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EVENT-TRIGGERED H_∞ CONTROL FOR MARKOVIAN JUMP SYSTEMS WITH QUANTIZATIONS

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Abstract. *The problem of event-triggered H_∞ control for Markovian jump systems with quantizations is presented in this paper. Both state and control input quantizations are considered. For different Markovian jumping modes, a dynamic discrete event-triggered communication scheme is presented to detect whether the latest sampled data should be triggered. Time-delay system analysis method is used to co-design the H_∞ controller and the event-triggered conditions. A numerical example is also given to show the effectiveness of the proposed method and the event-triggered control's capability of reducing the communication load.*
Keywords: Event-triggered, H_∞ control, Markovian jump systems, Quantization

1. **Introduction.** Due to its capacity of reducing signal transmission in networks, event-triggered control has received significant attention in recent years [1, 2]. In event-triggered sampling scheme, the necessary sampling is determined by the occurrence of an "event" rather than "time". So compared with traditional time-based sampling, event-based sampling can reduce the release times of the sensor and improve the limited network resources availability. However, in NCSs, the limited bandwidth leads to some challenges, such as network-induced delay, data dropout, and data disordering [3, 4], which may deteriorate the system's performance. To overcome this problem, quantization in control systems has become an active topic.

On the other hand, due to its powerful capacity of capturing the abrupt mode changes for the plant, Markovian jump system has received much attention [5, 6]. Event-based control for Markovian jump systems has been discussed in [7]. However, the effect of quantization in the context of event-triggered control has not been fully investigated. To the best of the authors' knowledge, there is no result reported in the open literature on the event-based H_∞ control for Markovian jump systems with quantizations. The theoretical results for such systems would be appealing and have wide practical use, and this motivates the research presented in this paper.

In this paper, the event-triggered quantized H_∞ control problem for Markovian jump systems is investigated. Dynamic discrete event-triggered scheme is proposed to detect whether to trigger the latest sampled-data for different Markovian jump modes. The effects of network-induced delays, state and control input quantizations, and event-triggered schemes are unified into an innovative delay system. By employing time-delay system method, H_∞ performance criterion is derived and the co-design method of the event-triggered condition and the H_∞ controller is also given. A numerical example is given to show the effectiveness of the proposed method.

The remaining of the paper is organized as follows. Section 2 formulates the problem under consideration. H_∞ control performance analysis and the co-design method of event-triggered condition and the controller are presented in Section 3. Illustrative examples are given in Section 4, and the paper is concluded in Section 5.

2. Problem Statement and Preliminaries.

2.1. **System description.** The framework of event-triggered control considered in this paper is shown in Figure 1. The plant is assumed to be described by the following Markovian jump system:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) + B_w(r(t))\omega(t) \\ z(t) = C(r(t))x(t) + D(r(t))u(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input and $z(t) \in \mathbb{R}^p$ is the controlled output, $\omega(t) \in \mathbb{R}^q$ is the external disturbance with $\omega(t) \in \mathcal{L}_2[0,\infty)$. Matrices $A(r(t))$, $B(r(t))$, $C(r(t))$, $B_\omega(r(t))$ and $D(r(t))$ are known real constant matrices with appropriate dimensions. $r(t)$ is a homogeneous finite-state Markov jump process with right continuous trajectories and taking discrete values in a given finite set $\wp = \{1, 2, \ldots, r\}$ with transition probability matrix $\Pi = (\lambda_{ij})$ $(i, j \in \wp)$ given by

$$Pr\left\{r(t+\Delta t) = j | r(t) = i\right\} = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t), & i \neq j \\ 1 + \lambda_{ii}\Delta t + o(\Delta t), & i = j \end{cases}$$

where $\lim\limits_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$, $\lambda_{ij} \geq 0$ $(i \neq j)$, and $\lambda_{ii} = -\sum\limits_{j=1, j\neq i}^{N} \lambda_{ij}$.

For the Markovian jump system shown in Figure 1, the following conditions are assumed.

1) The signal in the network is transmitted with a single packet and the data packet loss does not occur during the transmission.

2) Two quantizers ($f(\cdot)$ and $g(\cdot)$) are used and they are both logarithmic.

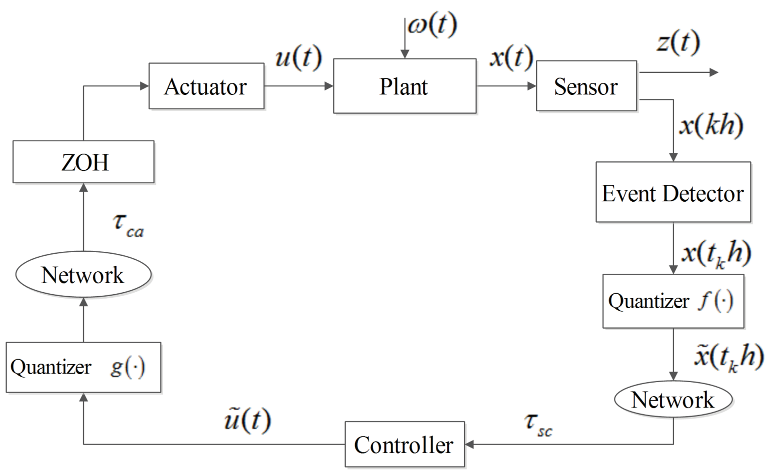

Figure 1. Framework of event-triggered control

2.2. **Event-triggered scheme.** The event detector is used to determine whether the newly sampled data should be sent out to the quantizer $f(\cdot)$ by using the following threshold condition:

$$[x(kh) - x(t_kh)]^T \Phi(r(kh)) [x(t_kh) - x(t_kh)] \geq \delta(r(kh))x^T(kh)\Phi(r(kh))x(kh) \quad (2)$$

where h is the sampling period, $x(kh)$, $(k = 0, 1, 2, \cdots)$ is current sampled state, $x(t_kh)$ is the latest transmitted data, $\delta(r(kh)) \in [0, 1)$ is a given scalar parameter, $\Phi(r(kh)) > 0$ is the event-triggered matrix to be designed. If the sampling data meets the event-triggered threshold condition (2), the data will be stored and sent to the quantizer $f(\cdot)$ at the same time.

When the data transmitted by the event monitor is forwarded to the controller, it incurs a communication delay, called the sensor-to-controller delay $\tau_{sc}(t_k)$. Similarly, the controller forwarding the actuation signals to the actuator incurs another communication delay, called the controller-to-actuator delay $\tau_{ca}(t_k)$. The total network-induced delay can be lumped together as the time-varying delay τ_{t_k}, and

$$\tau_{t_k} = \tau_{sc}(t_k) + \tau_{ca}(t_k), \quad 0 \leq \tau_m \leq \tau_{t_k} \leq \bar{\tau} \quad (3)$$

where τ_m and $\bar{\tau}$ denote the lower and upper delay bounds, respectively.

2.3. **Event-triggered quantized H_∞ control problem.** The problem of event-triggered H_∞ control with quantizations to be addressed in this paper is to design the state feedback controller

$$u(t) = K(r(t))x(t) \quad (4)$$

where $K(r(t))$ is the controller gain, such that

1) the resultant closed-loop system with $w(t) = 0$ is asymptotically stable, and

2) under zero initial conditions, for any nonzero $w(t) \in \mathcal{L}_2[0, \infty)$, the controlled output $z(t)$ satisfies $\|z(t)\|_2 \leq \gamma\|w(t)\|_2$, where γ is a prescribed performance index.

Considering the behavior of the ZOH, the input signal is

$$u(t) = g(Kf(x(t_kh))), \quad t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \quad (5)$$

By Figure 1, we now denote the quantized measurement of $x(t_kh)$ as $\tilde{x}(t_kh)$, and the control signal as $\tilde{u}(t)$ and the control input signal as $u(t)$. Then, at the release instant t_kh, we have: $\tilde{x}(t_kh) = f(x(t_kh))$, $\tilde{u}(t_kh+\tau_{sc}(t_k)) = K\tilde{x}(t_kh)$, and $u(t_kh+\tau_{t_k}) = g(\tilde{u}(t_kh+\tau_{sc}(t_k)))$. The quantizers $f(\cdot) = [f_1(\cdot), f_2(\cdot), \cdots, f_n(\cdot)]^T$ and $g(\cdot) = [g_1(\cdot), g_2(\cdot), \cdots, g_p(\cdot)]^T$ are assumed to be symmetric, that is, $f_j(-v) = -f_j(v)$ $(j = 1, 2, \cdots, n)$ and $g_m(-v) = -g_m(v)$ $(m = 1, 2, \cdots, p)$. Similar to [8, 9], the quantizers considered in this paper are logarithmic static and time-invariant. For each $f(\cdot)$, the set of quantized levels is described as in [9, 10] by:

$$\mathscr{U} = \left\{\pm u_i^{(j)}, u_i^{(j)} = \alpha_j^i u_0^{(j)}, i = \pm 1, \pm 2, \cdots\right\} \cup \left\{\pm u_0^{(j)}\right\} \cup \{0\},\ 0 < \alpha_j < 1,\ u_0^{(j)} > 0. \quad (6)$$

The associated quantizer $f_j(\cdot)$ is defined as

$$f_j(v) = \begin{cases} u_i^{(j)} & \text{if } \frac{1}{1+\sigma_{f_j}}u_i^{(j)} < v \leq \frac{1}{1-\sigma_{f_j}}u_i^{(j)},\ v > 0, \\ 0 & \text{if } v = 0, \\ -f_j(-v) & \text{if } v < 0, \end{cases}$$

where $\sigma_{f_j} = \frac{1-\alpha_j}{1+\alpha_j}$, and α_j is also called the quantization density of quantizer $f_j(\cdot)$. Similarly, the quantizer $g_j(\cdot)$ $(j = 1, 2, \ldots, p)$ is of quantization densities ρ_j and denote $\sigma_{g_j} = \frac{1-\rho_j}{1+\rho_j}$. For given logarithmic quantizer $f(\cdot)$, a sector bound condition was proposed as: $f(x) = (I + \Delta_f)x$, where $\Delta_f = \mathrm{diag}\{\Delta_{f_1}, \Delta_{f_2}, \cdots, \Delta_{f_n}\}$, and $\Delta_{f_n} \in [-\sigma_j, \sigma_j]$. For the quantizer on the controller side, the same definition can be applied and we have:

$g(\tilde{u}) = (I + \Delta_g)\tilde{u}$, where $\Delta_g = \mathrm{diag}\{\Delta_{g_1}, \Delta_{g_2}, \cdots, \Delta_{g_p}\}$ and $\Delta_{g_p} \in [-\pi_j, \pi_j]$. For simplicity, it is assumed that $\sigma_{f_j} = \sigma_f$ and $\sigma_{g_j} = \sigma_g$, where σ_f and σ_g are two constants. Then, we have

$$u(t_k h + \tau_{t_k}) = (I + \Delta_g)K(r(t))(I + \Delta_f)x(t_k h) = [K(r(t)) + \Delta K(r(t))]x(t_k h), \tag{7}$$

where $\Delta K(r(t)) = \Delta_g K(r(t)) + K(r(t))\Delta_f + \Delta_g K(r(t))\Delta_f$, and $t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$.

In order to facilitate the process of analysis, we convert the Markovian jump system (1) under event-triggered scheme (2) into a new time-delay system using the same technique as in [9]. Suppose there exists a finite positive integer q such that $t_{k+1} = t_k + q + 1$. The time-delay interval can be divided into the following $q+1$ subintervals: $[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_k+1}) = \bigcup_{n=0}^{q} \Omega_n$ with $\Omega_n = [t_k h + nh + \tau_{t_k+n}, t_k h + (n+1)h + \tau_{t_k+n+1})$. Define $\tau(t) = t - t_k h - nh$, and $e_k(t) = x(t_k h + nh) - x(t_k h)$, $t \in \Omega_n$. Then, it can be easily seen that

$$\begin{cases} 0 \le \tau_m \le \tau(t) \le h + \bar{\tau} = \tau_M \\ e_k^T(t)\Phi(r(t))e_k(t) \le \sigma(r(t))x^T(t - \tau(t))\Phi(r(t))x(t - \tau(t)) \end{cases} \tag{8}$$

Substituting (7) and (8) into (1) leads to the following closed-loop system:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))[K(r(t)) + \Delta(K(r(t)))][x(t - \tau(t)) - e_k(t)] + B_w(r(t)\omega(t) \\ z(t) = C(r(t))x(t) + D(r(t))[K(r(t)) + \Delta(K(r(t)))][x(t - \tau(t)) - e_k(t)] \end{cases} \tag{9}$$

where $x(t) = \phi(t)$, $t \in [t_0 - \tau_M, t_0]$, $\phi(t)$ is initial function of $x(t)$.

For notational simplicity, in this paper, when $r(t) = i$, $i \in \wp$, a matrix $M(r(t))$ will be denoted by M_i; for example, $A(r(t))$ is denoted by A_i, and $B(r(t))$ by B_i.

3. **Main Results.**

3.1. $\boldsymbol{H_\infty}$ **performance analysis.**

We first consider the H_∞ performance analysis of the Markovian jump system (9) under the event-triggered scheme (2) and quantizations (7).

Theorem 3.1. *For given scalars $\gamma > 0$, $0 \le \delta_i < 1$, τ_M, τ_m, and the feedback gain K_i, system (9) is asymptotically stable with an H_∞ performance index γ, if there exist real matrices $P_i > 0$, $Q_{li} > 0$, $\Phi_i > 0$ $(i \in \wp)$, $Q_l > 0$, $R_l > 0$ $(l = 1, 2)$, matrices M, N, and S_k $(k = 1, 2, 3, 4)$ with appropriate dimensions such that*

$$\begin{bmatrix} \Pi_{11} + \Gamma_1 + \Gamma_1^T & \Gamma_2 & \Gamma_3 & \Pi_{14}(l) \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\kappa R_2 \end{bmatrix} < 0, \quad \text{and} \quad \sum_{j=1}^{N} \lambda_{ij} Q_{lj} \le Q_l, \tag{10}$$

where

$$\Gamma_1 = \begin{bmatrix} 0 & N & M & -N & 0 & 0 \end{bmatrix}, \ \Gamma_2 = \begin{bmatrix} B_{wi}^T S_1^T & 0 & B_{wi}^T S_2^T & 0 & B_{wi}^T S_3^T & B_{wi}^T S_4^T \end{bmatrix}^T$$

$$\Gamma_3 = \begin{bmatrix} C_i & 0 & D_i\Theta_i & 0 & -D_i\Theta_i & 0 \end{bmatrix}^T, \ \kappa = \tau_M - \tau_m, \ \Theta_i = K_i + \Delta K_i,$$

$$\Pi_{14}(1) = \kappa M, \ \Pi_{14}(2) = \kappa N, \ \Pi_{11} = \begin{bmatrix} (1,1) & (1,2) \\ * & (2,2) \end{bmatrix}, \ (1,1) = \begin{bmatrix} \Lambda_1 & R_1 & \Lambda_2 \\ * & -Q_{1i} - R_1 & 0 \\ * & * & \Lambda_4 \end{bmatrix},$$

$$(1,2) = \begin{bmatrix} 0 & \Lambda_3 & P_i - S_1 + A_i^T S_4^T \\ 0 & 0 & 0 \\ 0 & \Lambda_5 & -S_2 + \Theta_i^T B_i^T S_4^T \end{bmatrix}, \ (2,2) = \begin{bmatrix} -Q_{2i} & 0 & 0 \\ * & \Lambda_6 & -S_3 + \Theta_i^T B_i^T S_4^T \\ * & * & \Lambda_7 \end{bmatrix}$$

$$\Lambda_1 = \sum_{j=1}^{N} \lambda_{ij} P_j + Q_{1i} + Q_{2i} + \tau_m Q_1 + \tau_M Q_2 - R_1 + S_1 A_i + A_i^T S_1^T$$

$$\Lambda_2 = S_1 B_i \Theta_i + A_i^T S_2^T, \ \Lambda_3 = -S_1 B_i \Theta_i + A_i^T S_4^T, \ \Lambda_4 = \delta_i \Phi_i - S_2 B_i \Theta_i + \Theta_i^T B_i^T S_2^T$$

$$\Lambda_5 = -S_2 B_i \Theta_i + \Theta_i^T B_i^T S_3^T, \ \Lambda_6 = -\Phi_i - S_3 B_i \Theta_i - \Theta_i^T B_i^T S_3^T,$$
$$\Lambda_7 = \tau_m^2 R_1 + \kappa R_2 - S_4 - S_4^T.$$

3.2. **Event-triggered H_∞ controller design.** Based on Theorem 3.1, we have the following event-triggered H_∞ controller design method.

Theorem 3.2. *For given $\gamma > 0$, τ_m, σ_f, σ_g, δ_i and ρ_j, if there exist positive definite symmetric matrices $\bar{P}_i$, $\bar{Q}_{li}$, $\bar{\Phi}_i$, $(i \in \wp)$, $\bar{Q}_l$, $\bar{R}_l$, $(l = 1, 2)$ and matrices $\bar{N}_k$, $\bar{M}_k$ $(k = 1, \ldots, 6)$, X, Y_i and scalar $\epsilon_j > 0$ $(j = 1, 2, 3)$, such that*

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) \\ * & (2,2) & (2,3) \\ * & * & (3,3) \end{bmatrix} < 0, \ \text{and} \ \begin{bmatrix} \lambda_{ii}\bar{Q}_{li} & \alpha_i \\ * & \beta_i \end{bmatrix} < \bar{Q}_l \tag{11}$$

where

$$(1,1) = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & -\bar{M}_1 & \Upsilon_{15} & \Upsilon_{16} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & \bar{N}_5^T & \bar{N}_6^T \\ * & * & \Upsilon_{33} & \Upsilon_{34} & \Upsilon_{35} & \Upsilon_{36} \\ * & * & * & \Upsilon_{44} & -\bar{M}_5^T & -\bar{M}_6^T \\ * & * & * & * & \Upsilon_{55} & \Upsilon_{56} \\ * & * & * & * & * & \Upsilon_{66} \end{bmatrix},$$

$$(2,2) = \begin{bmatrix} -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & -I & 0 & \epsilon_1\sigma_g D_i & 0 \\ * & * & -\kappa\bar{R}_2 & 0 & 0 \\ * & * & * & -\epsilon_1 I & 0 \\ * & * & * & * & -\epsilon_1 I \end{bmatrix}$$

$$(1,2) = \begin{bmatrix} B_{wi}^T & XC_i^T & \Upsilon_{19}(l) & \epsilon_1\sigma_g B_i & 0 \\ 0 & 0 & \Upsilon_{29}(l) & 0 & 0 \\ \rho_1 B_{wi} & Y_i^T D_i^T & \Upsilon_{39}(l) & \epsilon_1\sigma_g\rho_1 B_i & Y_i^T \\ 0 & 0 & \Upsilon_{49}(l) & 0 & 0 \\ \rho_2 B_{wi} & -Y_i^T D_i^T & \Upsilon_{59}(l) & \epsilon_1\sigma_g\rho_2 B_i & -Y_i^T \\ \rho_3 B_{wi} & 0 & \Upsilon_{69}(l) & \epsilon_1\sigma_g\rho_3 B_i & 0 \end{bmatrix}, \ (2,3) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \epsilon_2 D_i & 0 & \epsilon_3\sigma_g D_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(1,3) = \begin{bmatrix} \epsilon_2 B_i & 0 & \epsilon_3\sigma_g B_i & 0 & \vartheta_i \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon_2\rho_1 B_i & \delta_f Y_i^T & \epsilon_3\rho_1\delta_g B_i & \delta_f Y_i^T & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon_2\rho_2 B_i & -\delta_f Y_i^T & \epsilon_3\rho_2\delta_g B_i & -\delta_f Y_i^T & 0 \\ \epsilon_2\rho_3 B_i & 0 & \epsilon_3\rho_3\sigma_g B_i & 0 & 0 \end{bmatrix},$$

$$(3,3) = \begin{bmatrix} -\epsilon_2 I & 0 & 0 & 0 & 0 \\ * & -\epsilon_2 I & 0 & 0 & 0 \\ * & * & -\epsilon_3 I & 0 & 0 \\ * & * & * & -\epsilon_3 I & 0 \\ * & * & * & * & \nu_i \end{bmatrix}$$

$$\Upsilon_{11} = \lambda_{ii}\bar{P}_i + \bar{Q}_{1i} + \bar{Q}_{2i} + \tau_m\bar{Q}_1 + \tau_M\bar{Q}_2 - \bar{R}_1 + A_i X^T + XA_i^T, \ \Upsilon_{12} = \bar{R}_1 + \bar{N}_1,$$
$$\Upsilon_{13} = B_i Y_i + \rho_1 XA_i^T - \bar{N}_1 + \bar{M}_1, \ \Upsilon_{15} = -B_i Y_i + \rho_2 XA_i^T, \ \Upsilon_{16} = \bar{P}_i - X^T + \rho_3 XA_i^T,$$
$$\Upsilon_{22} = -\bar{Q}_{1i} - \bar{R}_1 + \bar{N}_2 + \bar{N}_2^T, \ \Upsilon_{23} = -\bar{N}_2 + \bar{N}_3^T + \bar{M}_2, \ \Upsilon_{24} = \bar{N}_4^T - \bar{M}_2$$
$$\Upsilon_{33} = \delta_i\bar{\Phi}_i + \rho_1 B_i Y_i + \rho_1 Y_i^T B_i^T - \bar{N}_3 + \bar{N}_3^T + \bar{M}_3 + \bar{M}_3^T, \ \Upsilon_{34} = -\bar{N}_4^T + \bar{M}_4^T$$

$$\Upsilon_{35} = -\rho_1 B_i Y_i + \rho_2 Y_i^T B_i^T - \bar{N}_5^T + \bar{M}_5^T, \ \Upsilon_{36} = -\rho_1 x^T + \rho_3 Y_i^T B_i^T - \bar{N}_6^T + \bar{M}_6^T$$
$$\Upsilon_{44} = -\bar{Q}_{2i} - \bar{M}_4 - \bar{M}_4^T, \ \Upsilon_{55} = -\bar{\Phi}_i - \rho_2 B_i Y_i - \rho_2 Y_i^T B_i^T,$$
$$\Upsilon_{56} = -\rho_2 x^T - \rho_3 Y_i^T B_i^T, \ \Upsilon_{66} = \tau_m \bar{R}_1 + \kappa \bar{R}_2 - \rho_3 X^T - \rho_3 X$$
$$\Upsilon_{19}(1) = \kappa \bar{M}_1, \ \Upsilon_{19}(2) = \kappa \bar{N}_1, \ \Upsilon_{29}(1) = \kappa \bar{M}_2, \ \Upsilon_{29}(2) = \kappa \bar{N}_2,$$
$$\Upsilon_{39}(1) = \kappa \bar{M}_3, \ \Upsilon_{39}(2) = \kappa \bar{N}_3, \ \Upsilon_{49}(1) = \kappa \bar{M}_4, \ \Upsilon_{49}(2) = \kappa \bar{N}_4,$$
$$\Upsilon_{59}(1) = \kappa \bar{M}_5, \ \Upsilon_{59}(2) = \kappa \bar{N}_5, \ \Upsilon_{69}(1) = \kappa \bar{M}_6, \ \Upsilon_{69}(2) = \kappa \bar{N}_6$$
$$\vartheta_i = \left[\sqrt{\lambda_{i1}}\bar{P}_1, \cdots, \sqrt{\lambda_{i,i-1}}\bar{P}_{i-1}, \sqrt{\lambda_{i,i+1}}\bar{P}_{i+1}, \cdots, \sqrt{\lambda_{iN}}\bar{P}_N\right]$$
$$\nu_i = diag\left\{-\bar{P}_1, \cdots, -\bar{P}_{i-1}, -\bar{P}_{i+1}, \cdots, -\bar{P}_N\right\}$$
$$\alpha_i = \left[\sqrt{\lambda_{i1}}\bar{Q}_{l1}, \cdots, \sqrt{\lambda_{i,i-1}}\bar{Q}_{l,i-1}, \sqrt{\lambda_{i,i+1}}\bar{Q}_{l,i+1}, \cdots, \sqrt{\lambda_{iN}}\bar{Q}_{lN}\right]$$
$$\beta_i = diag\left\{-\bar{Q}_{l1}, \cdots, -\bar{Q}_{l,i-1}, -\bar{Q}_{l,i+1}, \cdots, -\bar{Q}_{lN}\right\}$$

If the above conditions are feasible, the feedback gain matrix of the controller is given by

$$K_i = Y_i X^{-1}. \tag{12}$$

4. **Numerical Example.** The following numerical example is presented to illustrate the effectiveness of the proposed co-designed method.

Example 4.1. *Consider the Markovian jump system* (1) *with two operating modes* ($i \in \wp = \{1, 2\}$) *to capture the abrupt changes. The system parameters are described as follows:*

Mode 1:

$$A_1 = \begin{bmatrix} -6 & 0.7 \\ 0 & -3 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \ C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \ D_1 = 0.5, \ B_{\omega 1} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

Mode 2:

$$A_2 = \begin{bmatrix} -1 & 1 \\ 0.8 & -4 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1 & 0.6 \end{bmatrix}, \ D_2 = 0.1, \ B_{\omega 2} = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}.$$

We suppose that the sampling period $h = 0.1$s and the transition probability matrix is $\Pi = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}$. The initial condition $x_0 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T$ and the external disturbance is

$$\omega(t) = \begin{cases} 1, \ 0 \le t \le 10 \\ 0, \ t > 10. \end{cases}$$

We assume the quantization index $\sigma_f = \sigma_g = \sigma$. Letting $\delta_1 \neq \delta_2$, we consider that δ_1 varies, δ_2 is constant. When $\gamma = 2$, $\tau_m = 0$, $\delta_1 = 0.15$, $\delta_2 = 0.1$, Table 1 shows the maximum allowable delay τ_M for different σ. From Table 1, it is also clear that quantization degrades the transmission delay.

TABLE 1. τ_M for different σ when $\delta_1 = 0.15$, $\delta_2 = 0.1$

σ	0	0.1
τ_M	1.1723	1.1053

When $\delta_1 = 0.15$, $\delta_2 = 0.1$, $\gamma = 2$, $\sigma = 0.1$, $\tau_m = 0$. According to Theorem 3.2, we have $\tau_M = 1.1053$, the feedback controller $K_1 = \begin{bmatrix} 0.0218 & 0.0051 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.0400 & -0.0263 \end{bmatrix}$ and the event-triggered matrices $\Phi_1 = \begin{bmatrix} 0.1029 & -0.0003 \\ -0.0003 & 0.0595 \end{bmatrix}$, $\Phi_2 = \begin{bmatrix} 0.1029 & 0.0033 \\ 0.0033 & 0.0644 \end{bmatrix}$.

Giving a possible system mode' evolution as in Figure 2, the event-triggering release instants and intervals are shown in Figure 3, and the state responses of close-loop response are depicted in Figure 4. In the simulation times 30s, only 54 sample data are transmitted to the controller.

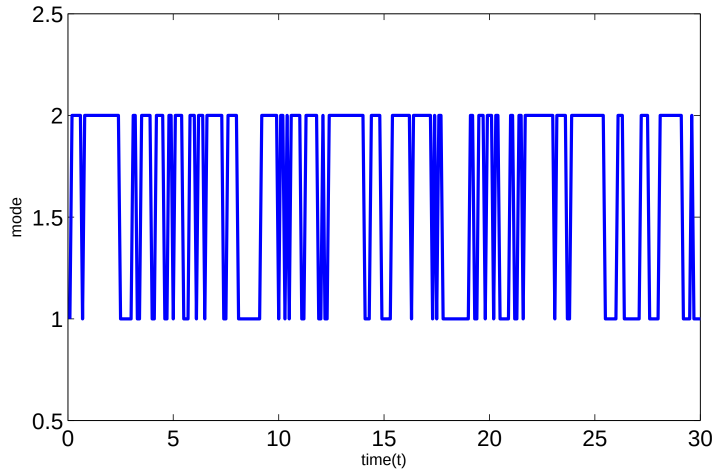

FIGURE 2. The mode of transition

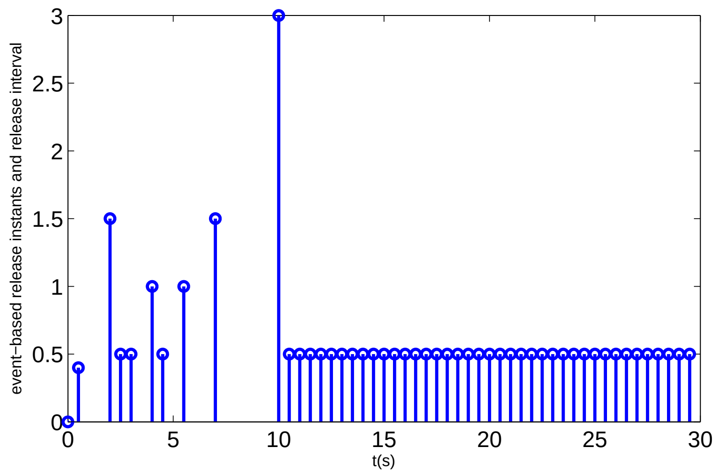

FIGURE 3. Release instants and release interval

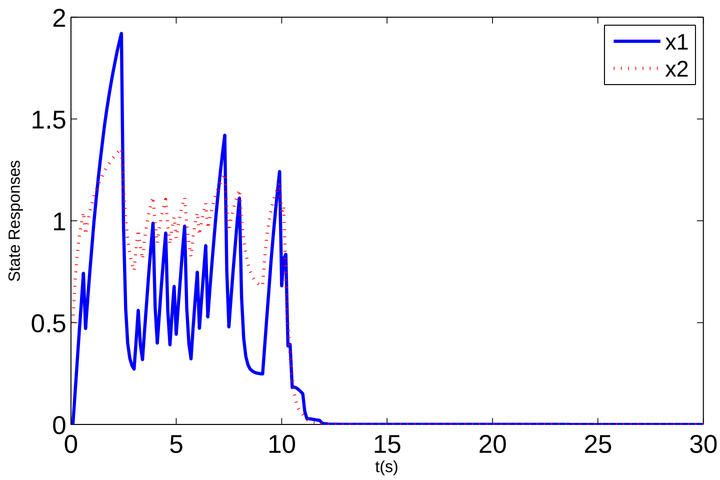

FIGURE 4. The state response $x(t)$ with $\sigma_i = 0.15$

Note that if using time-triggered scheme for this example, there are 300 times transmitted. So our event-triggered scheme triggered times are much less than the time-triggered communication scheme. The results indicate that our method can effectively mitigate the unnecessary waste of computation and communication resource.

5. **Conclusions.** The problem of event-triggered H_∞ control for Markovian jump systems with both state and control input quantizations is discussed in this paper. Considering the jumping character of Markovian jump system, a dynamic discrete event-triggered scheme is presented to determine when the sampled signals should be transmitted. Based on the analysis of network-induced delay intervals, a unified Markovian jump system with time-delay is constructed to describe the event-triggered scheme, network-induced delays, quantizations, and the Markovian jump system together. The criteria of stochastic stability with an H_∞ norm bound are obtained for this Markovian jump system with appropriate event-triggered parameters. The H_∞ controller and the event-triggered conditions are co-designed in the forms of linear matrix inequalities. The numerical example shows that the co-design method is effective and event-based communication can reduce the use of limited network resources greatly.

Considering unreliable communication networks and employing distributed event-triggered scheme for continuous or discrete Markovian jump systems are possible aims of future work.

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