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OPTIMAL PERFORMANCE OF NETWORKED CONTROL SYSTEMS WITH PACKET DROPOUTS AND BANDWIDTH CONSTRAINTS

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Abstract. *In this paper, we deeply investigate the optimal tracking performance of single-input single-output (SISO) networked control systems (NCSs) with data packet dropout and bandwidth constraints. The communication network is characterized by three parameters: the packet dropout, network bandwidth and channel noise. The explicit expression of the optimal tracking performance is obtained by applying the spectral factorization technique. The obtained results demonstrate that the optimal performance is influenced by the nonminimum phase zeros and unstable poles of the given plant, the characters of the reference signal, packet dropout probability, the bandwidth and channel noise of the communication channel. A typical example is given to illustrate the theoretical results.*
Keywords: Packet dropout probability, Bandwidth, Networked control systems, Unstable poles, Nonminimum phase zeros

1. **Introduction.** With the development of science and technology, networked control systems (NCSs) have been widely used in various fields in recent years [1, 2]. At the same time, NCSs have attracted much attention of scholars both at home and abroad. While NCSs have brought some convenience, they also raise new challenges due to inherent network-limited bandwidth and channel capacity. Time-delay and packet dropout can degrade the control performance of the NCSs, and even lead to the instability of the system. Some research achievements have been obtained in NCSs. The stability of networked system with packet dropouts and network bandwidth constraints has been investigated in [3]. In [4], the stability of NCSs and the design method of the controller are studied with considering the influence of packet dropout and time delay. In [5], a self-triggered sampling scheme (STS) is proposed for an NCS with consideration of data losses and communication delays.

It is well known that the research on the NCSs is not limited to this. In [6], the optimal modified performance of the single-input multiple-output (SIMO) linear time-invariant (LTI) systems was investigated. The optimal modified tracking performance of multi-input multi-output (MIMO) NCSs with bandwidth and channel noise constraints was studied in [7]. The optimal tracking performance of MIMO discrete-time NCSs with bandwidth and coding constraints was studied in [8], and the optimal tracking performance of NCSs was obtained by using spectral factorization technique and partial fraction.

[3] investigated the optimal regulation performance of networked system with packet dropouts and network bandwidth constraints. One-parameter controller was considered in [3]. The obtained result shows the relationship among the stability of networked system, structural characteristics of the given plant and communication network parameters. In [7], the optimal modified tracking performance of multi-input multi-output networked control systems with bandwidth and channel noise constraints was investigated, and in

this paper, both one-parameter controller and two-parameters controller were discussed. From this paper, the optimal modified performance is novel and reveals the relationship between the plant characteristics, the communication parameters, the modified factor and the optimal modified performance.

The main contributions of this paper are as follows. According to the existing research results, we know that the influence factors of the optimal tracking performance of the NCSs are the plant characteristics (unstable pole and nonminimum phase zero) and network parameters (packet dropout, bandwidth, noise and so on). In this paper, we studied the optimal tracking performance of NCSs with packet dropouts and bandwidth constraints, and the model is SISO NCSs with packet dropout and bandwidth constraints in the feedback channel, and the explicit expression of the optimal tracking performance is obtained by applying the spectral factorization technique. Result shows that the optimal tracking performance is dependent on the non-minimum phase zeros, unstable poles, packet dropouts probability, bandwidth, channel noise of the communication channel and the characters of the reference signal. The result may serve as guidelines for the design of NCSs.

This paper is organized as follows. Section 2 introduces the problem formulation. The optimal tracking performance of networked control systems with packet dropouts and bandwidth constraints is studied in Section 3. A typical example is given to illustrate the results in Section 4. The paper conclusion and future research directions are presented in Section 5.

2. **Problem Formulations.** In this paper, the symbols involved are standard symbols. For any vector u, we denote its conjugate transpose by u^H. For random variables N, expectations are denoted as $E\{N\}$. The open right-half plane and the open left-half plane are denoted by $C_+ := \{s : \text{Re}(s) > 0\}$ and $C_- := \{s : \text{Re}(s) < 0\}$, respectively. L_2 is defined as the Hilbert space, and it is well known that H_2 and $H_2^\perp$ are subspaces and form an orthogonal pair of L_2. Finally, let $\mathcal{RH}_\infty$ denote the set of all stable, proper, and rational transfer function.

We all know that the one-paremeter compensator is often difficult to meet the control objectives or it is difficult to obtain high control precision, and the two-paremeter compensator is more stable and better than that of the one-paremeter compensator, so we discussed the two-paremeter compensator in this paper. We establish a simple model of SISO NCSs as shown in Figure 1.

In Figure 1, $[K_1 \quad K_2]$ denotes the two-paremeter compensator, G denotes the plant model, and r and y denote the reference input and reference output, respectively. The communication network is characterized by three parameters: the packet dropout, network bandwidth and channel noise. F models the bandwidth, d_r models the packet dropout, n models the channel noise, and the channel noise variance is σ_2^2. We consider that reference signal r is a random process, and reference input signal variance is σ_1^2. The signal r is uncorrelated with n in this paper.

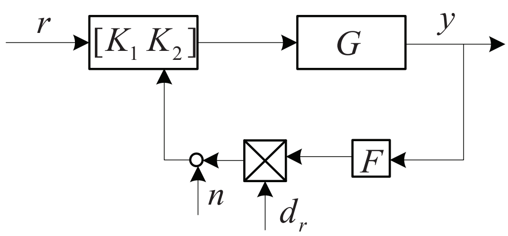

FIGURE 1. Networked control systems with packet dropout and bandwidth

Assumption (description of d_r): The signal d_r can be expressed as a Bernoulli distribution, namely

$$d_r = \begin{cases} 0 & \text{if the systems output is not successfully transmitted to the controller} \\ 1 & \text{if the systems output is successfully transmitted to the controller} \end{cases}$$

the distribution probability for d_r is: $P\{d_r = 1\} = 1 - q$, $P\{d_r = 0\} = q$, $0 \leq q < 1$, and q represents the packet dropout probability.

The tracking error of NCSs is $e = r - y$, and according to Figure 1, we can obtain

$$y = [rK_1 + K_2(yFd_r + n)]G \tag{1}$$

Then

$$e = r - y = (1 - K_1G)r + K_2Gn - K_2FGd_ry \tag{2}$$

According to [7], it can obtain

$$S_e\left(e^{jw}\right) = \frac{\left(1 - \alpha K_1\left(e^{jw}\right)G\left(e^{jw}\right)\right)S_{re}\left(e^{jw}\right) + K_2\left(e^{jw}\right)G\left(e^{jw}\right)S_{ne}\left(e^{jw}\right)}{1 + (1-q)G\left(e^{jw}\right)F\left(e^{jw}\right)K_2\left(e^{jw}\right)} \tag{3}$$

According to [9], we have

$$\sigma_e^2 = \left\|\frac{(1 - \alpha K_1G)}{1 + (1-q)GFK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{K_2G}{1 + (1-q)GFK_2}\right\|_2^2 \sigma_2^2 \tag{4}$$

Define $J := \sigma_e^2$, and then the optimal tracking performance is measured by the possible minimal tracking error achievable by all possible linear stabilizing controllers (denoted by $\mathcal{K}$), determined as

$$J^* = \inf_{K \in \mathcal{K}} \left\|\frac{(1 - \alpha K_1G)}{1 + (1-q)GFK_2}\right\|_2^2 \sigma_1^2 + \left\|\frac{K_2G}{1 + (1-q)GFK_2}\right\|_2^2 \sigma_2^2 \tag{5}$$

For the rational transfer function G, let G coprime factorization be given by

$$F(1-q)G = \frac{N}{M} \tag{6}$$

where $M, N \in \mathcal{RH}_\infty$, and satisfy the Bezout identify

$$MX - NY = 1 \tag{7}$$

where $X, Y \in \mathcal{RH}_\infty$. It is well known that every stabilizing compensator $\mathcal{K}$ can be characterized by Youla parameterization [10].

$$\mathcal{K} = \left\{K : K = [K_1 \quad K_2] = (X - RN)^{-1} \cdot [Q \quad Y - RM], Q \in \mathcal{RH}_\infty, R \in \mathcal{RH}_\infty\right\} \tag{8}$$

It is also well known that a nonminimum phase transfer function could factorize a minimum phase part and an all pass factor [11].

$$M = B_pM_m, \quad N = (1-q)L_zN_m \tag{9}$$

where B_p and L_z are all-pass factors, M_m and N_m are the minimum phase parts, L_z includes all non-minimum phase zeros z_i ($z_i \in \mathbb{C}_+$, $i = 1, \ldots, n$) of the given plant, and B_p includes all unstable poles p_j ($p_j \in \mathbb{C}_+$, $j = 1, \ldots, m$) of the given plant. They are defined as follows

$$B_p(s) = \prod_{j=1}^{m} \frac{s - p_j}{s + \bar{p}_j}, \quad L_z(s) = \prod_{i=1}^{n} \frac{s - z_i}{s + \bar{z}_i} \tag{10}$$

3. **Optimal Tracking Performance with Packet Dropouts and Bandwidth Constraints.** Consider a linear time-invariant feedback control system over communication channel with packet dropouts and bandwidth constraints depicted in Figure 1. According to (4), (6)-(9), we can get

$$J = \left\| 1 - \frac{NQ}{F(1-q)} \right\|_2^2 \sigma_1^2 + \left\| \frac{N(RM-Y)}{F(1-q)} \right\|_2^2 \sigma_2^2 \tag{11}$$

According to (5) and (11), we can rewrite J^*

$$J^* = \inf_{K \in \mathcal{K}} \left(\left\| 1 - \frac{NQ}{F(1-q)} \right\|_2^2 \sigma_1^2 + \left\| \frac{N(RM-Y)}{F(1-q)} \right\|_2^2 \sigma_2^2 \right) \tag{12}$$

Theorem 3.1. *For NCSs as shown in Figure 1, assume that the plant has many unstable poles $p_j \in \mathbb{C}_+$, $j = 1, \ldots, m$, and non-minimum phase zeros $z_i \in \mathbb{C}_+$, $i = 1, \ldots, n$, the optimal tracking performance can be expressed as*

$$J^* = \sum_{i=1}^{n_s} 2\mathrm{Re}(z_i)\sigma_1^2 + J_{11}\sigma_2^2$$

where

$$J_{11} = \sum_{i,j \in m} \frac{4\mathrm{Re}(p_j)\mathrm{Re}(p_i)}{(\bar{p}_j + p_i)\,\bar{b}_j b_j} \left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)\left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)^H.$$

Proof: In order to calculate the J^*, we denote

$$J_1^* = \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \left\| 1 - \frac{NQ}{F(1-q)} \right\|_2^2 \sigma_1^2, \quad J_2^* = \inf_{R \in \mathbb{R}\mathcal{H}_\infty} \left\| \frac{N(RM-Y)}{F(1-q)} \right\|_2^2 \sigma_2^2 \tag{13}$$

According to (6), (7) and (9), we can get

$$J_1^* = \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \|1 - L_z N_m Q\|_2^2 \sigma_1^2 \tag{14}$$

Because L_z is the all pass factors, we can rewrite J_1^* as

$$J_1^* = \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \left\| \left(L_z^{-1} - 1\right) + 1 - N_m Q \right\|_2^2 \sigma_1^2$$

Because $(L_z^{-1} - 1)$ is in $H_2^\perp$, and $(1 - N_m Q)$ is in H_2, conversely,

$$J_1^* = \sigma_1^2 \left(L_z^{-1} - 1\right) + \sigma_1^2 \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \|1 - N_m Q\|_2^2$$

According to [12], we have

$$\left(L_z^{-1} - 1\right) = \sum_{i=1}^{n_s} 2\mathrm{Re}(z_i) \tag{15}$$

Because N_m is the minimum phase parts, $Q \in \mathbb{R}\mathcal{H}_\infty$, then $\sigma_1^2 \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \|1 - N_m Q\|_2^2 = 0$. Hence,

$$J_1^* = \sum_{i=1}^{n_s} 2\mathrm{Re}(z_i)\sigma_1^2$$

According to the same method as J_1^*, we can obtain

$$J_2^* = \inf_{R \in \mathbb{R}\mathcal{H}_\infty} \left\| B_p^{-1} N_m Y F^{-1} - F^{-1} R M_m N_m \right\|_2^2 \sigma_2^2 \tag{16}$$

According to the partial factorization and (10), we have

$$B_p^{-1} N_m Y F^{-1} = \sum_{j=1}^{m} \frac{s + \bar{p}_j}{s - p_j} \frac{N_m(p_j) Y(p_j) F^{-1}(p_j)}{b_j} + R_1 \tag{17}$$

We can rewrite J_2^* as

$$J_2^* = \inf_{R\in\mathbb{RH}_\infty} \left\| \sum_{j=1}^{m} \left(\frac{s+\bar{p}_j}{s-p_j} - 1\right) \frac{N_m(p_j)Y(p_j)F^{-1}(p_j)}{b_j} + \frac{N_m(p_j)Y(p_j)F^{-1}(p_j)}{b_j} + R_1 - F^{-1}(p_j)RN_m(p_j)M_m(p_j) \right\|_2^2 \sigma_2^2.$$

Because M_m and N_m are the minimum phase parts, and at the same time $R_1 \in \mathbb{RH}_\infty$, $R \in \mathbb{RH}_\infty, b_j = \prod_{\substack{i\in N \\ i\neq j}} \frac{p_i - p_j}{\bar{p}_i + p_j}$, by choosing appropriate value, we can make

$$\inf_{R\in\mathbb{RH}_\infty} \left\| \frac{N_m(p_j)Y(p_j)F^{-1}(p_j)}{b_j} + R_1 - F^{-1}(p_j)RN_m(p_j)M_m(p_j) \right\|_2^2 = 0.$$

and then

$$\begin{aligned} J_2^* &= \inf_{R\in\mathbb{RH}_\infty} \left\| \sum_{j=1}^{m} \left(\frac{s+\bar{p}_j}{s-p_j} - 1\right) \frac{N_m(p_j)Y(p_j)F^{-1}(p_j)}{b_j} \right\|_2^2 \sigma_2^2 \\ &= \left\| \frac{2\mathrm{Re}(p_j)}{s-p_j} \frac{N_m(p_j)Y(p_j)F^{-1}(p_j)}{b_j} \right\|_2^2 \sigma_2^2 \end{aligned}$$

From (7) and $M(p_j) = 0$, we can get

$$Y(p_j) = -N^{-1} = -(1-q)^{-1}L_z^{-1}(p_j)N_m^{-1}(p_j)$$

and then

$$\begin{aligned} J_2^* &= \sum_{i,j\in m} \frac{4\mathrm{Re}(p_j)\mathrm{Re}(p_i)}{\bar{p}_j + p_i} \frac{1}{\bar{b}_j b_j} \left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)\left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)^H \sigma_2^2 \\ &= \sum_{i,j\in m} \frac{4\mathrm{Re}(p_j)\,\mathrm{Re}(p_i)}{(\bar{p}_j + p_i)\bar{b}_j b_j} \left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)\left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)^H \sigma_2^2 \end{aligned}$$

We have

$$J^* = \sum_{i=1}^{n_s} 2\mathrm{Re}(z_i)\sigma_1^2 + \sum_{i,j\in m} \frac{4\mathrm{Re}(p_j)\mathrm{Re}(p_i)}{(\bar{p}_j + p_i)\,\bar{b}_j \cdot b_j} \cdot \left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)\left(F^{-1}(p_j)(1-q)^{-1}L_z^{-1}(p_j)\right)^H \sigma_2^2$$

The proof is now completed.

4. **Numerical Example.** Consider the unstable plant model described by $G(s) = \frac{s-k}{(s-1)(s+0.3)}$. It has a nonminimum phase zero at $s = k$, and the unstable pole is located at $p = 1$. The LTI filters are used to model the finite bandwidth $F(s)$ of the communication, and the filters are chosen to be low-pass Butterworth with different values of 10, 30, 100

$$F_1 = \frac{10}{p+10}, \quad F_2 = \frac{30}{p+30}, \quad F_3 = \frac{100}{p+100}.$$

If $q = 0.5$, from Theorem 3.1, we can calculate the J^*

$$J^* = 2k + 2\left(\frac{2}{F}\frac{k+1}{k-1}\right)^2$$

The optimal tracking performance of the NCSs with different unstable poles is shown in Figure 2. It can be seen from Figure 2 that the optimal tracking performance has been degraded when the bandwidth decreases. It can also be seen that the tracking performance tends to be infinity when the unstable poles move closer to the non-minimum

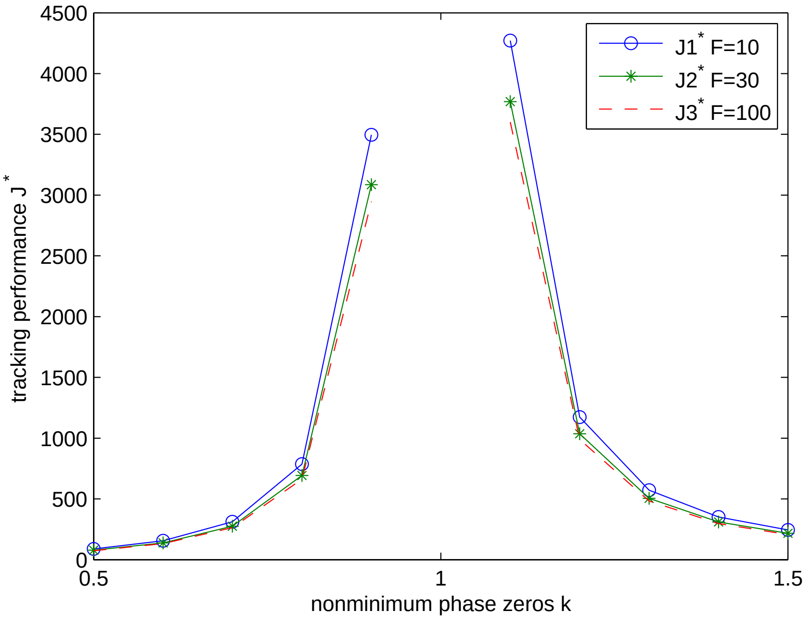

Figure 2. Optimal performance of NCSs with bandwidth

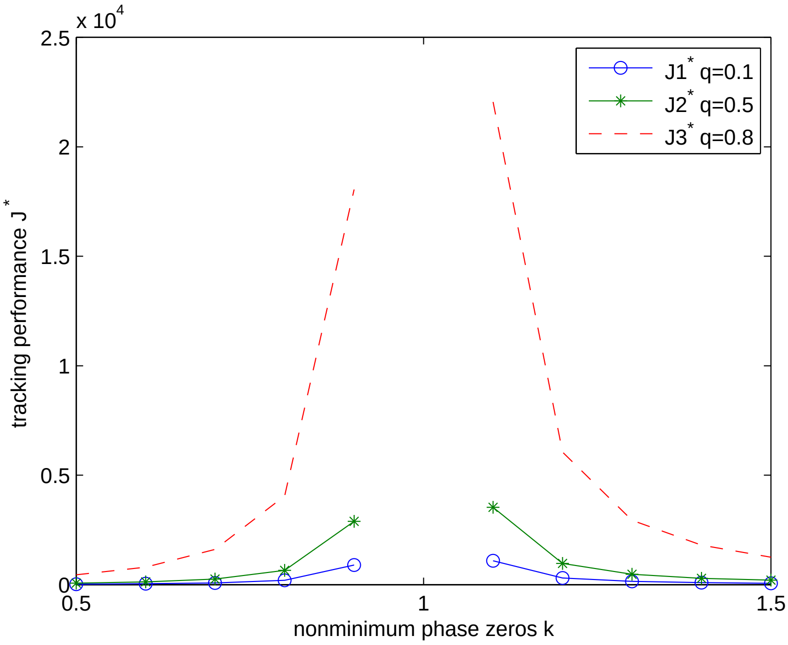

Figure 3. Optimal performance of NCSs with packet dropout ratio

phase (NMP) zeros. It is clear that the optimal tracking performance of the NCSs will be seriously degraded when the bandwidth F decreases.

If $F = 1$, the packet dropout ratio q for three different values of 0.1, 0.5, 0.8, from Theorem 3.1, we can calculate the J^*

$$J^* = 2k + 2\left(\frac{1}{1-q}\frac{k+1}{k-1}\right)^2$$

The optimal tracking performance of the NCSs with different packet dropout ratio q is shown in Figure 3.

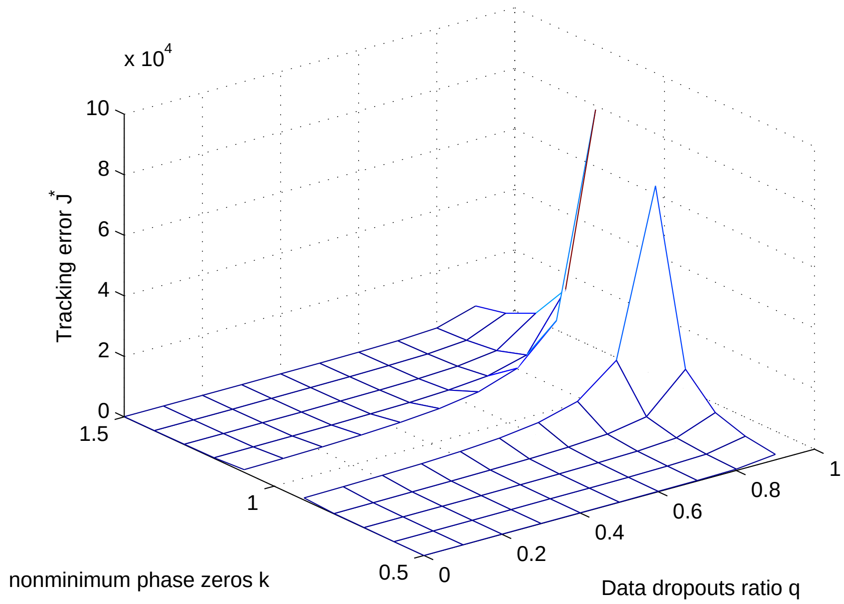

FIGURE 4. Optimal performance of NCSs with packet dropout and unstable poles

The same situation can be observed from Figure 3, which shows the optimal performance for different values of q. It is clear that the optimal tracking performance will increase with the increase of q.

If $F = 1$, and q is unknown value, from Theorem 3.1, we can calculate the J^*

$$J^* = 2k + 2\left(\frac{1}{1-q}\frac{k+1}{k-1}\right)^2$$

In the influence of different packet dropout ratio q and unstable poles p, we can get the optimal tracking performance of NCSs like Figure 4.

The optimal tracking performance of NCSs with different packet dropout ratios and unstable poles are shown in Figure 4. It can be seen from Figure 4 that the optimal tracking performance has been degraded when the packet dropout increases. It can also be seen that the tracking performance tends to be infinity when the unstable poles move closer to the non-minimum phase (NMP) zeros.

5. **Conclusion.** In this paper, the optimal tracking performance of NCSs based on packet dropouts and bandwidth constraints is studied. The packet dropout and bandwidth limitations that exist in the feedback channel are considered. An explicit expression for the optimal performance of NCSs is obtained by using a method of spectral factorization technique. The obtained results show that the optimal performance of NCSs is influenced by unstable poles, packet dropouts and bandwidth constraints. A typical example is given to illustrate the theoretical results.

In this paper, we studied with the SISO NCSs with data packet dropout and bandwidth constraints, and possible future extensions to this work include study on MIMO, time delay and these problem.

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