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ACCURATE MOVING TARGETS POSITIONING WITH ANISOTROPIC GAUSSIAN SURFACE FITTING MODEL

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Abstract. *The identification of stars and satellites is one of the most important applications of space surveillance system. However, the features for target recognition are difficult to extract for star maps because of the point target imaging property and the influence of the dark space background. As a result, the position information of the targets becomes a key feature for target recognition. Gaussian surface fitting is one of the most widely used centroid extraction techniques. By theoretical analysis and tests, this paper shows that the traditional Gaussian surface fitting method has significant systematic error for moving satellites. This paper puts forward a new centroid extraction method based on anisotropic Gaussian surface fitting (AGSF) model which employs two different Gaussian fuzzy parameters and a rotation factor. AGSF can characterize the features for different directions and thus is suitable for the random direction blur caused by motion of satellites. The numerical results from real data demonstrate that, AGSF can extract the centers of the targets accurately, with positioning error 0.04 for real data.*
Keywords: Moving target positioning, Gaussian surface fitting model, Star map, Positioning accuracy, Anisotropic Gaussian surface fitting

1. **Introduction.** The location information is a key feature for star-map targets recognition. For instance, the position of stars can be used to identify stars by star library matching [1,2]; the satellites location information is always used to estimate the orbit information, which can be further used to identify satellites and orbit prediction.

However, it is of great difficulty to locate targets accurately in star maps. The star-map imaging process has a significant point spread fuzzy (PSF) effect [3,4] caused by factors such as hardware limitations of the sensor, the scattering effect of the lens and the interference of the atmosphere. As a result, star-map targets appear as spots centered by themselves but with certain degree of gray distribution to neighbor pixels. The commonly used algorithms for the target location in star maps include the grayscale centroid (GC) method [5-7], the weighted grayscale centroid (WGC) method [8-10], the Gaussian surface fitting (GSF) method [11-13] and the Parabolic Surface Fitting (PSF) method [12]. The GSF method is currently the most accurate centroid extraction technology. However, it is only suitable for round-shape (or nearly round-shape) objects, and for instance stars, since the model describes the point target diffusion effect. The traditional Gaussian model cannot be used to extract the centroid of satellites because satellites have obvious linear features due to the motion effect.

To address this issue, this paper proposes a new centroid extraction method based on anisotropic Gaussian surface fitting (AGSF) model. The model employs two different Gaussian fuzzy parameters to describe the linear characteristics, and also introduces a rotation factor to characterize the random moving direction of satellites. It is shown that the traditional Gaussian surface fitting model is a special case of our model with two equal Gaussian fuzzy parameters and zero rotation angle. Based on the advanced model, the proposed method enables discovery of moving objects and unknown objects without a priori knowledge about the target location, orientation, length, or surface brightness, which the conventional techniques can hardly accomplish. We present the results from the application of this method to real star maps. The result shows that the ASGF can be applied to precisely positioning both the stars and the moving satellites and it outperforms the compared techniques in terms of accuracy.

The rest of the paper is organized as follows. A detailed information of the proposed method is provided in Section 2. The evaluation experiments on real star maps are conducted in Section 3. Finally, Section 4 shows some remarking conclusions.

2. Anisotropic Gaussian Surface Fitting Centroid Extraction Method.

Assuming that $f(x, y)$ is the brightness value of pixel (x, y) in image I and (x_0, y_0) is the center of the target. The point spread function for GSF can be expressed as follows:

$$f(x, y) = A \cdot \exp\left\{-\frac{1}{2\sigma^2}\left[(x - x_0)^2 + (y - y_0)^2\right]\right\} \tag{1}$$

where A is a fixed coefficient, which equals the brightness of the target center; σ is the Gaussian function mean square error determined by the imaging conditions. The GSF method has higher accuracy than other methods [14,15] and can achieve sub-pixel accuracy. However, the conventional GSF method is only suitable for round-shape objects.

Figure 1 shows a real star map, in which the moving satellite targets are framed by the rectangles. We can see that the targets have clear linear feature, which cannot be characterized by Formula (1). To address this issue, this paper presents the anisotropic Gaussian surface fitting method.

Star-map consists of mainly two categories of targets, namely, stars and moving satellites. The stars have circular (or nearly circular) shapes since they can be considered stationary in imaging time, as shown in Figure 2(a). On the other hand, satellites have obvious linear features due to the large displacement in imaging time. Moreover, the direction of the satellite trajectory is uncertain as the moving direction is random, which may appear as the ones shown in Figure 2(b) or other angles.

From Figure 2, we can see that, to extract the centroid of the moving objects accurately, two problems need to be addressed: (1) the moving targets have linear feature, and thus the model has to quantitatively characterize the linear feature; (2) the moving direction of targets is random, and thus the model needs to adapt to different moving directions.

In the traditional Gaussian surface fitting model, as shown in Formula (1), σ controls the degree of the point diffusion effect, which determines the size of the diffused target. In

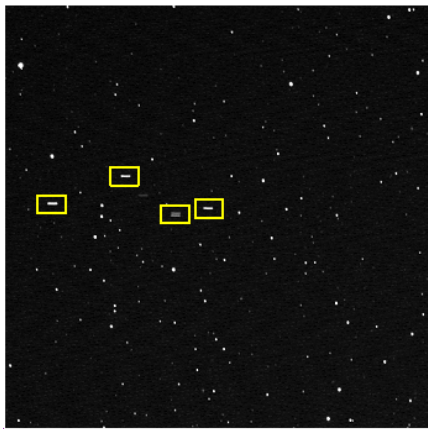
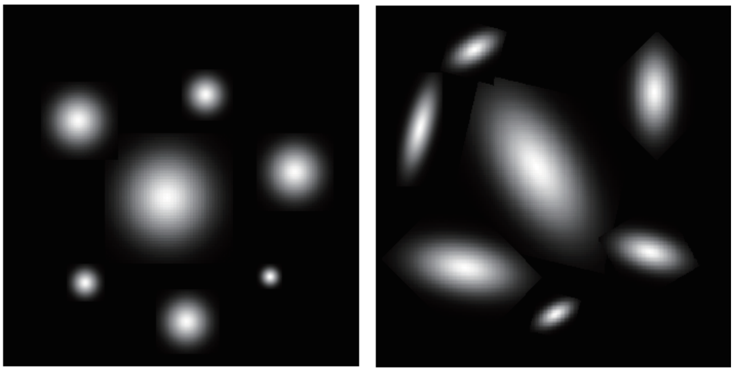


Figure 1. Real star map



(a) (b)

FIGURE 2. Typical targets of star maps: (a) stars of round shape with different sizes, (b) satellites having linear features with different directions and different sizes

order to describe the linear feature of the target, we can use different σ parameters for x direction and y direction respectively, which can make the target have different diffusion effects in x direction and y direction. In this way, the vertical or horizontal linear feature can be characterized by the different sizes of x direction and y direction.

Regardless of the moving direction, we first construct the centroid extraction model with two different diffusion parameters in x and y directions, as follows:

$$f(x, y) = A \exp\left\{-\frac{1}{2}\left[\frac{(x-x_0)^2}{\sigma_x^2} + \frac{(y-y_0)^2}{\sigma_y^2}\right]\right\} \tag{2}$$

where A is a fixed coefficient, which equals the brightness of the target center; σ_x and σ_y are the Gaussian function mean square errors of x direction and y direction respectively, and (x_0, y_0) is the target centroid.

Taking the randomness of the moving direction into consideration, we introduce the rotating coordinate system to enable the model to adapt to different moving directions. Without loss of generality, we take the 60° moving direction as the example. The original coordinate system is x-y; rotate the original coordinate system counterclockwise by 60°, and we obtain a new coordinate system $\hat{x}$-$\hat{y}$. In the new coordinate system, the moving direction of the target is horizontal, as shown in Figure 3. As for the other moving directions, we can simply adjust the rotation angles correspondingly.

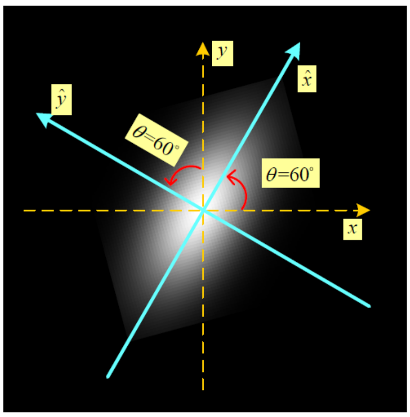


FIGURE 3. The random moving directions that can be characterized by rotating coordinate system

Assuming counterclockwise rotation is positive, the transform matrix from the original coordinates to the new coordinate system is

$$\mathbf{R}_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}. \tag{3}$$

By Formula (3), we can see that the coordinates of point (x, y) in the original coordinates system is $(x\cos\theta + y\sin\theta, x\sin\theta - y\cos\theta)$ in the new coordinate system. Substitute the new coordinates into Formula (2), and we have:

$$\begin{aligned} &f(x,y) \\ &= A\exp\left\{-\frac{1}{2}\left[\frac{((x-x_0)\cos\theta + (y-y_0)\sin\theta)^2}{\sigma_x^2} + \frac{((x-x_0)\sin\theta - (y-y_0)\cos\theta)^2}{\sigma_y^2}\right]\right\}, \end{aligned} \tag{4}$$

where θ is the counterclockwise angle between the target moving direction and the horizontal direction.

From the centroid extraction model expressed in Formula (4), it can be seen that this model uses different diffusion parameters for the x direction and the y direction respectively, which can characterize the linear feature of the target. At the same time, the rotation angle θ is employed to describe the random moving directions of the target. Interestingly, we can see that, when $\sigma_x = \sigma_y$ and $\theta = 0°$, this model degrades to the traditional Gaussian model as Formula (1). Therefore, the traditional Gaussian surface fitting model is in fact a special case of our new model which is anisotropic and can be applied to extracting the location of arbitrary star-map targets (stars or satellites).

In order to obtain the target centroid (x_0, y_0), we need to estimate all the parameters in Formula (4) by observed star maps. In the following, we discuss the solving method for Formula (4). Calculate the logarithm for both sides for Equation (4), and we have:

$$\ln(f) = \ln(A) - \frac{1}{2}\left[\frac{((x-x_0)\cos\theta + (y-y_0)\sin\theta)^2}{\sigma_x^2} + \frac{((x-x_0)\sin\theta - (y-y_0)\cos\theta)^2}{\sigma_y^2}\right]. \tag{5}$$

It can be seen that Formula (5) is a quadratic function of x and y, and thus Equation (5) can be expressed as:

$$\ln(f) = t_0 + t_1 x + t_2 y + t_3 xy + t_4 x^2 + t_5 y^2, \tag{6}$$

where parameters t_0, t_1, t_2, t_3, t_4, t_5 can be obtained by Equation (5). When t_0, t_1, t_2, t_3, t_4, t_5 are known, the six parameters in the model (Equation (4)) can be calculated as follows:

$$\begin{cases} x_0 = \dfrac{t_2 t_3 - 2t_1 t_5}{4t_4 t_5 - t_3^2} \\ y_0 = \dfrac{t_1 t_3 - 2t_2 t_4}{4t_4 t_5 - t_3^2} \\ \sigma_x^2 = \dfrac{t_4 + t_5 + \sqrt{(t_4 - t_5)^2 + t_3}}{t_3 - 4t_4 t_5} \\ \sigma_y^2 = \dfrac{t_4 + t_5 - \sqrt{(t_4 - t_5)^2 + t_3}}{t_3 - 4t_4 t_5} \\ \cos\theta = \dfrac{1}{2}(t_5 - t_4)\sqrt{(t_4 - t_5)^2 + t_3}(t_4 - t_5)^2 + t_3 \\ A = \exp\left\{t_0 - \dfrac{t_1 t_2 t_3^3 + t_1^2 t_4 t_5^2 + t_2^2 t_4^2 t_5 - 3t_1 t_2 t_3 t_4 t_5}{(t_4 t_5 - t_3^2)^2}\right\} \end{cases}. \tag{7}$$

Therefore, solving t_0, t_1, t_2, t_3, t_4, t_5 is the key point of the centroid extraction process. At least six pixels are required to participate in the computation, since there are six parameters unknown. Generally, the number of target and its dispersion pixels is greater than six. As a result, Equation (6) is over-determined; we use the least square method to calculate t_0, t_1, t_2, t_3, t_4, t_5. Assuming the coordinates of n target pixels are (x_i, y_i), $i = 1, 2, \ldots, n$ and the corresponding gray values are f_i, we construct the following $n \times 6$ matrix:

$$\mathbf{B} = \begin{bmatrix} 1 & x_1 & y_1 & x_1y_1 & x_1^2 & y_1^2 \\ 1 & x_2 & y_2 & x_2y_2 & x_2^2 & y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & y_n & x_ny_n & x_n^2 & y_n^2 \end{bmatrix}. \tag{8}$$

Let $\mathbf{t} = [t_0, t_1, t_2, t_3, t_4, t_5]^T$, and then Equation (6) can be expressed as follows:

$$\mathbf{Bt} = \begin{bmatrix} 1 & x_1 & y_1 & x_1y_1 & x_1^2 & y_1^2 \\ 1 & x_2 & y_2 & x_2y_2 & x_2^2 & y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & y_n & x_ny_n & x_n^2 & y_n^2 \end{bmatrix} \begin{bmatrix} t_0 \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix} = \begin{bmatrix} \ln f_1 \\ \ln f_2 \\ \vdots \\ \ln f_n \end{bmatrix}. \tag{9}$$

Let $\mathbf{f} = [\ln f_1 \ \ln f_2 \ \cdots \ \ln f_n]^T$, and we can solve $\mathbf{t}$ according to Equation (9) as follows:

$$\mathbf{t} = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{f}. \tag{10}$$

Based on the intermediate parameter t_0, t_1, t_2, t_3, t_4, t_5, the target centroid can be finally calculated using Formula (7).

In summary, the steps of using AGSF to extract the target centroid are listed as follows:

1) constructing the parameter matrix $\mathbf{B}$ by using Formula (8);

2) computing the logarithm of the pixels to construct the vector $\mathbf{f}$;

3) calculating the intermediate parameters t_0, t_1, t_2, t_3, t_4, t_5 according to Formula (10);

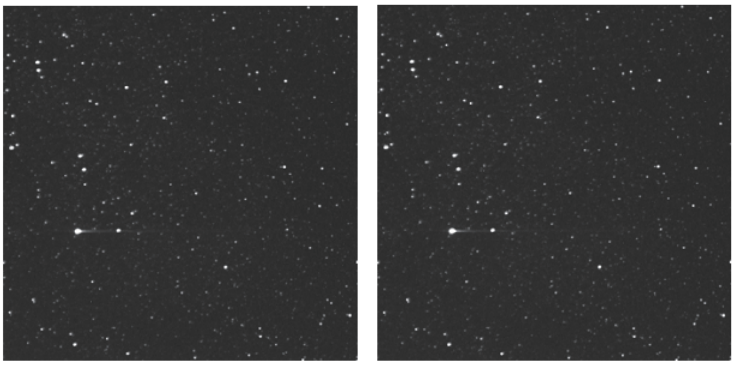
4) calculating the model parameters according to Formula (7), where (x_0, y_0) is the centroid coordinate of the target.

It is worth noting that this method requires calculating six parameters in the anisotropic Gaussian model. Thus the targets need to occupy at least six pixels, and in most cases, they can meet the requirement. If the target is less than six pixels, the bilinear interpolation method [16] can be used first to interpolate the target pixels, and then this method can be used to extract the centroid.

3. **Real Data Experiments.**

The real star maps used in this experiment are acquired by the large field (4.8°) 600mm aperture optical telescope of the Chinese Academy of Sciences. The software platform is Matlab 2010, and the hardware configuration is Intel dual-core 2.4G CPU and 1G memory. The astronomical positioning computer is a portable industrial computer equipped with P4 3.0 CPU. Levelling and north-seeking operation have been done to the equipment before acquiring the star maps. The data used in this experiment contains 60 frames of CCD images, with spatial size of 512×512 pixels. Two denoised frames are shown in Figures 4(a) and 4(b).

In this paper, the performances of the centroid technologies are compared in the following way. The centroid extraction results of each method have different levels of errors, which are random in different frames. Therefore, for the same stationary target (star), the method with larger positioning error must obtain more dispersive results from the 60-frame images, which is characterized by larger variances. Thus, the variance of the



(a) (b)

FIGURE 4. The original map: (a) the first frame, (b) the second frame

results from the 60-frame images for each method is adopted as the metric to measure the performance of these techniques. Smaller variance corresponds to higher accuracy.

It is noteworthy that the above comparison method can be used only to compare the accuracy of the stationary star targets. As for the satellite targets, the comparison is performed as follows. The motion trajectory of satellites can be approximately deemed as a straight line. Thus we first use the results obtained by these three methods to linearly fit the motion trajectory of the satellites. The estimated positions of the satellites in each image are regarded as the "true" positions. Then the error between the extracted positions and the real positions for each method is calculated, which can be used as the measure of the positioning accuracy of the satellite targets.

There are totally 317 targets in this data, among which 315 are stars and 2 are moving satellites. Each method has 60 centroid extraction results for each target. The results for one star are shown in Figure 5(a). The results for one satellite and trajectory fitting are shown in Figure 5(b). The positioning errors of the satellite for different methods are shown in Figure 5(c).

It can be seen from Figure 5(a) that, all the methods have some errors in terms of positioning the star. The error of the WGC method is the largest while that of the AGSF method is the least, implying that our method can improve the positioning accuracy of the stars. From Figure 5(b), we can see that, the trajectory of the satellite is approximately a straight line, and it is difficult to evaluate these methods only from the trajectory figure. The positioning errors shown in Figure 5(c) demonstrate that, WGC and GSF have larger errors in satellite positioning and the error of AGSF is much lower. To quantitatively evaluate the performances of these techniques, we calculate the mean values of the variances of positioning results of 315 stars and positioning errors of two satellites, as shown in Figure 6.

As shown in Figure 6, the positioning errors of stars for WGC, GSF and AGSF are 0.2647, 0.1153 and 0.0401 respectively while that of moving satellite targets are 0.2970, 0.3178 and 0.0425 respectively. It also can be seen that, compared to stars, the positioning accuracy of these methods on satellites are lower, which can be attributed to the motion effects of satellites. Moreover, the positioning error of satellites is significantly higher than that of the stars for GSF. As previously analyzed, the reason is that the GSF model does not adapt to moving satellite targets which leads to notable systematic errors.

4. **Conclusions.** This paper proposes a new centroid extraction algorithm for star-map targets based on anisotropic Gaussian surface fitting (AGSF). By using an anisotropic Gaussian model, AGSF can be applicable to both the moving satellites and the stationary stars, overcoming the problem of the traditional Gaussian surface fitting model that it is

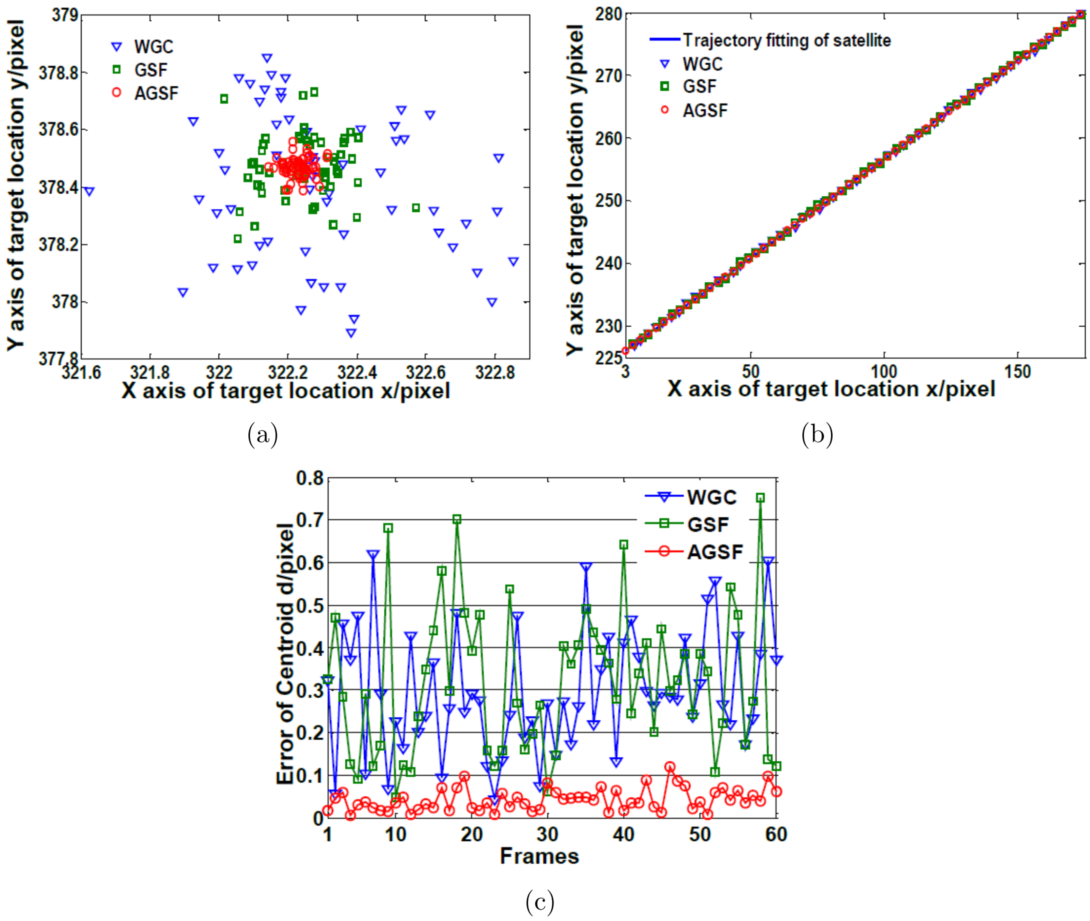


FIGURE 5. Results of real data: (a) center extraction results from different methods for one star, (b) the positioning results for one satellite and the trajectory fitting result, (c) centroid errors of each method for the satellite

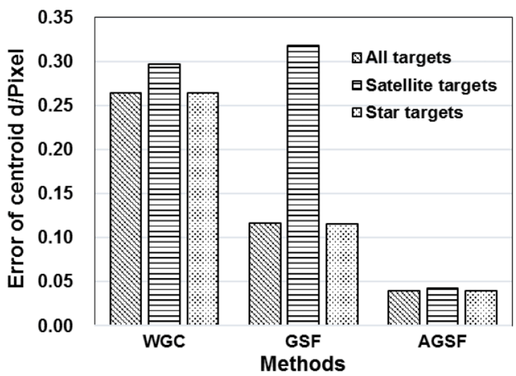


FIGURE 6. Real data test: from left to right the average error of all targets, the satellite targets and the star targets of each method

suitable only for circular targets. The real data experiments demonstrate that the AGSF can accurately extract the centroids of star targets and satellite targets in star maps, with the overall positioning accuracy 0.04 pixel. Compared with traditional methods, AGSF greatly improves the positioning accuracy of star-map targets, especially for satellites. In addition, the quantitative evaluation method for real data which has no true values is also a valuable reference for other studies. Future study may focus on the efficiency issues since the AGSF model has higher computational complexity than traditional methods.

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