [0, 1]$.

Output: Best interval parameters and corresponding compression degree.

Step 1. According to formal context, obtain the number of attribute: n, and set up the length of step: $\lambda = 1/n$. Initialize the interval parameters: $\alpha_0 = 1/n$, $\beta_0 = 1$, then build concept set CS_0', and according to compression algorithm, obtain Rd_1.

Step 2. Make $\alpha_1 = \alpha_0 + \lambda$, $\beta_0 = 1 - \lambda$. According to update algorithm, update CS_0' to CS_1'.

Step 3. According to compression algorithm, obtain Rd_2. If $Rd_2 \leq Rd_1$, turn to Step 2; else turn to Step 4.

Step 4. Output α, β and Rd_2.

4. **Example Analysis.** For simplicity, we set parameter β value to 1 and explore the effect of α on compression degree of interval concept lattice. The formal context is shown in Table 1.

Table 1. Formal context

Object	a	b	c	d	e	Object	a	b	c	d	e
1	0	0	1	1	0	5	0	0	0	1	1
2	0	1	0	1	1	6	0	1	1	1	0
3	1	0	0	0	1	7	1	0	0	0	0
4	1	1	0	0	0	8	0	1	0	0	0

4.1. **Model validation.** According to parameters optimization algorithm of interval concept lattice based on compression degree, find the best parameters.

(1) Calculate similarity degree of binary relation pairs in R. From formal context, obtain: $t_1 = (1, c)$, $t_2 = (1, d)$, $t_3 = (2, b)$, $t_4 = (2, d)$, $t_5 = (2, e)$, $t_6 = (3, a)$, $t_7 = (3, e)$, $t_8 = (4, a)$, $t_9 = (4, b)$, $t_{10} = (5, d)$, $t_{11} = (5, e)$, $t_{12} = (6, b)$, $t_{13} = (6, c)$, $t_{14} = (6, d)$, $t_{15} = (7, a)$, $t_{16} = (8, b)$. Obtain $R = \{t_1, t_2, \ldots, t_{16}\}$. Calculate similarity degree of t_i and t_j. The matrix is as Table 2.

(2) Set initial parameters $[1/5, 1]$ and $\gamma = 0.55 \in [0, 1]$. Obtain γ similarity class from similarity matrix, and then get C_R. Based on C_R, calculate $N(t)$, $N(t)^*$ and $N(t)^*_{1/5}$. The neighborhood of binary relation pairs is shown by Table 3.

Some $N(t)^*$: $N(t_1)^* = \{1, 6\}$, $N(t_2)^* = \{1, 2, 5, 6\}$, $N(t_3)^* = \{2, 4, 6, 8\}$, $N(t_4)^* = \{1, 2, 5, 6\}, \ldots, N(t_{15})^* = \{3, 4, 7\}$, $N(t_{16})^* = \{2, 4, 6, 8\}$. Some $N(t)^*_{1/5}$: $N(t_1)^*_{1/5} = \{1, 6\}$, $N(t_2)^*_{1/5} = \{1, 2, 5, 6\}$, $N(t_3)^*_{1/5} = \{2, 4, 6, 8\}$, $N(t_4)^*_{1/5} = \{1, 2, 5, 6\}, \ldots, N(t_{14})^*_{1/5} = \{1, 2, 5, 6\}$, $N(t_{15})^*_{1/5} = \{3, 4, 7\}$, $N(t_{16})^*_{1/5} = \{2, 4, 6, 8\}$, $N(t_{16})^*_{1/5} = \{2, 4, 6, 8\}$.

(3) According to Definition 2.10, calculate $N(x)$ and $N(x)'$. For examples: $N(1) = \cap N^*_{1/5}(t_1) \cap N^*_{1/5}(t_2) = \{1,6\}$, $N(2) = \cap N^*_{1/5}(t_3) \cap N^*_{1/5}(t_4) \cap N^*_{1/5}(t_5) = \{2\}$, $N(3) = \cap N^*_{1/5}(t_6) \cap N^*_{1/5}(t_7) = \{3\}$. Results are as Table 4.

(4) Build original concept sets CS_1 under $[1/5,1]$, as Table 5.

According to Definition 2.11, compress $[1/5,1]$ concept lattice. For example, $C3 = (\{1,6\},\{1,6\},\{c\})$ and its intension $Y = c$. Upper extension is $\{1,6\}$, and lower extension is $\{1,6\}$; $\because N(1)' = \{c,d\}$, $N(6)' = \{b,c,d\}$, $|N(1)' \cap Y|/|Y| = |cd \cap c|/|c| =$

Table 2. Similarity matrix

	t_1	t_2	t_3	...	t_{14}	t_{15}	t_{16}
t_1	1.00	0.75	0.23	...	0.58	0.00	0.10
t_2	0.75	1.00	0.29	...	0.83	0.00	0.17
t_3	0.23	0.29	1.00	...	0.42	0.08	0.67
...	...	...	...	...	...	...	...
t_{14}	0.58	0.83	0.42	...	1.00	0.00	0.33
t_{15}	0.00	0.00	0.08	...	0.00	1.00	0.08
t_{16}	0.00	0.17	0.67	...	0.33	0.08	1.00

Table 3. $N(t)$ of binary relation pairs

Binary relation pairs t_i	Neighborhood $N(t_i)$	Binary relation pairs t_i	Neighborhood $N(t_i)$	Binary relation pairs t_i	Neighborhood $N(t_i)$
t_1	$\{t_1,t_2,t_{13},t_{14}\}$	t_7	$\{t_7\}$	t_{12}	$\{t_{12}\}$
t_2	$\{t_2,t_{14}\}$	t_8	$\{t_8\}$	t_{13}	$\{t_{13},t_{14}\}$
t_3	$\{t_3\}$	t_9	$\{t_9\}$	t_{14}	$\{t_{14}\}$
t_4	$\{t_4\}$	t_{10}	$\{t_{10}\}$	t_{15}	$\{t_6,t_8,t_{15}\}$
t_5	$\{t_5\}$	t_{11}	$\{t_{11}\}$	t_{16}	$\{t_3,t_9,t_{12},t_{16}\}$
t_6	$\{t_6,t_8,t_{15}\}$				

Table 4. $N(x)$ and $N(x)'$ under $[1/5,1]$

x_i	$N(x_i)$	$N(x_i)'$	x_i	$N(x_i)$	$N(x_i)'$
1	$\{1,6\}$	$\{c,d\}$	5	$\{2,5\}$	$\{d,e\}$
2	$\{2\}$	$\{b,e,d\}$	6	$\{6\}$	$\{b,c,d\}$
3	$\{3\}$	$\{a,e\}$	7	$\{3,4,7\}$	$\{a\}$
4	$\{4\}$	$\{a,b\}$	8	$\{2,4,6,8\}$	$\{b\}$

Table 5. $[1/5,1]$ concept sets

Concept	Upper extension	Lower extension	Intension	Concept	Upper extension	Lower extension	Intension
C1	$\{3,4,7\}$	$\{3,4,7\}$	a	C11	$\{1,2,5,6\}$	$\{1,6\}$	cd
C2	$\{2,4,6,8\}$	$\{2,4,6,8\}$	b	C12	$\{1,2,3,5,6\}$	$\{2,5\}$	de
C3	$\{1,6\}$	$\{1,6\}$	c	C13	$\{1,2,3,4,6,7,8\}$	$\emptyset$	abc
C4	$\{1,2,5,6\}$	$\{1,2,5,6\}$	d	C14	$\{2,3,4,5,6,7,8\}$	$\emptyset$	abe
C5	$\{2,3,5\}$	$\{2,3,5\}$	e	C15	$\{1,2,4,5,6,8\}$	$\{6\}$	bcd
C6	$\{2,3,4,6,7,8\}$	$\{4\}$	ab	C16	$\{1,2,3,4,5,6,8\}$	$\{2\}$	bde
C7	$\{1,3,4,6,7\}$	$\emptyset$	ac	C17	$\{1,2,3,5,6\}$	$\emptyset$	cde
C8	$\{2,3,4,5,7\}$	$\{3\}$	ae	C18	$\{1,2,3,4,5,6,7\}$	$\emptyset$	$acde$
C9	$\{1,2,4,5,6,8\}$	$\{2,6\}$	bd	C19	$\{1,2,3,4,5,6,8\}$	$\emptyset$	$bcde$
C10	$\{2,3,4,6,8\}$	$\{2\}$	be	C20	$\{1,2,3,4,5,6,7,8\}$	$\emptyset$	$abcde$

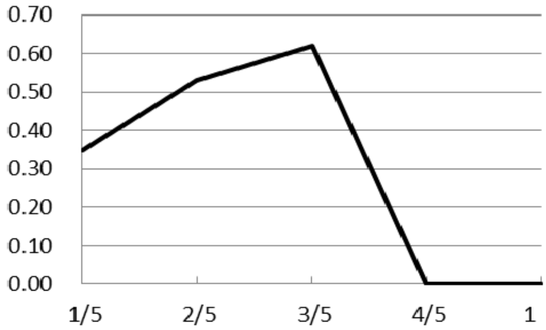

FIGURE 1. The relation of α and compression degree

1, $|N(6)' \cap Y|/|Y| = |bcd \cap c|/|c| = 1$. And $\because$ $\alpha = 1/5$, $\beta = 1$, $1/5 < 1 \leq 1$. $\therefore$ C3 $= (\{1,6\},\{1,6\},\{c\})$ is the concept after compressed. Concept set after compressed is $\{C1, C2, C3, C4, C5, C7, C8, C9, C10, C11, C12, C15, C16\}$ and compression degree is $Rd = (20 - 13)/20 = 0.35$.

(5) Continue to calculate the compression degree under $[2/5, 1]$, $[3/5, 1]$, $[4/5, 1]$ and $[1, 1]$. Draw the figure as Figure 1.

4.2. **Model comparison.** From Figure 1, we can see when $\alpha = 3/5$, $\beta = 1$, the compression degree is maximum 0.62, namely under $[3/5, 1]$ redundancy is minimum, remove the weak association, retain the strong association, and obtain relatively simple lattice structure. Compared with the parameters optimization model in [8], their conclusions are about the same. Further verify the reliability and validity of the model.

5. **Conclusion.** This paper analyzes interval parameters based on the compression theory. Building binary relation pairs, forming similarity matrix, obtaining neighborhood space and solving neighborhood of each object with different parameters, realize the compression. When the compression degree is maximum (redundant is the least), find the best parameters (roughly 0.5). Combining the theory of information entropy to optimize the interval parameters could be considered in future studies.

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