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EVALUATION OF CLASSIC HEDGING MODELS – OLS, ECM AND GARCH MODEL

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Abstract. *This paper compared the hedging ratio and its corresponding hedging effects of OLS, ECM and GARCH models separately in six stages. We found that the applicability of the three models is determined by their algorithm and assumptions. The OLS model is the most suitable when the spot and futures prices change is relatively stable and has a strong linear characteristic. When prices changed a lot but do not show clustering characteristics, the ECM model behaves better. GARCH model is the best when ARCH effect occurs. So there is no absolute most superior model throughout the whole hedging period. We should consider the different changing characteristics of basis and prices when making hedging portfolios. Models are just some kinds of tools. We should select tools dynamically, rather than relying on some dynamic tools.*
Keywords: Hedging ratio, Hedging effects, OLS, ECM, GARCH

1. **Introduction.** The key issue of commodity futures hedging is the determination of the optimal hedging ratio, that is to say, how many units of futures positions correspond to 1 unit spot position. The early analysis of optimal hedging ratio for futures was based on the Markowitz mean-variance framework and developed by Working [1], Johnson [2], and Stein [3]. They considered hedging as making portfolios between spot market and futures market. The existing research can be divided into two categories: the static hedging model and the dynamic hedging model.

Most common static hedging methods use the slope of the spot-to-futures linear regression as the optimal hedging ratio. Those static models assume that the optimal hedging ratio does not change with time. However, the constant hedging ratio cannot reflect the time-varying characteristics of spot and futures prices. Ederington [4] calculated the optimal hedging ratio by OLS method and proposed a way to measure the effectiveness of hedging. Ghosh [5] and Sim and Zurbruegg [6] have shown that the error correction model (ECM) is a good way to further characterize the responses of the spot and futures prices to the error correction term in different systems. However, the ECM cannot handle stationary time series data and also assumes that the hedging ratio does not change with time.

Researches on dynamic hedging ratio were mainly based on the autoregressive conditional heteroskedasticity (ARCH) model and the generalized autoregressive conditional heteroscedasticity (GARCH) model [7], because the financial time series data always has time-varying second moment. The ARCH and GARCH model are widely used to observe the impact of time-varying characteristics and the cointegration between spot and futures prices on hedging ratio. There are also a lot of researches on the optimal hedging ratio in china, such as Wang et al. [8], Peng and Ye [9], Shao [10]. In recent years, the static models, such as OLS, ECM or VAR develop slowly. Scholars preferred an accurate description of the volatility characteristics of asset prices and time-varying of hedging ratio. So a lot of dynamic hedging models that are based on GARCH were developed, such as CCC-GARCH (1990), BGARCH (1993), MRS-GARCH (2004), DADCC-GARCH (2011), Copula-ECM-GARCH (2011) and MRS-DCC-GARCH (2015). The principle of all these models is the combinations of GARCH and various distribution functions to describe the volatility characteristics of the residual series of the return on assets. The purpose of doing so is to accurately depict the volatility of asset prices. However, in practice, these models ignore an important issue, that is, the changing frequency of spot prices and futures prices is inconsistent. Spot prices are more low-frequency data, and futures prices are high-frequency data. Therefore, in practice, the accurate description of prices fluctuations is meaningless. The pursuit on precise description of time-varying is the same.

We can find that the static hedging method is mainly OLS and ECM models and the dynamic methods are mainly GARCH and its extended model. In other words, OLS, ECM and GARCH models are the most widely and the basic models used in hedging. It can be also found that, the existing researches calculated the hedging ratio within a given time frame, but ignored the impact of different stages of futures and spot prices on their hedging effect, and the resulting changes of the basis. Because the psychological state of investors is completely different when the prices rise, fall or stable, the different stages of prices change have a great impact on hedging practices. This will directly affect the hedging ratio and hedging effect.

Therefore, this paper will analyze and compare the hedging effects of three models in three different prices stages and three different basis stages. Then, we will analyze the applicability of the three models in different stages.

2. **Introduction of Three Models.**

(1) OLS

Ordinary least square method is the main method of traditional regression model which was proposed by Witt et al. [11]:

$$\begin{aligned} \Delta S_t &= \alpha + h\Delta F_t + \varepsilon_t \\ h &= Cov(\Delta S_t, \Delta F_t)/Var(\Delta F_t) \end{aligned} \tag{1}$$

In Equation (1), ΔS_t and ΔF_t represent the spot and futures prices changes; α is the intercept term of the regression function. h is the slope of the regression function, which also means the hedging ratio (minimum variance hedge ratio). ε_t is the random error. OLS model only depicts the static relationship between futures and spot prices, which assumes that the changes in futures and spot prices are stable and residual items subject to the white noise process.

(2) ECM

ECM takes account of the non-stationary, long-run equilibrium and short-run dynamic relationships between futures and spot prices.

$$\begin{aligned} \Delta S_t &= C_s + \lambda_s Z_{t-1} + \sum_{i-1}^{l} \alpha_{si}\Delta S_{t-i} + \sum_{i-1}^{l} \beta_{si}\Delta F_{t-l} + \varepsilon_{st} \\ \Delta F_t &= C_f + \lambda_f Z_{t-1} + \sum_{i-1}^{l} \alpha_{fi}\Delta S_{t-1} + \sum_{i-1}^{l} \beta_{fi}\Delta F_{t-1} + \varepsilon_{ft} \\ \Delta S_t &= \alpha + h\Delta F_t + \sum_{i-1}^{m} Y_i\Delta S_{t-1} + \sum_{j-1}^{n} \theta_j\Delta F_{t-j} + \omega Z_{t-1} + \varepsilon_t \end{aligned} \tag{2}$$

Here Z_{t-1} is error correction term and is a stationary linear combination of spot prices and futures prices. At least one of λ_s or λ_f is no zero. The regression coefficient h of ΔF_t is the hedging ratio to be estimated. ECM incorporates the cointegration relationship of the futures and spot prices and the error correction term that includes the long-run equilibrium mechanism and the short-term non-equilibrium. It can also describe the short-term volatility of the variable.

(3) GARCH model

GARCH model can describe the volatility clustering, that is, the GARCH model can be applied when the data series show ARCH effect. The variance equation of a normal GARCH (p, q) model is:

$$\begin{aligned} &\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \gamma_1 \sigma_{t-1}^2 + \cdots + \gamma_p \sigma_{t-p}^2 \\ &\varepsilon_t = v_t \sqrt{\sigma_t^2} \end{aligned} \tag{3}$$

Here p is the autoregressive order of σ_t^2, q is the lag order of ε_t^2, v_t is a white noise, and ε_t is disturbance term. We use the widely used GARCH $(1, 1)$ model in this paper to estimate hedging ratio. The variance equation is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 \tag{4}$$

Here $\sigma_t^2 > 0$, $\alpha_0 > 0$, $\alpha_1 > 0$, $\gamma_1 > 0$. This assumption also makes the GARCH model ignore the residual asymmetric effects.

3. **Data Selection and Statistical Description.**

(1) Division of three stages

Copper is the most expensive transaction variety in Chinese futures market, and its trading volume is relatively high, which made it a "benchmark" in the mind of professional investors. Therefore, we choose copper as the research tool to compare the hedging effect of OLS, ECM and GARCH models in Chinese futures market. Because the time characteristic is often active in a few months before the delivery date of futures contracts, we choose the futures contract of CU1606 and the data from November 2, 2015 to May 3, 2016 and the spot prices of the corresponding time span.

It can be seen from Figure 1 that during this period the futures and spot prices of copper have three stages which are rising, stable and falling. The solid line represents the spot price, and the dotted line indicates the futures prices. From the early November 2015 to the late November 2015 spot and futures prices were in the short-term continuous decline, and from the end of November 2015 to mid-January 2016 spot and futures prices were in the short-term relatively stable stage, and the rising stage was from mid-January 2016 to mid-March. So we set the data into three stages, which were decline stage from November 2, 2015 to November 23, 2015, stable stage from November 24, 2015 to January 19, 2016 and rising stage from January 20, 2016 to March 17, 2016. In this paper we will calculate the hedging ratio and the hedging effect of OLS, ECM and GARCH models separately in each stage.

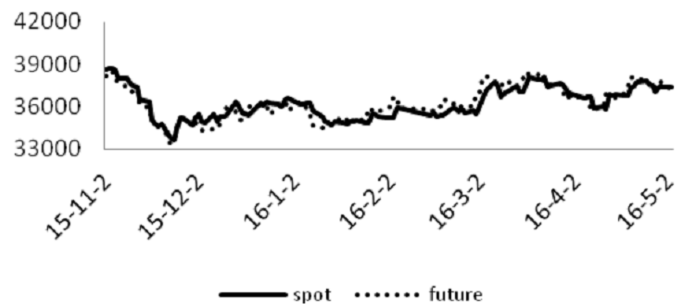

Figure 1. Spot and futures prices trend of copper

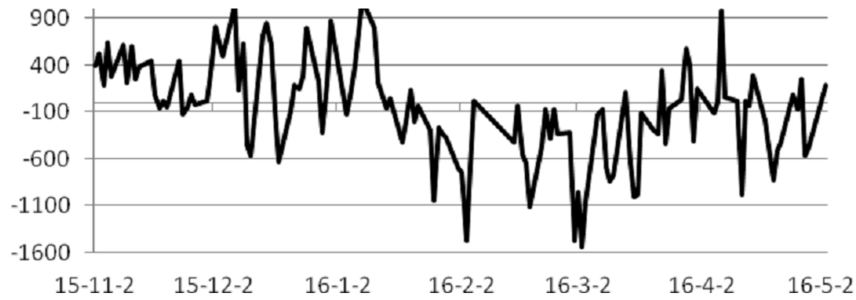

FIGURE 2. Basis of copper

TABLE 1. Cointegration test results

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−5.775665	0.0000
Test critical values:	1% level	−3.484198	
	5% level	−2.885051	
	10% level	−2.579386	

Changing of the basis has a direct impact on the hedging effect. Hedging is actually a replacement of the basis risks with spot prices volatility risks. In theory, if the basis has not changed from the beginning to the end of hedging, it is possible to achieve a full hedging. So basis is a variable that should be closely watched in the course of a transaction. There are three situations of basis which are negative, positive and zero. Its changes can also be divided into three stages which are going weak, strong and stable.

Figure 2 shows that during this period the basis of copper has three stages of negative, positive and zero. The positive stage was from November 2, 2015 to January 7, 2016. The changing stage was from January 8, 2016 to February 3, 2016 and negative stage was from February 4, 2016 to March 28, 2016. We will also calculate the hedging ratio and the hedging effect of OLS, ECM and GARCH models separately in those stages.

(2) Cointegration between spot and futures prices

Firstly, we use ADF to do the unit root test of spot prices and futures prices of copper. The results of ADF unit root test show that futures prices and basis are all nonstationary. Then we use ADF to test the first-order difference of spot prices and futures prices and found that they are all first order single integer sequence. So we need to use cointegration test to check whether those sequence has a long-term equilibrium relationship, which can avoid the pseudo-regression that is caused by the original nonstationary sequence. The test results are shown in Table 1.

We can see from Table 1 that there are cointegration relationships between spot prices and futures prices. It means a long-term equilibrium between spot and futures prices.

4. **Comparison of Three Models.**

(1) Hedging effect

Hedging effect refers to the percentage of variance reduction of rate of returns before and after hedging. Assume that R_H is the rate of return of a spot and futures portfolio. We can get the following equation:

$$R_H = R_P - hR_F \tag{5}$$

In Equation (5), R_F and R_P are the earning rate of futures and spot in the portfolio, and h means the hedging ratio. The variance of the portfolio returns is:

$$\sigma^2(R_H) = \sigma^2(R_P) + h^2 \times \sigma^2(R_F) - 2hCov(R_P, R_F) \tag{6}$$

The first-order condition of Equation (6) is:

$$h = Cov(R_P, R_F)/\sigma^2(R_F) \quad (7)$$

Then we substitute Equation (7) into Equation (6) and get the variance of return under risk minimization:

$$\sigma^2_{\min}(R_H) = \sigma^2(R_P)\left[1 - \rho^2(R_P, R_F)\right] \quad (8)$$

In Equation (8), $\rho(R_P, R_F)$ means the correlation coefficient of rate of return between spot and futures. Then we get the hedging effect:

$$H_e = 1 - \sigma^2_{\min}(R_H)/\sigma^2(R_P) = \rho^2(R_P, R_F) = h\sigma(R_F)/\sigma(R_P) \quad (9)$$

Now we can get the results in six stages in Table 2.

Table 2. Hedging effect of three models in six stages

Stages	Models	Hedging ratio	Hedging effect
Rising stage of prices	OLS	0.763472	0.827095
	ECM	0.781468	0.84659
	GARCH	0.792673	0.858729
Stable stage of prices	OLS	0.802343	0.827784
	ECM	0.803457	0.828933
	GARCH	0.795146	0.820358
Decline stage of prices	OLS	1.066742	1.25468
	ECM	1.094853	1.287744
	GARCH	1.064667	1.25224
Negative stage of basis	OLS	0.49481	0.509167
	ECM	0.45256	0.465691
	GARCH	0.421624	0.433857
Changing stage of basis	OLS	0.132714	0.189451
	ECM	0.219204	0.312916
	GARCH	0.131158	0.18723
Positive stage of basis	OLS	0.569718	0.777743
	ECM	1.058457	1.444939
	GARCH	0.131247	0.17917

(2) Analysis of calculation results

We use these three models to calculate the optimal hedging ratio respectively, and put the results in Table 2. There are barely noticeable differences among the optimal hedging ratio that is calculated by three models during the period of price decline. During the stable stage of the spot and futures prices, the optimal hedging ratio was significantly lower than the decline stage. In the period of prices increase, the hedging ratio is lower than the previous two stages. The optimal hedging ratio of three models differ greatly when divided according to basis. When the expected value of the basis is greater than zero, the hedging ratio of the ECM model is the highest, while the GARCH model is very low. When the basis changed from positive to negative in a short term, the hedging ratios are all relatively low and basically the same. When the expected value of the basis is less than zero, the results of three models are basically the same and stable. It should be noted that this time period is near the delivery date, and the basis converges from negative to zero gradually. Whether it is long hedge or short hedge, the investor's portfolio tends to be stable.

The results show that the hedging effect of GARCH model is the best in the rising stage of prices. It can be seen intuitively from Figure 1 that the futures and spot prices show a clustering and autocorrelation characteristics. The hedging effect of ECM model is the best in stable stage of prices. At this point there is a divergence between the futures and spot price and the basis changes have occurred. While the ECM takes account of the cointegration relationship between the spot and futures prices through the introduction of the error correction term in this case, the hedging effect of the ECM model is relatively better than the others in the decline stage of prices. OLS model and ECM are better than GARCH model in this stage. Because the spot and futures prices do not show obvious volatility clustering characteristics, that is, there is no ARCH effect, the spot and futures prices in this stage have an obvious linear relationship, which makes the hedging effect calculated by static model better than the dynamic model here.

When the basis is negative, the hedging effect of OLS model is the best. The average value of the basis in this stage can be deemed as fluctuation around 0 near the delivery day. It can be considered that the spot and futures prices change is relatively stable and has a strong linear characteristic here. So the OLS model is the most suitable at this time. The hedging effect of ECM is the best in changing stage and positive stage of basis. The spot and futures prices do not show clustering characteristics and ARCH effect, but they changed a lot in these two stages. So the cointegration relationship needs to be considered here and the ECM model behaves better in this case.

5. **Conclusions.** First, the empirical results show that the model with a high hedging ratio also has a relatively good hedging effect, if this relationship is a coincidence when only considering the minimum risk and if it is a reasonable relationship when the benefits are considered together. All those are the follow-up study of this article.

Secondly, the results of this study show that different hedging models have different hedging effects under different standards of classification when the other conditions are the same. This is determined by the algorithm and assumptions of hedging model. For example, the ECM model takes account of the equilibrium relationship between the spot and futures prices, and can reflect the short-term imbalance characteristics. The GARCH model is a dynamic model, which focuses on the clustering characteristics of data, while the OLS model only takes account of the linear relationship between the spot and futures prices. Essentially, the estimation algorithms of these three models are all based on the least squares method. This determines the applicability of different models in different stages. It also shows that investors should choose the hedging model dynamically according to the changing stage of prices and their risk acceptance level, rather than just selecting one dynamic model.

Finally, the good or bad hedging effect is only a relative concept. Although each hedging model aims at finding the optimal hedging ratio, they all have shortcomings in algorithm, so there is no absolute most superior model throughout the whole hedging period. There are three problems to be solved in future. The first is to further analyze the applicability of models in different volatility periods of asset prices from the algorithm and assumptions aspects. The second is to design a low frequency time-varying features hedging ratio model system. The third is to solve the mismatch problem of low frequency spot prices fluctuations and high frequency futures prices fluctuations.

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