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THE DYNAMIC VERTICAL MERGING ALGORITHM OF MULTI INTERVAL CONCEPT LATTICE

Baoxiang Liu*, Ru Zhang, Liya Wang and Chunying Zhang

College of Sciences
North China University of Science and Technology
No. 46, Xinhua West Road, Tangshan 063009, P. R. China
*Corresponding author: Liubx5888@126.com; { zhangru1266; wang_liya; hblg_zcy }@126.com

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Abstract. *Facing the real-time data updating under the background of big data, how to deal with the data effectively which are generated at any time has become one of the key issues. And interval concept lattice is an effective tool for data representation and analysis. Therefore, this paper makes an in-depth study of dynamic vertical merging algorithm for multi interval concept lattice. So propose the concept of consistency of interval concept lattice firstly, and then come up with the related theorems of vertical merger according to the intent characteristics of interval concepts. Further improve the algorithm of building lattice. Categorize interval concepts and store them in the structure of mode in order to improve the efficiency of judging category of concept when interval concept lattices are merged. Then discuss the specific situations which have consistent intensions after merging, then propose the corresponding judging theorems. Finally, using the strategy of breadth-first search design the vertical merging algorithm of concept lattice and prove the correctness and efficiency of the algorithm through examples.*
Keywords: Interval concept lattice, Vertical merger, Dynamic, Real time update, Breadth-first

1. **Introduction.** Interval concept lattice [1] is a collection of objects which has a certain number or percentage of attributes in the connotation. Compared with the classical concept lattice and rough concept lattice, the application of interval concept lattice is stronger.

With the advent of the era of big data [2], the data processing in time and space complexity have become increasingly demanding. Real-time updating of data requires efficient processing in dynamic data. For example, every day a huge amount of transaction information is generated. If we build the interval concept lattice of the daily trading information, we can only tap the local association rules. It cannot provide timely and accurate decision-making for the decision-makers from the overall in the supermarket shopping system. Therefore, it is very necessary to carry out the research of the merging of interval concept lattice to realize the data aggregation. At present, the merging algorithms of the classical concept lattice [3] are the incremental double sequence algorithm [4], the vertical union algorithm of concept lattice [5] and so on [6,7,12]. In classic concept lattice and its expanded concept lattices [8-10], the extent contains the objects which meet all the attributes in the intent. Therefore, these methods cannot be applied to interval concept lattice directly. In addition, if we construct interval concept lattices after accumulating many days trading information will lead to the lag of information. Aiming at the above-mentioned problems we propose the research which is the dynamic vertical merging algorithm of multi interval concept lattices. It aims to efficiently merge it with consistency. And this algorithm achieves data summarizing fast and effectively. Further, the effective information is mined by decision makers so as to develop targeted marketing and occupating opportunities.

The structure of the thesis is as follows. The basic concepts of interval concept lattice and improved ICAICL [11] are introduced in Section 2. The basic concepts and theorems of the vertical merging of interval concepts are proposed in Section 3. And based on the improved incremental generation algorithm of interval concept lattice, the vertical merging algorithm of interval concept lattice is designed, and the algorithm analysis is carried out. The efficiency of the algorithm is demonstrated through an example in Section 4. Section 5 shows conclusions.

2. **Basic Concepts.**

2.1. **Interval concept lattice.**

Definition 2.1. *Suppose* (U, A, R) *has two interval concepts,* $\left(M_1^\alpha, M_1^\beta, Y_1\right)$ *and* $\left(M_2^\alpha, M_2^\beta, Y_2\right)$*. If* $\left(M_1^\alpha, M_1^\beta, Y_1\right)$ *and* $\left(M_2^\alpha, M_2^\beta, Y_2\right)$ *meet* $Y_1 \subseteq Y_2$*,* $|Y_2| - |Y_1| = 1$*,* $M_1^\alpha = M_2^\alpha$ *and* $M_1^\beta = M_2^\beta$*, then* $\left(M_1^\alpha, M_1^\beta, Y_1\right)$ *is called the redundant concept.*

Definition 2.2. *Suppose* (U, A, R) *has an interval concept,* $\left(M^\alpha, M^\beta, Y\right)$*. If it meets* $M^\alpha = M^\beta = \varnothing$*, then* $\left(M^\alpha, M^\beta, Y\right)$ *is called the empty concept.*

Definition 2.3. *Suppose* (U, A, R) *has an interval concept,* $C = \left(M^\alpha, M^\beta, Y\right)$*. If* C *is neither the redundant concept nor the empty concept, then* C *is called the existence concept.* $L_\alpha^\beta(U, A, R)$ *is a collection of all the existence concepts.*

Definition 2.4. $\overline{L_\alpha^\beta(U, A, R)}$ *refers to all the* $[\alpha, \beta]$ *interval concepts, which includes existence concepts, redundant concepts and empty concepts, that is:*

$$\left(M_1^\alpha, M_1^\beta, Y_1\right) \leq \left(M_2^\alpha, M_2^\beta, Y_2\right) \Leftrightarrow Y_1 \supseteq Y_2 \tag{1}$$

Then "$\leq$*" is called the partial order relationship of* $\overline{L_\alpha^\beta(U, A, R)}$*.*

2.2. **Improved algorithm of the ICAICL.**

In order to retain all the interval concepts and improve the merging efficiency of interval concept lattice effectively, we need to improve the existing ICAICL [11], and propose vertical merger interval concept lattice on this basis.

In order to distinguish different interval concepts, the concept node is defined and stored in a structural manner. It is expressed as follows.

$$\boxed{flag \mid M^\alpha \mid M^\beta \mid Y \mid parent \mid children \mid no} \tag{2}$$

Define the form as:

Struct concept {String M_i^α; M_i^β; Y_i; Struct Y; parent; children; Int flag;}

The concept of category is marked by flag. When flag = 1, flag = 2 and flag = 3, stored concept is existence concept, redundant concept and empty concept separately.

Algorithm: Improved ICAICL

Input: formal context (U, A, R)

Output: interval concept lattice L_α^β and $\overline{L_\alpha^\beta}$

(1) Calculate power set of attribute set $P(A)$ to determine the intent of concept, generate concept node of initialization G.

(2) Determine upper bound extent M_i^α and lower bound extent M_i^β, and make flag of empty concept be 3 and the other be 1.

(3) According to the partial order relationship, determine the level of the node and the parent-child relationship, and make flag of redundant concept be 2.

The method of finding out redundant concept is Romove-redun(Ch,Gi).

Remove-redun(Ch,Gi)//find out redundant concept, sign and store, and delete it.

$\{$ for each child Ch in Gi //*Ch pointer points to every child of Gi*
$\{$ If $(\text{Gi}.M_i^\alpha = \text{Ch}.M_i^\alpha,\ \text{Gi}.M_i^\beta = \text{Ch}.M_i^\beta)$
$\{$ Flag = 2;
Delete Gi from $L_\alpha^\beta\}\}\}$

(4) To concept of no = 1, structure the root node, and then insert other nodes into lattice according to the parent-child relationship successively. Eventually form the structure of interval concept lattice.

3. The Dynamic Vertical Merging of Multi Interval Concept Lattices Principle and Algorithm.

3.1. Basic principle of vertical merger.

Definition 3.1. *Suppose the formal contexts (U_1, A_1, R_1) and (U_2, A_2, R_2) are consistent and they compose interval concept lattice $L_{\alpha_1}^{\beta_1}(U_1, A_1, R_1)$ and $L_{\alpha_2}^{\beta_2}(U_2, A_2, R_2)$ respectively. When they meet $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$, then $L_\alpha^\beta(U_1, A_1, R_1)$ and $L_\alpha^\beta(U_2, A_2, R_2)$ call consistent; otherwise they are not consistent.*

Definition 3.2. *If interval concept lattices $L_\alpha^\beta(U_1, A_1, R_1)$ and $L_\alpha^\beta(U_2, A_2, R_2)$ are consistent and meet $A_1 = A_2 = A$, we get $L_\alpha^\beta(U, A, R)$ when we combine them. $L_\alpha^\beta(U, A, R)$ call vertical merger of $L_\alpha^\beta(U_1, A, R_1)$ and $L_\alpha^\beta(U_2, A, R_2)$. Suppose $C_1 \in L_\alpha^\beta(U_1, A, R_1)$, $C_2 \in L_\alpha^\beta(U_2, A, R_2)$, and then $C = C_1 \oplus C_2$ is merger of interval concept.*

Theorem 3.1. *Suppose $L_\alpha^\beta(U_1, A_1, R_1)$ and $L_\alpha^\beta(U_2, A_2, R_2)$ are consistent and meet $A_1 = A_2 = A$. At the same time suppose $\left(M_1^\alpha, M_1^\beta, Y_1\right)$ and $\left(M_2^\alpha, M_2^\beta, Y_2\right)$ are interval concepts in $\overline{L_\alpha^\beta(U_1, A, R_1)}$ and $\overline{L_\alpha^\beta(U_2, A, R_2)}$ respectively. If they meet $Y = Y_1 = Y_2$, $M^\alpha = M_1^\alpha \cup M_2^\alpha$ and $M^\beta = M_1^\beta \cup M_2^\beta$, then $\left(M^\alpha, M^\beta, Y\right)$ is interval concept after vertical merger.*

Proof: We can divide formal contexts into two parts respectively, that is the same object subset X and the different object subset (X_1^* and X_2^*). In terms of upper extension, according to $\frac{|f(X)\cap Y|}{|Y|} \geq \alpha$, the extension of the upper bound of X, X_1^* and X_2^* are M^α, $M_1^{\alpha*}$ and $M_2^{\alpha*}$ respectively. So upper extension of $\overline{L_\alpha^\beta(U_1, A, R_1)}$ is $M^\alpha \cup M_1^{\alpha*} = M_1^\alpha$ and upper extension of $\overline{L_\alpha^\beta(U_2, A, R_2)}$ is $M^\alpha \cup M_2^{\alpha*} = M_2^\alpha$.

Merge two interval concept lattices, and then $M_1^\alpha \cup M_2^\alpha$ is the upper bound of the same meaning concepts merged extension. It is the same as the extension of lower bound.

Suppose $L_\alpha^\beta(U_1, A, R_1)$ and $L_\alpha^\beta(U_2, A, R_2)$ being interval concept lattices which have consistent property. All their interval concepts are $\overline{L_\alpha^\beta(U_1, A, R_1)}$ and $\overline{L_\alpha^\beta(U_2, A, R_2)}$ respectively. If we want do vertical merger to $L_\alpha^\beta(U_1, A, R_1)$ and $L_\alpha^\beta(U_2, A, R_2)$, at the same time suppose $L_\alpha^\beta(U, A, R)$ is the interval concept lattice after vertical merger, the following several theorems related to are shown as follows:

Theorem 3.2. *If $C_1 \in L_\alpha^\beta(U_1, A, R_1)$, $C_2 \in L_\alpha^\beta(U_2, A, R_2)$, then $C = C_1 \oplus C_2 \in L_\alpha^\beta(U, A, R)$.*

Proof: Suppose $\left(M_1^\alpha, M_1^\beta, Y\right) \in L_\alpha^\beta(U_1, A, R_1)$, $\left(M_2^\alpha, M_2^\beta, Y\right) \in L_\alpha^\beta(U_2, A, R_2)$. Children concept of $\left(M_1^\alpha, M_1^\beta, Y\right)$ and $\left(M_2^\alpha, M_2^\beta, Y\right)$ refer to $\left(M_{child1}^\alpha, M_{child1}^\beta, Y_{child}\right)$ and $\left(M_{child2}^\alpha, M_{child2}^\beta, Y_{child}\right)$ respectively. Because $\left(M_1^\alpha, M_1^\beta, Y\right)$ and $\left(M_2^\alpha, M_2^\beta, Y\right)$ are existence concepts, $M_1^\alpha \neq M_{child1}^\alpha$ or $M_1^\beta \neq M_{child1}^\beta$ and $M_2^\alpha \neq M_{child2}^\alpha$ or $M_2^\beta \neq M_{child2}^\beta$. The corresponding children concept is $\left(M_{child1}^\alpha \cup M_{child2}^\alpha, M_{child1}^\beta \cup M_{child2}^\beta, Y_{child}\right)$. Combine upper and lower bound extent of two existence concepts. By the above method we can get $M_1^\alpha \cup M_2^\alpha \neq M_{child1}^\alpha \cup M_{child2}^\alpha$ or $M_1^\beta \cup M_2^\beta \neq M_{child1}^\beta \cup M_{child2}^\beta$.

Corollary 3.1. *If* $C_1 \in L_\alpha^\beta(U_1, A, R_1)$*,* $C_2 \notin L_\alpha^\beta(U_2, A, R_2)$*, then* $C = C_1 \oplus C_2 \in L_\alpha^\beta(U, A, R)$*.*

Proof: It is similar to the proof of Theorem 3.2.

Theorem 3.3. *Suppose* $C_1 \notin L_\alpha^\beta(U_1, A, R_1)$*,* $C_2 \notin L_\alpha^\beta(U_2, A, R_2)$*, at the same time they are redundant concepts. If connotation of children concept of* C_1C_2 *are the same,* $C = C_1 \oplus C_2 \notin L_\alpha^\beta(U, A, R)$*, then* C *is redundant concept. If connotation of children concept of* C_1C_2 *are different,* $C = C_1 \oplus C_2 \in L_\alpha^\beta(U, A, R)$*, then* C *is existence concept.*

Proof: According to Theorem 3.2, when their intents of children are the same, then $M_1^\alpha \cup M_2^\alpha = M_{child1}^\alpha \cup M_{child2}^\alpha$ and $M_1^\beta \cup M_2^\beta = M_{child1}^\beta \cup M_{child2}^\beta$. So the merged concept is always redundant concept. When their intent of children is different, the merged concept is existence concept.

Theorem 3.4. *Suppose* $C_1 \notin L_\alpha^\beta(U_1, A, R_1)$*,* $C_2 \notin L_\alpha^\beta(U_2, A, R_2)$*, they are redundant concept and empty concept respectively, when the child concept attribute of empty concept is the same as the child concept attribute of redundant concept and the child concept of empty concept is empty concept, then* C *is redundant concept. Otherwise,* C *is existence concept.*

Proof: In the first case, $M_1^\alpha = M_{child1}^\alpha$ and $M_1^\beta = M_{child1}^\beta$, $M_2^\alpha = M_2^\beta = \varnothing$ and $M_{child2}^\alpha = M_{child2}^\beta = \varnothing$, the C and its child concept are $\left(M_1^\alpha, M_1^\beta, Y\right)$. Otherwise, the C and its child concept are different.

Theorem 3.5. *Suppose* $C_1 \notin L_\alpha^\beta(U_1, A, R_1)$*,* $C_2 \notin L_\alpha^\beta(U_2, A, R_2)$*, at the same time they are empty concepts,* $C = C_1 \oplus C_2 \notin L_\alpha^\beta(U, A, R)$*, then* C *is empty concept.*

Proof: On the basis of $C_1 = (\varnothing, \varnothing, Y)$ and $C_2 = (\varnothing, \varnothing, Y)$, we can obtain $C = (\varnothing, \varnothing, Y)$.

3.2. **Vertical merger algorithm.** There are existence concepts , redundant concepts and empty concepts in interval concept lattice. So we need take different measures to merge in different situations, and finally get the interval concept lattice.

Algorithm: DVM (Dynamic Vertical Merger)

Input: L_α^β, $\overline{L_\alpha^\beta}$, $L_\alpha^\beta 1$, $\overline{L_\alpha^\beta 1}$, $L_\alpha^\beta 2$, $\overline{L_\alpha^\beta 2} \ldots$ $L_\alpha^\beta i$, $\overline{L_\alpha^\beta i} \ldots$

Output: $L_\alpha^{\beta^*}$ *and* $\overline{L_\alpha^{\beta^*}}$

(1) Set $L_\alpha^\beta = L_\alpha^\beta 1$ *and* $\overline{L_\alpha^\beta} = \overline{L_\alpha^\beta 1}$.

(2) Merge $L_\alpha^\beta i$, $\overline{L_\alpha^\beta i}$ $(i = 2, 3, \ldots, n)$ *and* L_α^β, $\overline{L_\alpha^\beta}$*.*

The result of the merger is assigned to $L_\alpha^{\beta^*}$, $\overline{L_\alpha^{\beta^*}}$. Among them, the merging steps of the two interval concept lattices are as follows.

By the principle of breadth-first, scan two interval concept lattices. We illustrate the merging algorithm with the merging of two concept lattices as an example.

Algorithm: Union-$\overline{L_\alpha^\beta}\,\overline{L_\alpha^\beta i}\left(\overline{L_\alpha^\beta}, \overline{L_\alpha^\beta i}, \overline{L_\alpha^{\beta^*}}\right)$

Input: L_α^β, $\overline{L_\alpha^\beta}$, $L_\alpha^\beta i$, $\overline{L_\alpha^\beta i}$

Output: interval concept lattice $L_\alpha^{\beta^*}$ *and* $\overline{L_\alpha^{\beta^*}}$

Union-$\overline{L_\alpha^\beta}\,\overline{L_\alpha^\beta i}\left(\overline{L_\alpha^\beta}, \overline{L_\alpha^\beta i}, \overline{L_\alpha^{\beta^*}}\right)$

For $\left(\overline{L_\alpha^\beta}.no = 1;\ \overline{L_\alpha^\beta}.no <= n;\ no++\right)$

$\{number = \overline{L_\alpha^\beta}.no$

$C_i = \overline{L_\alpha^\beta}$ $//C_i$ *is node which number is no.*

$C_j = \overline{L_\alpha^\beta i}$ $//C_j$ *is node which number is no.*

If $\left(C_i \text{ in } \overline{L_\alpha^\beta}.flag = 1 || \ C_j \text{ in } \overline{L_\alpha^\beta i}.flag = 1\right)$ /* *merge existence and other concept* */

Merge-interval concept $\left(\overline{L_\alpha^\beta}, C_i, \overline{L_\alpha^\beta i}, C_j, \overline{L_\alpha^{\beta*}}, C_k^*\right)$

If $\left(C_i \text{ in } \overline{L_\alpha^\beta}.flag = 2,\ C_j \text{ in } \overline{L_\alpha^\beta i}.flag = 2\right)$/* *merge two redundant concepts* */

If $(C_i.CH.Y \neq C_j.CH.Y)$

$\left\{\right.$ *Merge-interval concept* $\left(\overline{L_\alpha^\beta}, C_i, \overline{L_\alpha^\beta i}, C_j, \overline{L_\alpha^{\beta*}}, C_k^*\right)$

$Flag = 1$ $\left.\right\}$

Else $\left\{\right.$ *Merge-interval concept* $\left(\overline{L_\alpha^\beta}, C_i, \overline{L_\alpha^\beta i}, C_j, \overline{L_\alpha^{\beta*}}, C_k^*\right)$ $Flag = 2 \left.\right\}$

If $\left(C_i \text{ in } \overline{L_\alpha^\beta}.flag = 2,\ C_j \text{ in } \overline{L_\alpha^\beta i}.flag = 3 ||\right.$

$\left.C_i \text{ in } \overline{L_\alpha^\beta}.flag = 3,\ C_j \text{ in } \overline{L_\alpha^\beta i}.flag = 2\right)$/* *merge redundant and empty concept* */

$\left\{\right.$ *Merge-interval concept* $\left(\overline{L_\alpha^\beta}, C_i, \overline{L_\alpha^\beta i}, C_j, \overline{L_\alpha^{\beta*}}, C_k^*\right)$ $Flag = 2 \left.\right\}$

If $\left(C_i \text{ in } \overline{L_\alpha^\beta}.flag = 3,\ C_j \text{ in } \overline{L_\alpha^\beta i}.flag = 3\right)$/* *merge two empty concepts* */

$\left\{C_k^*.M_k^\alpha = \varnothing, C_k^*.M_k^\beta = \varnothing, C_k^*.Y = Y \quad Flag = 3 \right\}\}$

Merge-interval concept (C_i, C_j, C_k^*)/* *merge two interval concepts* */

$\left\{C_k^*.M_k^{\alpha*} = C_i.M_i^\alpha \cup C_j.M_j^\alpha, C_k^*.M_k^{\beta*} = C_i.M_i^\beta \cup C_j.M_j^\beta, C_k^*.Y = C_i.Y \cup C_j.Y \right\}$

When we come to the concept lattice, we repeat the above process to achieve the vertical merger process of the concept lattice. All the merging of the interval concept is covered in this algorithm. Therefore, the algorithm is correct and complete. The time complexity of this algorithm is $O(n)$. However, the way which we first merge formal background and then apply the ICAICL algorithm to construct it is more than $O(n)$ in second step. So the efficiency of this algorithm is proved through analysis.

4. **Examples.** Know formal context as shown in Table 1 and Table 2.

(1) Suppose $\alpha = 0.6$, $\beta = 0.7$. The lattice structure of formal context is generated by the improved ICAICL. And it is shown in Figure 1 and Figure 2. All generated concepts by the way are shown in Table 3. Redundant concepts and empty concepts of $L_\alpha^\beta 1$ and $L_\alpha^\beta 2$ which have been stored are shown in Table 3.

(2) According to the Union-$L_\alpha^\beta 1$ $L_\alpha^\beta 2$, the merged interval concept lattice $L_\alpha^\beta(U, A, R)$ is shown in Figure 3.

5. **Conclusions.** Interval concept lattice has a certain amount or proportion of connotation of the lattice structure of object set. Its vertical merger is different from other forms of concept lattices. We design the dynamic vertical merging algorithm for multi interval

TABLE 1. Formal context of $L_\alpha^\beta(U_1, A, R_1)$

	a	b	c	d
1	1	1	0	0
2	0	0	1	0
3	1	0	1	1

Table 2. Formal context of $L_{\alpha}^{\beta}(U_2, A, R_2)$

	a	b	c	d
[1]	0	0	0	1
[2]	0	1	1	0
[3]	1	0	1	1

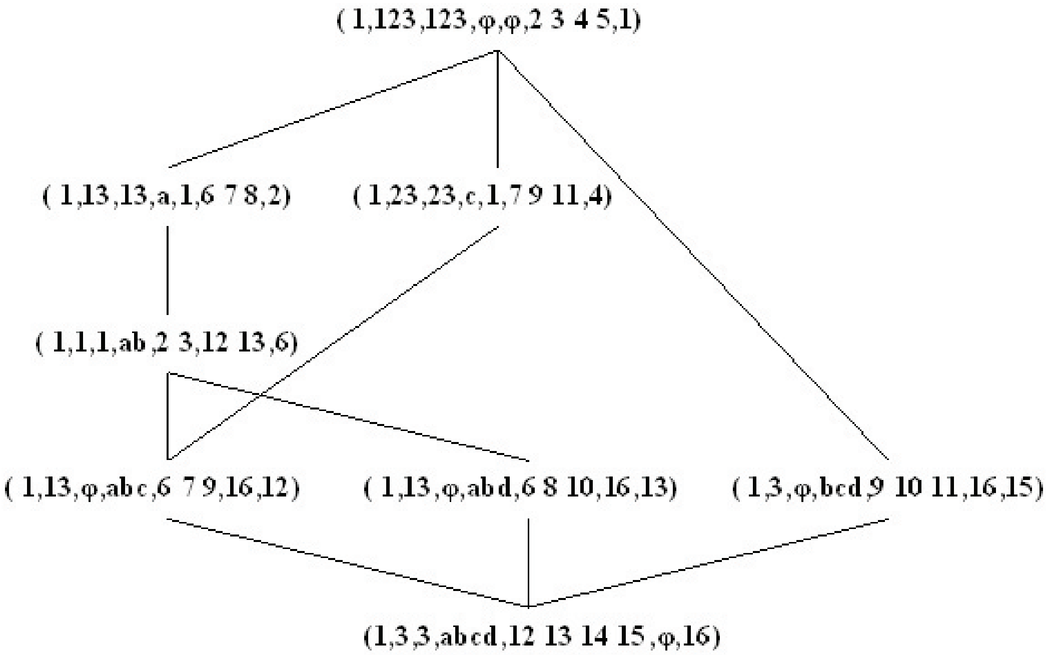

Figure 1. Interval concept lattice of $L_{\alpha}^{\beta}(U_1, A, R_1)$

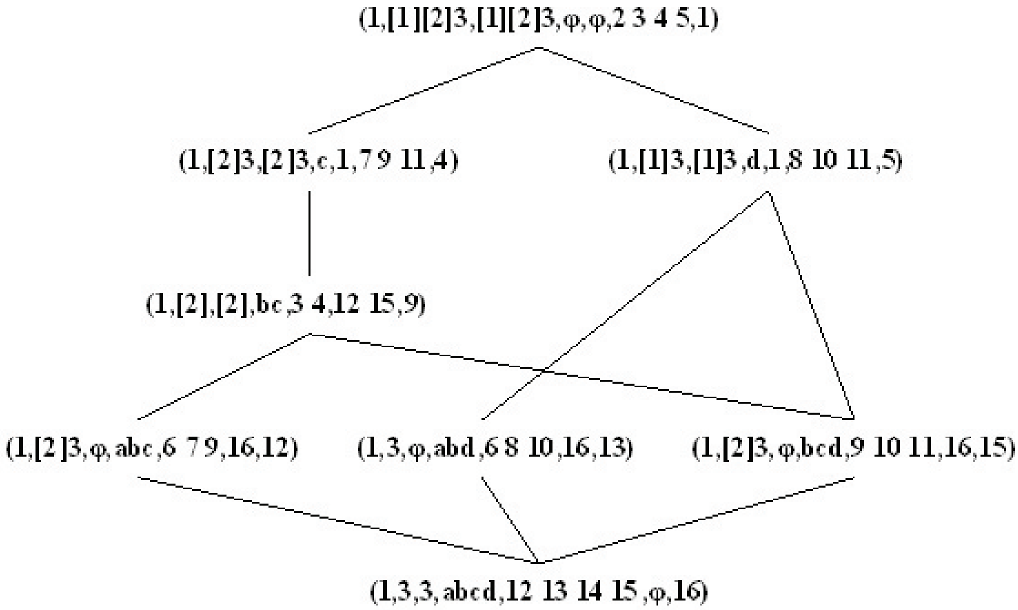

Figure 2. Interval concept lattice of $L_{\alpha}^{\beta}(U_2, A, R_2)$

concept lattices based on breadth first according to the requirements of the era of big data and its own characteristics. In real life, the summary of information and merging interval concept lattices efficiently provide operability for the further mining of association rules. Therefore, the dynamic vertical merging algorithm of interval concept lattices

Table 3. Redundant concepts and empty concepts in $L_\alpha^\beta 1$ and $L_\alpha^\beta 2$

$L_\alpha^\beta 1$	$L_\alpha^\beta 2$
(2,1,1,b,1,6 9 10,3)	(2,3,3,a,1,6 7 8,2)
(2,3,3,d,1,8 10 11,5)	(2,[2],[2],b,1,6 9 10,3)
(2,3,3,ac,2 4,12 14,7)	(2,3,3,ac,2 4,12 14,7)
(2,3,3,ad,2 5,13 14,8)	(2,3,3,ad,2 5,13 14,8)
(2,3,3,cd,4 5,14 15,11)	(2,3,3,cd,4 5,14 15,11)
(2,3,3,acd,7 8 11,16,14)	(2,3,3,acd,7 8 11,16,14)
(3,φ,φ,bc,3 4,12 15,9)	(3,φ,φ,ab,2 3,12 13,6)
(3,φ,φ,bd,3 5,13 15,10)	(3,φ,φ,bd,3 5,13 15,10)

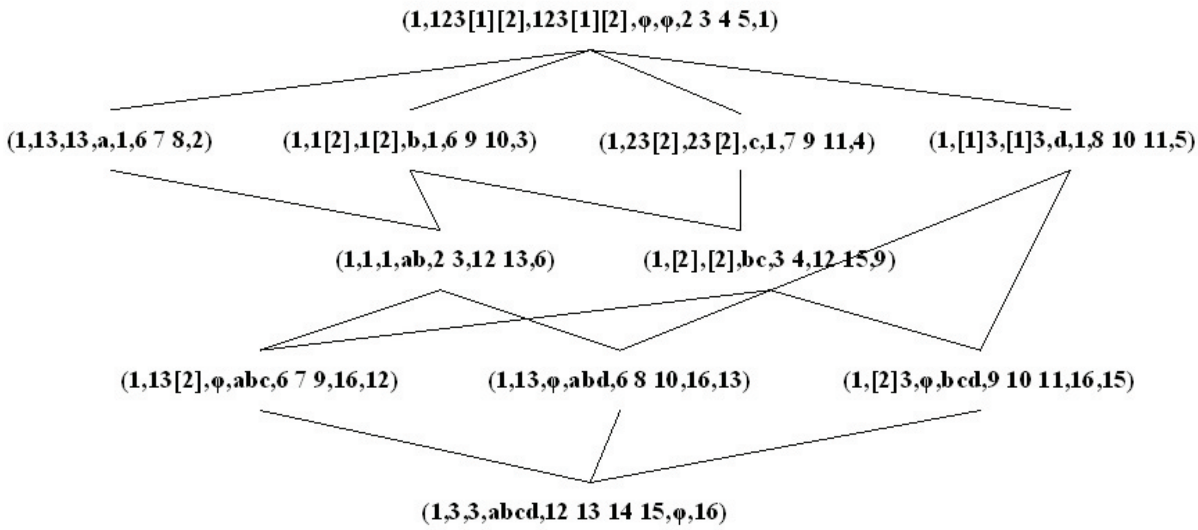

Figure 3. Interval concept lattice of $L_\alpha^\beta(U, A, R)$

has profound theoretical value and practical significance. In the next step, we study the dynamic horizontal merger of interval concept lattices mainly.

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