

## ADAPTIVE NEURAL NETWORKS OUTPUT-FEEDBACK CONTROL FOR INDUCTION MOTORS IN ELECTRIC VEHICLE DRIVE SYSTEMS

CHENG FU, YUMEI MA, JINPENG YU\*, HAISHENG YU AND YAQIANG HU

College of Automation and Electrical Engineering  
Qingdao University

No. 308, Ningxia Road, Qingdao 266071, P. R. China

\*Corresponding author: yjp1109@hotmail.com

Received October 2016; accepted January 2017

**ABSTRACT.** *This paper focuses on the problem of position tracking control of induction motors (IMs) with parameter uncertainties in electric vehicle drive systems. An adaptive neural networks output-feedback control method is proposed to deal with this problem. Neural networks are utilized to approximate the nonlinearities and adaptive backstepping technique is used to construct controllers. By first-order filters, the dynamic surface solves the trouble of “explosion of complexity” problem of classical backstepping. The algorithm in this paper ensures that all the closed-loop signals are bounded. Finally, the simulation results illustrate the effectiveness of the proposed control algorithm.*

**Keywords:** Induction motors, Output-feedback, Neural networks, Backstepping

1. **Introduction.** Induction motors (IMs) have been increasingly applied in electric vehicles because of their simple and robust construction, low cost, and high reliability. However, the control of IMs is complex due to highly nonlinear, multivariable dynamic model. Hence, many control techniques have been developed to control IMs, such as sliding mode control [1], backstepping control [2] and other control methods [3]. Backstepping is considered to be a powerful tool for design of controllers for nonlinear systems. However, the problem of “certain functions must be linear” and “explosion of complexity” limit the scope of the classical backstepping’s application. In another research front line, adaptive NNs control approaches give a system methodology of solving the first problem lying in the adaptive backstepping method in which NNs are utilized to approximate the uncertain nonlinear functions [4]. However, the proposed NNs controllers combining the backstepping technology have not taken account of the case of nonlinear systems with unknown states. To cope with this, the NNs based output-feedback was applied in induction motors [5,6] to estimate some unknown state variables which cannot be measured directly. However, the “explosion of complexity” problem for the adaptive backstepping cannot be solved by the above approximation-based adaptive output-feedback control.

In order to solve the above problems, an output-feedback and adaptive neural networks (NNs) based dynamic surface control method is proposed for induction motors in electric vehicle drive systems. The dynamic surface control (DSC) technique is proposed to deal with “explosion of complexity” problem inherent in conventional backstepping design, in which differentiations are replaced by low-pass filters and virtual control of each step is composed using filtered signal. Moreover, the output-feedback is utilized to estimate the angle speed. In addition, the NNs are exploited to approximate the unknown nonlinear functions. The proposed NNs control scheme not only guarantees the boundedness of all of the signals in the closed-loop system, but also reduces the number of adaptive parameters which alleviates the computational burden. Finally, simulation results illustrate the effectiveness of the proposed approach.

The rest of the paper is organized as follows. Section 2 describes the mathematical model of the position drive system for induction motors. The adaptive neural networks output-feedback controllers are designed in Sections 3 and 4. Section 5 presents simulation studies. Finally, Section 6 draws some conclusions.

**2. Mathematical Model of Induction Motors Drive Systems.** The dynamic model of IMs with iron losses based on the  $d$ - $q$  axis is displayed as follows:

$$\begin{aligned}\frac{d\Theta}{dt} &= \omega_r, & \frac{d\omega_r}{dt} &= \frac{n_p L_m}{L_{1r} J} \psi_d i_{qm} - \frac{T_L}{J} \\ \frac{di_{qm}}{dt} &= \frac{R_{fe}}{L_m} i_{qs} - \frac{(L_m + L_{1r}) R_{fe}}{L_{1r} L_m} i_{qm} + i_{dm} \omega_r + \frac{L_m R_r}{L_{1r}} \frac{i_{qm} i_{dm}}{\psi_d} \\ \frac{di_{qs}}{dt} &= -\frac{R_s + R_{fe}}{L_{1s}} i_{qs} + \frac{L_m R_r}{L_{1r}} \frac{i_{ds} i_{qm}}{\psi_d} + i_{ds} \omega_r + \frac{(L_m + L_{1r}) R_{fe}}{L_{1r} L_{1s}} i_{qm} + \frac{1}{L_{1s}} u_{qs} \\ \frac{d\psi_d}{dt} &= -\frac{R_r}{L_{1r}} \psi_d + \frac{L_m}{L_{1r}} R_r i_{dm} \\ \frac{di_{dm}}{dt} &= \frac{R_{fe}}{L_m} i_{ds} + \frac{R_{fe}}{L_{1r} L_m} \psi_d - \frac{(L_m + L_{1r}) R_{fe}}{L_{1r} L_m} i_{dm} + \frac{L_m R_r}{L_{1r}} \frac{i_{qm}^2}{\psi_d} + i_{qm} \omega_r \\ \frac{di_{ds}}{dt} &= -\frac{R_s + R_{fe}}{L_{1s}} i_{ds} + \frac{L_m R_r}{L_{1r}} \frac{i_{qs} i_{qm}}{\psi_d} + i_{qs} \omega_r + \frac{(L_m + L_{1r}) R_{fe}}{L_{1r} L_{1s}} i_{dm} - \frac{R_{fe}}{L_{1s}^2} \psi_d + \frac{1}{L_{1s}} u_{ds}\end{aligned}$$

The dynamic model of IMs with iron losses has been simplified in [7] as follows:

$$\begin{aligned}\dot{x}_1 &= x_2, & \dot{x}_2 &= \frac{1}{J} a_1 x_3 x_5 - \frac{T_L}{J}, & \dot{x}_3 &= b_1 x_4 - b_2 x_3 + b_3 \frac{x_3 x_6}{x_5} + x_2 x_6 \\ \dot{x}_4 &= c_1 u_{qs} - c_2 x_4 + x_2 x_7 + c_3 \frac{x_3 x_7}{x_5} + c_4 x_3, & \dot{x}_5 &= d_1 x_5 + d_2 x_6 \\ \dot{x}_6 &= \bar{e}_1 x_7 + \bar{e}_2 c x_5 - \bar{e}_3 x_6 + \bar{e}_4 \frac{x_3^2}{x_5} + x_2 x_3 \\ \dot{x}_7 &= g_1 u_{ds} - g_2 x_7 + g_3 \frac{x_3 x_4}{x_5} + x_2 x_4 - g_4 x_5 + g_5 x_6.\end{aligned}\tag{1}$$

**3. The NNs Output-Feedback State Observer Design.** In this section, since the state variables are not available, a state observer should be introduced to approximate the states. Therefore, rewrite Equation (1) as follows:

$$\begin{aligned}\dot{X} &= AX + \sum_{i=2}^4 E_i \left( f_i(\hat{X}) + \Delta f_i \right) + D_i u_{qs} \\ y &= C^T X\end{aligned}\tag{2}$$

where  $\Delta f_i = f_i(X) - f_i(\hat{X})$ ,  $\hat{X} = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]$  are the estimates of  $X$ ;  $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & b_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ,

$E_i = [0, \dots, 1, \dots, 0]$ ,  $C^T = [1, 0, 0, 0]$ ,  $D_i = [0, 0, 0, c_1]^T$ .

By the approximation property of radial basis function (RBF) neural networks in [4], we can get  $\hat{f}_i(\hat{X}) = \hat{\varphi}_i^T P_i(\hat{X})$  where  $\hat{\varphi}_i$  ( $i = 1, 2, 3, 4$ ) are the estimation of the unknown optimal parameter vector  $\varphi_i$  which are defined as

$$\varphi_i = \arg \min_{\hat{\varphi}_i \in \Omega_i} \left[ \sup_{\hat{X} \in U_i} \left| \hat{f}_i(\hat{X} | \hat{\varphi}_i) - f_i(\hat{X}) \right| \right]$$

where  $\Omega_i, U_i$  are compact regions for  $\hat{\varphi}_i, \hat{X}$ , respectively. Also the neural networks minimum approximation error  $\delta_i$  is defined by

$$\delta_i = f_i(\hat{X}) - \hat{f}_i(\hat{X}|\varphi_i)$$

where  $\delta_i$  satisfies  $|\delta_i| < \varepsilon_i$ , with  $\varepsilon_i > 0$ . Rewrite (2) as

$$\begin{aligned} \dot{X} &= A_0X + \sum_{i=2}^4 E_i \left( f_i(\hat{X}) + \Delta f_i \right) + D_i u + Ky \\ y &= C^T X \end{aligned} \tag{3}$$

where  $K = [k_1, k_2, k_3, k_4]^T$  and  $A_0 = A - KC^T$ . The vector  $K$  that will be chosen should ensure  $A_0$  is Hurwitz matrix. Then, if given a  $Q^T = Q > 0$ , there exists a positive definite matrix  $P^T = P > 0$  which satisfies

$$A_0^T P + P A_0 = -Q.$$

Applying the RBF NNs, state observer can be design as

$$\begin{aligned} \dot{\hat{X}} &= A_0 \hat{X} + \sum_{i=2}^4 E_i \hat{f}_i(\hat{X}|\hat{\varphi}_i) + D_i u + Ky \\ \hat{y} &= C^T \hat{X}. \end{aligned} \tag{4}$$

Define the observer errors  $e = X - \hat{X}$ , and we can obtain the following equations with (3)(4)

$$\dot{e} = A_0 e + \sum_{i=2}^4 E_i \left[ \tilde{\varphi}_i^T S_i(\hat{X}) + \delta_i + \Delta f_i \right]$$

with  $\tilde{\varphi}_i = \varphi_i - \hat{\varphi}_i$ . Choose  $V_0 = e^T P e$ , and then

$$\dot{V}_0 = \dot{e}^T P e + e^T P \dot{e} = -e^T Q e + 2e^T P \sum_{i=2}^4 E_i \left[ \tilde{\varphi}_i^T S_i(\hat{X}) + \delta_i + \Delta f_i \right].$$

By Young's inequality, we have

$$\begin{aligned} 2e^T P \sum_{i=2}^4 E_i \left[ \tilde{\varphi}_i^T S_i(\hat{X}) + \delta_i \right] &\leq 6e^T e + \|P\|^2 \sum_{i=2}^4 E_i (\tilde{\varphi}_i^T \tilde{\varphi}_i + \varepsilon_i^2) \\ 2e^T P \sum_{i=2}^4 E_i \Delta f_i &\leq \left( 3 + \|P\|^2 \sum_{i=2}^4 h_i^2 \right) e^T e. \end{aligned} \tag{5}$$

Substituting (5) into (4), we can get

$$\dot{V}_0 \leq - \left( \lambda_{\min}(Q) - 9 - \|P\|^2 \sum_{i=2}^4 h_i^2 \right) e^T e + \|P\|^2 \sum_{i=2}^4 E_i [\tilde{\varphi}_i^T \tilde{\varphi}_i + \varepsilon_i^2].$$

**4. Adaptive NNs Dynamic Surface Controllers Design.** At each step, a virtual control function  $\alpha_i$  ( $i = 1, 2, 3, 4, 5$ ) is constructed by using an appropriate Lyapunov function. Finally, the real control laws  $u_{qs}$  and  $u_{ds}$  are constructed to control the system. The tracking error variables are designed as follows:

$$\begin{aligned} z_1 &= x_1 - x_{1d}, & z_2 &= \hat{x}_2 - \alpha_{1d}, & z_3 &= \hat{x}_3 - \alpha_{2d}, & z_4 &= \hat{x}_4 - \alpha_{3d} \\ z_5 &= x_5 - x_{5d}, & z_6 &= x_6 - \alpha_{4d}, & z_7 &= x_7 - \alpha_{5d}. \end{aligned}$$

**Step 1:** Consider the following Lyapunov function as  $V_1 = V_0 + \frac{1}{2}z_1^2$ , and then

$$\dot{V}_1 = \dot{V}_0 + z_1 \dot{z}_1 = \dot{V}_0 + z_1(\hat{x}_2 - \dot{x}_{1d} + \dot{e}_2).$$

Construct the virtual control  $\alpha_1$  as  $\alpha_1 = -\lambda_1 z_1 + \dot{x}_{1d}$ . Let  $\alpha_1$  pass a first-order filter based on DSC techniques, and then we can get  $h_1 \dot{\alpha}_{1d} + \alpha_{1d} = \alpha_1$ ,  $\alpha_{1d}(0) = \alpha_1(0)$ . From the above we can get

$$\dot{V}_1 = \dot{V}_0 + z_1(z_2 + (\alpha_{1d} - \alpha_1) + \alpha_1 + e_2 - \dot{x}_{1d}). \quad (6)$$

By using Young's inequality  $z_1 e_2 \leq \frac{1}{2} z_1^2 + \frac{1}{2} \|e\|^2$ , then (6) can be rewritten as

$$\dot{V}_1 \leq \dot{V}_0 + \frac{1}{2} \|e\|^2 + z_1(\alpha_{1d} - \alpha_1) - \lambda_1 z_1^2 + z_1 z_2. \quad (7)$$

**Step 2:** The second Lyapunov function may be chosen as  $V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2r_2} \tilde{\varphi}_2^T \tilde{\varphi}_2$ , and the derivative of  $V_2$  is calculated as follows

$$\begin{aligned} \dot{V}_2 \leq & \dot{V}_0 + \frac{1}{2} \|e\|^2 + z_1(\alpha_{1d} - \alpha_1) - \lambda_1 z_1^2 + z_1 z_2 + \frac{\tilde{\varphi}_2^T}{r_2} \left( r_2 z_2 P_2(\hat{X}) - \dot{\varphi}_2 \right) \\ & + z_2 \left( z_3 + (\alpha_{2d} - \alpha_2) + \alpha_2 + \hat{\varphi}_2^T P_2(\hat{X}) + k_2 e_1 - \dot{\alpha}_{1d} - \tilde{\varphi}_2^T P_2(\hat{X}) \right). \end{aligned}$$

By means of Young's inequality, we can obtain  $-z_2 \tilde{\varphi}_2^T P_2(\hat{X}) \leq \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{\varphi}_2^T \tilde{\varphi}_2$ . The virtual controller and adaptive law are constructed as

$$\alpha_2 = -z_1 - \frac{1}{2} z_2 - \hat{\varphi}_2^T P_2(\hat{X}) - k_2 e_1 + \dot{\alpha}_{1d} - \lambda_2 z_2, \quad \dot{\varphi}_2 = r_2 z_2 P_2(\hat{X}) - \sigma_2 \hat{\varphi}_2. \quad (8)$$

Then, the derivative of  $V_2$  can be simplified as

$$\begin{aligned} \dot{V}_2 \leq & \dot{V}_0 + \frac{1}{2} \tilde{\varphi}_2^T \tilde{\varphi}_2 + \frac{\sigma_2}{r_2} \tilde{\varphi}_2^T \hat{\varphi}_2 + \frac{1}{2} \|e\|^2 - \lambda_1 z_1^2 - \lambda_2 z_2^2 + z_1(\alpha_{1d} - \alpha_1) \\ & + z_2(\alpha_{2d} - \alpha_2) + z_2 z_3. \end{aligned}$$

**Step 3:** The third Lyapunov function is considered as  $V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2r_3} \tilde{\varphi}_3^T \tilde{\varphi}_3$ , and then

$$\begin{aligned} \dot{V}_3 = & \dot{V}_2 + z_3 \dot{z}_3 - \frac{1}{r_3} \tilde{\varphi}_3^T \dot{\varphi}_3 \leq \dot{V}_2 - z_2 z_3 + z_2 z_3 + z_3 \left( b_1 z_4 + b_1(\alpha_{3d} - \alpha_3) \right. \\ & \left. + b_1 \alpha_3 + \hat{\varphi}_3^T P_3(\hat{X}) + k_3 e_1 - \dot{\alpha}_{2d} - \tilde{\varphi}_3^T P_3(\hat{X}) \right) + \frac{\tilde{\varphi}_3^T}{r_3} \left( r_3 z_3 P_3(\hat{X}) - \dot{\varphi}_3 \right). \end{aligned}$$

We can get  $-z_3 \tilde{\varphi}_3^T P_3(\hat{X}) \leq \frac{1}{2} z_3^2 + \frac{1}{2} \tilde{\varphi}_3^T \tilde{\varphi}_3$ , by means of Young's inequality. The virtual controller and adaptive law are constructed as

$$\alpha_3 = \frac{1}{b_1} \left( -z_2 - \frac{1}{2} z_3 - \hat{\varphi}_3^T P_3(\hat{X}) - k_3 e_1 + \dot{\alpha}_{2d} - \lambda_3 z_3 \right), \quad \dot{\varphi}_3 = r_3 z_3 P_3(\hat{X}) - \sigma_3 \hat{\varphi}_3. \quad (9)$$

Then  $\dot{V}_3$  can be rewritten as

$$\dot{V}_3 \leq \dot{V}_2 - z_2 z_3 + \frac{1}{2} \tilde{\varphi}_3^T \tilde{\varphi}_3 + \frac{\sigma_3}{r_3} \tilde{\varphi}_3^T \hat{\varphi}_3 - \lambda_3 z_3^2 + b_1 z_3(\alpha_{2d} - \alpha_2) + b_1 z_3 z_4. \quad (10)$$

**Step 4:** Similarly, the fourth Lyapunov function is considered as  $V_4 = V_3 + \frac{1}{2} z_4^2 + \frac{1}{2r_4} \tilde{\varphi}_4^T \tilde{\varphi}_4$ , and the time derivative of  $V_4$  is given as

$$\begin{aligned} \dot{V}_4 = & \dot{V}_3 + z_4 \dot{z}_4 - \frac{1}{r_4} \tilde{\varphi}_4^T \dot{\varphi}_4 \leq \dot{V}_3 - b_1 z_3 z_4 + b_1 z_3 z_4 + z_4 \left( c_1 u_{qs} + \hat{\varphi}_4^T P_4(\hat{X}) \right. \\ & \left. + k_4 e_1 - \dot{\alpha}_{3d} - \tilde{\varphi}_4^T P_4(\hat{X}) \right) + \frac{\tilde{\varphi}_4^T}{r_4} \left( r_4 z_4 P_4(\hat{X}) - \dot{\varphi}_4 \right). \end{aligned}$$

By Young's inequality,  $-z_4 \tilde{\varphi}_4^T P_4(\hat{X}) \leq \frac{1}{2} z_4^2 + \frac{1}{2} \tilde{\varphi}_4^T \tilde{\varphi}_4$ . Construct the real controller

$$u_{qs} = \frac{1}{c_1} \left( -b_1 z_3 - \lambda_4 z_4 - \hat{\varphi}_4^T P_4(\hat{X}) - k_4 e_1 + \dot{\alpha}_{3d} \right), \quad \dot{\varphi}_4 = r_4 z_4 P_4(\hat{X}) - \sigma_4 \hat{\varphi}_4.$$

Then  $\dot{V}_4$  can be rewritten as

$$\dot{V}_4 \leq \dot{V}_3 - b_1 z_3 z_4 + \frac{1}{2} \tilde{\varphi}_4^T \tilde{\varphi}_4 + \frac{\sigma_4}{r_4} \tilde{\varphi}_4^T \hat{\varphi}_4 - \lambda_4 z_4^2. \quad (11)$$

**Step 5:** Similarly,  $V_5 = V_4 + \frac{1}{2} z_5^2$ , and then the time derivative of  $V_5$  is given as

$$\dot{V}_5 = \dot{V}_4 + z_5 \dot{z}_5 = \dot{V}_4 + z_5 (d_1 x_5 + d_2 x_6 - \dot{x}_{5d}). \quad (12)$$

Construct the virtual controller as

$$\alpha_4 = \frac{1}{d_2} (-d_1 x_5 + \dot{x}_{5d} - \lambda_5 z_5).$$

Then (12) can be rewritten as

$$\dot{V}_5 \leq \dot{V}_4 - \lambda_5 z_5^2 + d_2 z_5 (\alpha_{4d} - \alpha_4) + d_2 z_5 z_6 \quad (13)$$

**Step 6:** The Lyapunov function is chosen as  $V_6 = V_5 + \frac{1}{2} z_6^2$ , and  $V_6$  is given as

$$\dot{V}_6 \leq \dot{V}_5 - d_2 z_5 z_6 + d_2 z_5 z_6 + \left( z_6 \bar{e}_1 x_7 + \bar{e}_2 x_5 - \bar{e}_3 x_6 + \bar{e}_4 \frac{(\hat{x}_3 + e_3)^2}{x_5} + (\hat{x}_2 + e_2)(\hat{x} + e_3) \right).$$

Using Young's inequality,

$$\frac{2\bar{e}_4 \hat{x}_3 e_3}{x_5} \leq \frac{\bar{e}_4^2 \hat{x}_3^2}{x_5^2} + \|e\|^2, \quad \hat{x}_3 e_2 \leq \frac{1}{2} \hat{x}_3^2 + \frac{1}{2} \|e\|^2, \quad \hat{x}_2 e_3 \leq \frac{1}{2} \hat{x}_2^2 + \frac{1}{2} \|e\|^2, \quad e_2 e_3 \leq \|e\|^2.$$

Thus,  $\dot{V}_6$  can be simplified as  $\dot{V}_6 \leq \dot{V}_5 - d_2 z_5 z_6 + z_6 (\bar{e}_1 x_7 + f_6)$ , where

$$f_6 = d_2 z_5 + \bar{e}_2 x_5 - \bar{e}_3 x_6 + \frac{\bar{e}_4 \hat{x}_3^2}{x_5} + \frac{\bar{e}_4 \hat{x}_3^2}{x_5^2} + \hat{x}_2 \hat{x}_3 + \frac{1}{2} \hat{x}_3^2 + \frac{1}{2} \hat{x}_2^2 - \dot{\alpha}_{4d} + \left( 3 + \frac{\bar{e}_4}{x_5} \right) \|e\|^2.$$

There exist RBF NNs  $\phi_6^T P_6(Z_6)$  such that  $f_6 = \phi_6^T P_6(Z_6) + \delta_6(Z_6)$ , where  $\delta_6(Z_6)$  is the approximation error satisfying  $|\delta_6| \leq \varepsilon_6$ . Consequently, we can show the inequality

$$z_6 f_6 \leq \frac{1}{2l_6^2} z_6^2 \|\phi_6\|^2 P_6^T P_6 + \frac{1}{2} l_6^2 + \frac{1}{2} z_6^2 + \frac{1}{2} \varepsilon_6^2.$$

Then the virtual control  $\alpha_5$  is constructed as

$$\alpha_5 = \frac{1}{\bar{e}_1} \left( -\lambda_6 z_6 - \frac{1}{2} z_6 - \frac{1}{2l_6^2} z_6 \hat{\eta} P_6^T P_6 \right).$$

Then  $\dot{V}_6$  can be rewritten as

$$\begin{aligned} \dot{V}_6 \leq & \dot{V}_5 - d_2 z_5 z_6 - \lambda_6 z_6^2 + \frac{1}{2} l_6^2 + \frac{1}{2} \varepsilon_6^2 + \bar{e}_1 z_6 z_7 \\ & + \frac{1}{2l_6^2} z_6^2 (\|\phi_6\|^2 - \hat{\eta}) P_6^T P_6 + \bar{e}_1 z_6 (\alpha_{5d} - \alpha_5). \end{aligned} \quad (14)$$

**Step 7:** Similarly, the Lyapunov function is considered as  $V_7 = V_6 + \frac{1}{2} z_7^2$ , and the time derivative of  $V_7$  can be simplified as

$$\begin{aligned} \dot{V}_7 = & \dot{V}_6 - \bar{e}_1 z_6 z_7 + \bar{e}_1 z_6 z_7 + z_7 \left( g_1 u_{ds} - g_2 x_7 + g_3 \frac{(\hat{x}_3 + e_3)(\hat{x}_4 + e_4)}{x_5} \right. \\ & \left. + (\hat{x}_2 + e_2)(\hat{x}_4 + e_4) - g_4 x_5 + g_5 x_6 - \dot{\alpha}_{5d} \right). \end{aligned} \quad (15)$$

According to Young's inequality,

$$\begin{aligned} g_3 \frac{\hat{x}_4 e_3}{x_5} & \leq \frac{g_3^2 \hat{x}_4^2}{2x_5^2} + \frac{1}{2} \|e\|^2, \quad g_3 \frac{\hat{x}_3 e_4}{x_5} \leq \frac{g_3^2 \hat{x}_4^2}{2x_5^2} + \frac{1}{2} \|e\|^2 \\ e_3 e_4 & \leq \|e\|^2, \quad e_2 e_4 \leq \|e\|^2, \quad \hat{x}_h e_4 \leq \frac{1}{2} \hat{x}_h^2 + \frac{1}{2} \|e\|^2, \quad h = 2, 4. \end{aligned}$$

Thus, (15) can be simplified as

$$\dot{V}_7 \leq \dot{V}_6 - \bar{e}_1 z_6 z_7 + z_7 (g_1 u_{ds} + f_7)$$

There exist RBF NNs  $\phi_7^T P_7(Z_7)$  such that

$$f_7 = e_1 z_6 - g_2 x_7 + g_3 \frac{\hat{x}_3 \hat{x}_4}{x_5} + \frac{g_2^2 \hat{x}_3^2}{2x_5^2} + \frac{g_3^2 \hat{x}_4^2}{2x_5^2} + \hat{x}_2 \hat{x}_4 + \frac{1}{2} \hat{x}_2^2 + \frac{1}{2} \hat{x}_4^2 - g_4 x_5 + g_5 x_6 - \dot{\alpha}_{5d} + 4\|e\|^2.$$

There exist RBF NNs  $\phi_7^T P_7(Z_7)$  such that  $f_7 = \phi_7^T P_7(Z_7) + \delta_7(Z_7)$ , where  $\delta_7(Z_7)$  is the approximation error satisfying  $|\delta_7| \leq \varepsilon_7$ , for given  $\varepsilon_7 > 0$ . Consequently, we can show the following inequality:

$$z_7 f_7 \leq \frac{1}{2l_7^2} z_7^2 \|\phi_7\|^2 P_7^T P_7 + \frac{1}{2} l_7^2 + \frac{1}{2} z_7^2 + \frac{1}{2} \varepsilon_7^2.$$

Construct the virtual control as

$$u_{ds} = \frac{1}{g_1} \left( -\lambda_7 z_7 - \frac{1}{2} z_7 - \frac{1}{2l_7^2} z_7 \hat{\eta} P_7^T P_7 \right).$$

Then (15) can be rewritten as

$$\begin{aligned} \dot{V}_7 \leq & \dot{V}_0 - \sum_{i=1}^7 \lambda_i z_i^2 + z_1(\alpha_{1d} - \alpha_1) + z_2(\alpha_{2d} - \alpha_2) + b_1 z_3(\alpha_{3d} - \alpha_3) + d_2 z_5(\alpha_{4d} - \alpha_4) \\ & + \bar{e}_1 z_6(\alpha_{5d} - \alpha_5) + \sum_{i=2}^4 \left( \frac{1}{2} \tilde{\varphi}_i^T \tilde{\varphi}_i + \frac{\sigma_i}{r_i} \tilde{\varphi}_i^T \hat{\varphi}_i \right) + \sum_{i=6}^7 \frac{1}{2l_i^2} z_i^2 (\|\phi_i^T\|^2 - \hat{\eta}) P_i^T P_i \\ & + \sum_{i=6}^7 \left( \frac{1}{2} l_i^2 + \frac{1}{2} \varepsilon_i^2 \right) + \frac{1}{2} \|e\|^2. \end{aligned}$$

**Step 8:** Define  $y_i = \alpha_{id} - \alpha_i$ ,  $i = 1, 2, \dots, 5$ . Then we have

$$\dot{y}_i = \alpha_{id} - \alpha_i = -\frac{\alpha_{id} - \alpha_i}{h_i} - \dot{\alpha}_i = -\frac{y_i}{h_i} + B_i,$$

where  $B_i = -\dot{\alpha}_i$ . Finally, the Lyapunov function of whole system is chosen as  $V = V_7 + \sum_{i=1}^5 \frac{1}{2} y_i^2 + \frac{\tilde{\eta}^2}{2r_8}$ , with  $r_8$  being positive constant. We obtain

$$\begin{aligned} \dot{V} \leq & \dot{V}_0 - \sum_{i=1}^7 \lambda_i z_i^2 + z_1 y_1 + z_2 y_2 + b_1 z_3 y_3 + d_2 z_5 y_4 + \bar{e}_1 z_6 y_5 + \sum_{i=2}^4 \left( \frac{1}{2} \tilde{\varphi}_i^T \tilde{\varphi}_i + \frac{\sigma_i}{r_i} \tilde{\varphi}_i^T \hat{\varphi}_i \right) \\ & + \sum_{i=6}^7 \left( \frac{1}{2} l_i^2 + \frac{1}{2} \varepsilon_i^2 \right) + \frac{1}{2} \|e\|^2 + \frac{\tilde{\eta}}{r_8} \left( \dot{\eta} - \frac{r_8}{2l_6^2} z_6^2 P_6^T P_6 - \frac{r_8}{2l_7^2} z_7^2 P_7^T P_7 \right) + \sum_{i=1}^5 y_i \dot{y}_i. \quad (16) \end{aligned}$$

On the basis of (16), the adaptive law is constructed as

$$\dot{\hat{\eta}} = \frac{r_8}{2l_6^2} z_6^2 P_6^T P_6 + \frac{r_8}{2l_7^2} z_7^2 P_7^T P_7 - \sigma_8 \hat{\eta}.$$

Then we can get

$$\begin{aligned} \dot{V} \leq & \dot{V}_0 - \sum_{i=1}^7 \lambda_i z_i^2 + \sum_{i=2}^4 \left( \frac{1}{2} \tilde{\varphi}_i^T \tilde{\varphi}_i + \frac{\sigma_i}{r_i} \tilde{\varphi}_i^T \hat{\varphi}_i \right) + z_1 y_1 + z_2 y_2 + b_1 z_3 y_3 \\ & + d_2 z_5 y_4 + \bar{e}_1 z_6 y_5 + \sum_{i=6}^7 \left( \frac{1}{2} l_i^2 + \frac{1}{2} \varepsilon_i^2 \right) + \frac{1}{2} \|e\|^2 + \sum_{i=1}^5 y_i \dot{y}_i - \frac{\tilde{\eta}^T \hat{\eta}}{r_8}. \quad (17) \end{aligned}$$

$|B_i|$  has a maximum  $B_{im}$  on compact set  $|\Omega_i|$ ,  $i = 1, 2, 3, 4, 5$ ,  $|B_i| \leq B_{im}$ . Therefore, we can get

$$y_i \dot{y}_i \leq -\frac{y_i^2}{h_i} + |B_i| |y_i| \leq -\frac{y_i^2}{h_i} + \frac{1}{2\tau} B_i^2 y_i^2 + \frac{\tau}{2}, \quad z_i y_i \leq \frac{1}{4} y_i^2 + z_i^2, \quad (18)$$

$$\tilde{\varphi}_i^T \dot{\varphi}_i = \tilde{\varphi}_i^T (\varphi_i - \tilde{\varphi}_i) \leq \frac{\varphi_i^T \varphi_i}{2} - \frac{\tilde{\varphi}_i^T \tilde{\varphi}_i}{2}, \quad \tilde{\eta}^T \dot{\eta} = \tilde{\eta}^T (\eta - \tilde{\eta}) \leq \frac{\eta^T \eta}{2} - \frac{\tilde{\eta}^T \tilde{\eta}}{2}.$$

According to (17) and (18), we can obtain

$$\begin{aligned} \dot{V} \leq & - \left( \lambda_{\min}(Q) - 9 - \|P\|^2 \sum_{i=2}^4 h_i^2 - \frac{1}{2} \right) e^T e - \sum_{i=1}^5 (\lambda_i - 1) z_i^2 - \lambda_7 z_7^2 - \lambda_6 z_6^2 \\ & - \sum_{i=2}^4 \left( \frac{\sigma_i}{2r_i} - \|P\|^2 - \frac{1}{2} \right) \tilde{\varphi}_i^T \tilde{\varphi}_i - \sum_{i=1}^5 \left( \frac{y_i^2}{h_i} - \left( \frac{1}{4} + \frac{1}{2\tau} B_{iM}^2 \right) \right) y_i^2 - \frac{1}{2} \tilde{\eta}^2 \\ & + \frac{1}{2} \eta^2 + \sum_{i=2}^4 \frac{\sigma_i \varphi_i^T \varphi_i}{2r_i} + \sum_{i=6}^7 \left( \frac{1}{2} l_i^2 + \frac{1}{2} \varepsilon_i^2 \right) + \|P\|^2 (\varepsilon_2^{*2} + \varepsilon_3^{*2} + \varepsilon_4^{*2}) \leq -aV + b. \end{aligned} \quad (19)$$

where  $\lambda_{\min}(Q) - 9 - \frac{1}{2} - \|P\|^2 \sum_{i=1}^n h_i^2 > 0$  and  $\frac{\sigma_i}{2r_i} - \|P\|^2 - \frac{1}{2} > 0$

$$\begin{aligned} a = \min & \left\{ \frac{\lambda_{\min}(Q) - 9 - \|P\|^2 \sum_{i=2}^4 h_i^2 - \frac{1}{2}}{\lambda_{\max}(P)}, 2(\lambda_1 - 1), 2(\lambda_2 - 1), \dots, 2(\lambda_5 - 1), 2\lambda_6, 2\lambda_7, \right. \\ & \left. 2r_2 \left( \frac{\sigma_2}{2r_2} - \|P\|^2 - \frac{1}{2} \right), \dots, \frac{\sigma_4}{2r_4} - \|P\|^2 - \frac{1}{2}, 2 \left( \frac{y_\kappa^2}{h_\kappa} - \left( \frac{1}{4} + \frac{1}{2\tau} B_{\kappa M}^2 \right) \right) \right\} \\ b = & \|P\|^2 (\varepsilon_2^{*2} + \varepsilon_3^{*2} + \varepsilon_4^{*2}) + \sum_{i=2}^4 \frac{\sigma_i \varphi_i^T \varphi_i}{2r_i} + \sum_{i=6}^7 \left( \frac{1}{2} l_i^2 + \frac{1}{2} \varepsilon_i^2 \right) + \frac{1}{2} \eta^2 + \frac{5}{2} \tau \end{aligned}$$

where  $\kappa = 1, 2, 3, 4, 5$ . Furthermore, (19) implies that

$$V_n(t) \leq \left( V_n(t_0) - \frac{b}{a} \right) e^{-a(t-t_0)} + \frac{b}{a} V_n(t_0) + \frac{b}{a}, \quad \forall t \geq t_0. \quad (20)$$

The solution exists for  $t \in [0, \infty)$ , so we have

$$\lim_{t \rightarrow \infty} |z_1| \leq \sqrt{\frac{2b}{a}}. \quad (21)$$

By choosing the suitable parameters, the tracking error  $z_1$  can converge to a small area of the origin.

**5. Simulation Results.** In order to illustrate the effectiveness of the proposed method, the simulation is run for the IMs with the parameters:  $J = 0.0586 \text{kgm}^2$ ,  $B = 1.158 * 10^{-3} \text{N}\cdot\text{m}/(\text{rad}/\text{s})$ ,  $R_s = 0.1\Omega$ ,  $R_r = 0.15\Omega$ ,  $R_{fe} = 30\Omega$ ,  $L_{1s} = L_{1r} = 0.1\text{H}$ ,  $L_m = 0.068\text{H}$ ,  $n_p = 1$ . The simulation is carried out under the zero initial conditions. The RBF NNs are chosen in the following way. The NNs  $\phi_i^T P_i(Z)$  contain eleven nodes with centers spaced evenly in the interval  $[-9, 9]$  and widths being equal to 2. The reference signals are taken as  $x_{1d} = 0.5 \sin(t) + 0.3 \sin(0.5t)$  and  $x_{5d} = 1$ . The proposed adaptive NNs controllers are used to control the induction motors. The control parameters are chosen as:  $\lambda_1 = 30$ ,  $\lambda_2 = 1$ ,  $\lambda_3 = 15$ ,  $\lambda_4 = 85$ ,  $\lambda_5 = 325$ ,  $\lambda_6 = 30$ ,  $\lambda_7 = 50$ ,  $r_8 = r_4 = r_3 = r_2 = 0.05$ ,  $l_6 = l_7 = 0.25$ .

Figure 1 shows the tracking performance of  $\Theta$  and  $x_{1d}$ , and Figure 2 displays the trajectories of  $\psi_d$  and  $x_{5d}$ . Figure 1 and Figure 2 show that the given signals can be tracked very well by the method proposed in this paper. Figures 3 and 4 show the curves of  $u_{qs}$ ,  $u_{ds}$ . It can be seen that the controllers are bounded. According to the above

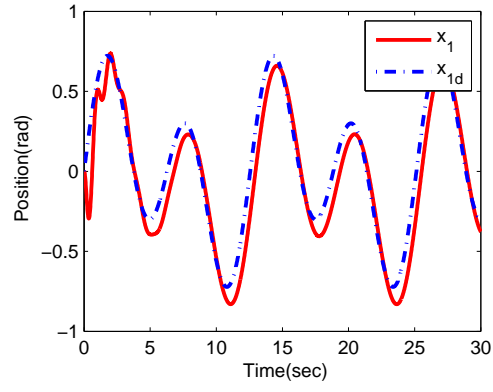


FIGURE 1.  $x_1$  and  $x_{1d}$

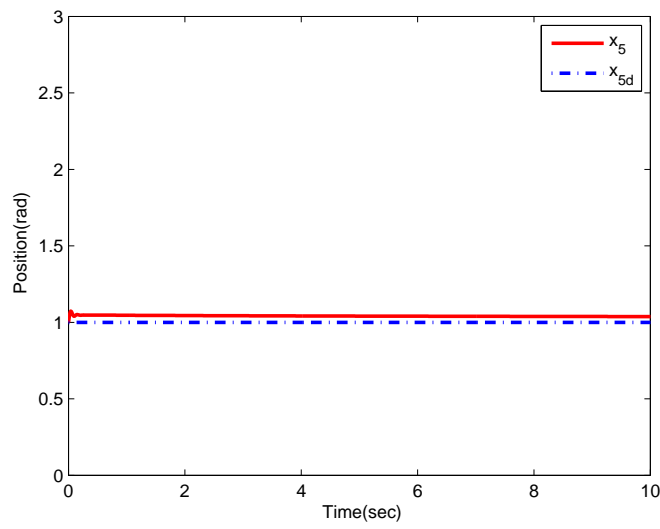


FIGURE 2.  $x_5$  and  $x_{5d}$

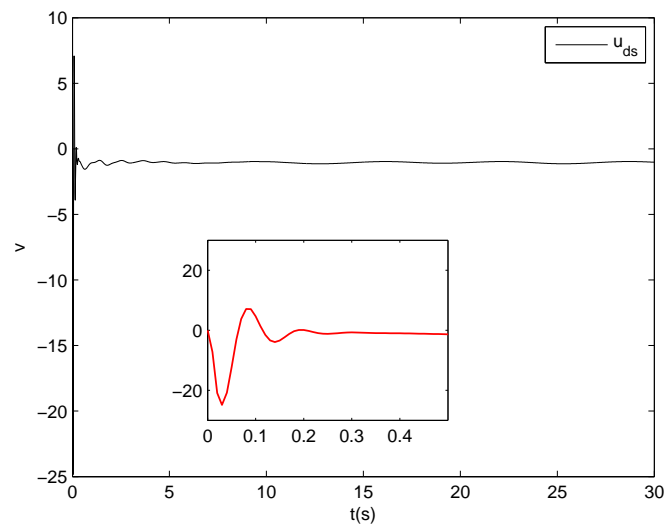
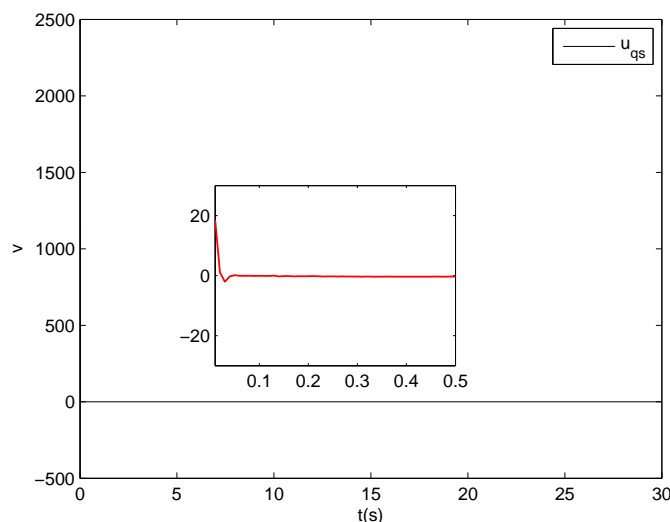


FIGURE 3.  $u_{ds}$



FIGURE 4.  $u_{qs}$ 

simulation results, we can conclude that the proposed controllers can track the given signals very well even under unknown states and uncertain parameters.

**6. Conclusion.** In this paper, adaptive neural networks output-feedback control algorithm has been introduced with parametric uncertainty. Neural networks are applied to approximate the unknown nonlinear functions and the output-feedback is designed to estimate the unmeasured states. Furthermore, the problem of explosion of complexity can be avoided by employing the first-order filters. Simulation results testify its effectiveness in the IMs drive systems.

**Acknowledgment.** This work was partially supported by the National Natural Science Foundation of China (61573204, 61573203, 61501276, 61603204), Shandong Province Outstanding Youth Fund (ZR2015JL022), Taishan Scholar Special Project Found, the China Postdoctoral Science Foundation (2014T70620, 2013M541881, 201303062, 2016M592139) and Qingdao Postdoctoral Application Research Project (2015120).

## REFERENCES

- [1] D. Chen, Y. Liu and X. Ma, Sliding mode position control for real-time control of induction motors, *Nonlinear Dynamics*, vol.67, no.1, pp.893-901, 2012.
- [2] Y. Li, S. Tong, T. Li and X. Jing, Adaptive fuzzy control of uncertain stochastic nonlinear systems with unknown dead zone using small-gain approach, *Fuzzy Sets and Systems*, vol.235, no.1, pp.1-24, 2014.
- [3] D. Swaroop, J. Hedrick, P. Yip and J. Gerdes, Dynamic surface control of nonlinear systems, *Proc. of American Control Conference*, vol.5, pp.3028-3034, 1997.
- [4] J. Yu, P. Shi, W. Dong, B. Chen and C. Lin, Neural network-based adaptive dynamic surface control for permanent magnet synchronous motors, *IEEE Trans. Neural Networks & Learning Systems*, vol.26, no.3, pp.640-645, 2015.
- [5] M. Simoes and B. Bose, Neural network based estimation of feedback signals for a vector controlled induction motor drive, *IEEE Trans. Industry Applications*, vol.31, no.3, pp.620-629, 2011.
- [6] Y.-C. Luo, Y.-H. Chen, Y.-P. Kuo and C.-T. Tsai, Sensorless vector controlled induction motor drive with full-order rotor flux observer speed identification, *ICIC Express Letters, Part B: Applications*, vol.7, no.9, pp.2013-2020, 2016.
- [7] J. Yu, Y. Ma, H. Yu and C. Lin, Adaptive fuzzy dynamic surface control for induction motors with iron losses in electric vehicle drive systems via backstepping, *Information Science*, vol.376, no.3, pp.172-189, 2016.