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AN INTERFEROMETER DIRECTION FINDING METHOD WITH FEW SNAPSHOTS UNDER MULTIPLE SOURCES

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Abstract. *As with the number of emitters in electronic magnetic environment (EME) increasing, the possibility that multiple signals are received by receiver simultaneously is higher and higher. When the live time of signals are short, the classical signal separation technique based on Fourier transform cannot separate each signals in frequency domain and the interferometer direction finding technique cannot estimate the direction of arrival (DOA) of signals. Aiming at the DOA estimations that multiple signals impinge on the array at the same time, an interferometer multiple signals direction finding algorithm is proposed. With the proposed method, DOA of multiple signals can be obtained with limited snapshots. Finally, we demonstrate the validity of the algorithm by simulations and experiments.*

Keywords: Convex optimization, Frequency super-resolution, Interferometer direction finding, DOA, Sparse reconstruction

1. **Introduction.** Array signal processing using antenna arrays has been successfully applied to many engineering fields including wireless communications and radar systems [1, 2, 3]. Many direction finding methods have been developed. Until now, the interferometer direction finding method has been the most developed method because of its lower cost and faster DOA estimation speed. It has an important position in the engineering application field.

In the existing interferometer direction finding (DF) method, we use the instantaneous frequency measure (IFM) method to estimate the signal frequency firstly, and then obtain the DOA estimation based on the derived frequency estimation [4]. However, the instantaneous frequency measure method cannot estimate frequencies rightly when there are multiple signals exiting in the same receiver channel. Therefore, an existing solution to obtain the DOA estimations of multiple signals is based on the Fourier transform. Firstly, it estimates the frequency of each signal by the Fourier transform. And then the DOA estimation of each signal based on the frequency estimation is derived. However, as we all know, with the existing solution, the frequency resolution of multiple signals is limited to the snapshot number. When the signal snapshot number is insufficient, this method is invalid for multiple signals' DOA estimations. Therefore, in order to obtain the DOA estimations of multiple signals, we must develop a frequency estimation method of supper frequency resolution property. Fortunately, as one of the research hotspots in signal processing field, the compressive sensing (CS) has been applied in DOA estimation, radar imaging and so on [5, 6, 7]. Its core thought is depending on the sparsity of signal to reconstruct signal with insufficient signal samples, which is suitable to the frequency estimation of multiple signals. Sparse signal processing has been widely used in radar imaging [8], communication algorithm developing [9], and DOA estimation [10], etc. As far as we know, there is not any published literature addressing the multiple signals' DOA estimations with limited samples. Aiming at this issue, based on the fact that the frequencies of multiple signals are sparse in the whole frequency domain and the inverse Fourier transform matrix is of the restricted isometric property (RIP), we propose an interferometer multiple signals direction finding method. Firstly, we calculate the frequency support by sparse reconstruction technology based on the signal time samples and the inverse Fourier transforms matrix. And then we obtain the values of all frequency components based on signal samples and Fourier transform matrix. At last, we calculate the DOA of multiple signals based on each recovered frequency component.

The organization of this paper is as follows. Section 2 is the problem formulation of multiple signals direction finding with interferometer. Section 3 is the problem solution of multiple signal interferometer direction finding. The validity of the proposed method in this paper is addressed in Section 4 by computer simulations. Finally, the conclusion of the paper and the future work is presented in Section 5. In this paper, the scalar signal, vector signal and matrix signal are represented by lower case letters, bold lower case letters and upper bold case letters, respectively. $\|\cdot\|_1$, $\|\cdot\|_2$ and $(\cdot)^T$ are vector ℓ_1-norm, vector ℓ_2-norm and transpose operation, respectively.

2. **Problem Formulation.** As shown in Figure 1, two narrowband far field signals $\{s_1(n),\ s_2(n)\}$ impinge on an array with four sensors which are labeled by 1#, 2#, 3#, 4# and we consider the 1# antenna as the reference. We define $\{\theta_1, \theta_2\}$ as the DOAs of signals $\{s_1(n), s_2(n)\}$, separately. In this paper, the noise is additive white Gaussian noise. By assumption, not only are the noise and the incident signals uncorrelated, but also the noises between receiver channels are uncorrelated. The maximum frequency of receiver is B. Suppose the array output signals are sampled at the rate of Fs and the largest sampling number is T. Thus, the received signal of m# sensor at the n-th sample time can be written as

$$x_m(n) = \sum_{k=1}^{2} g_m(\theta_k) \exp\left(-j2\pi d_m \sin(\theta_k)/\lambda\right) s_k(n) + e_m(n), \tag{1}$$

where the antenna number $m = 1, 2, 3, 4$, the sampling time $n = 1, 2, \ldots, T$, $g_m(\theta_k)$ denotes the antenna pattern of m# sensor at θ_k, d_m denotes the distance between the reference antenna and m# antenna and $e_m(n)$ denotes the additive complex Gaussian noise of the m# receiver channel. The matrix formulation of (1) can be written as

$$\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{e}(n), \tag{2}$$

where

$$\mathbf{x}(n) = [x_1(n), x_2(n), x_3(n), x_4(n)]^T, \tag{3}$$

$$\mathbf{A}(\theta) = \begin{bmatrix} a_{1,1} & a_{2,1} & a_{3,1} & a_{4,1} \\ a_{1,2} & a_{2,2} & a_{3,2} & a_{4,2} \end{bmatrix}^T, \tag{4}$$

$$\mathbf{s}(n) = [s_1(n), s_2(n)]^T, \tag{5}$$

$$a_{m,k} = g_m(\theta_k) \exp\left(-j2\pi d_m \sin(\theta_k)/\lambda\right), \tag{6}$$

$$\mathbf{e}(n) = [e_1(n), e_2(n), e_3(n), e_4(n)]^T, \tag{7}$$

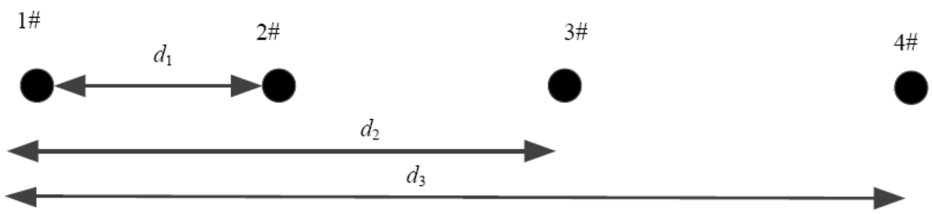

FIGURE 1. The linear array with 4 antennas

Based on the above hypothesis, we will address the procedures of how to get the DOAs of the two incident signals with interferometer direction finding method.

3. Problem Solution of Multiple Signals Direction Finding.

3.1. The super-resolution frequency estimation. In practical setting, we can select the signal of any receiver channel to calculate the frequency estimations. Without loss of generality, the signal of 1# receiver channel can be written as

$$\mathbf{y}(n) \triangleq [x_1(1), x_1(2), \ldots, x_1(T)]^T \tag{8}$$

Split the frequency scoped in $[0, B]$ into Q $(Q > T)$ frequency bins. The frequency interval of each frequency bin is B/Q. According to inverse Fourier transform, we can get

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{h} \tag{9}$$

In Equation (9), each element in the inverse Fourier matrix can be denoted as

$$\mathbf{H}(t, q) = \frac{1}{T} \exp\left\{ j\frac{2\pi}{N}(q-1)(t-1) \right\} \tag{10}$$

where $t = 1, 2, \ldots, T$, $q = 1, 2, \ldots, Q$, $\mathbf{h} \in \mathrm{C}^{Q\times 1}$ denotes the frequency components of signal. Because the number of variables is larger than the number of equation, it is underdetermined, and hence there are infinite solutions. However, the Fourier matrix has the RIP and the frequency components of multiple signals are limited in the whole frequency domain, namely the frequency components $\mathbf{h} \in \mathrm{C}^{Q\times 1}$ are sparse. The details of RIP can refer to article [11, 12, 13]. For Equation (10), if the matrix $\mathbf{H} \in \mathrm{C}^{T\times Q}$ has the RIP, we can recover the sparse vector $\mathbf{h} \in \mathrm{C}^{Q\times 1}$ with a high probability through searching for the optimum solution as follows [5, 6, 7],

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \|\mathbf{h}\|_1 \ \text{ s.t. } \mathbf{y} = \mathbf{H} \cdot \mathbf{h} \ \text{ (Ideal Case)} \tag{11}$$

or

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \|\mathbf{h}\|_1 \ \text{ s.t. } \|\mathbf{y} - \mathbf{H} \cdot \mathbf{h}\|_2 \leq \eta \ \text{ (Noisey Case)} \tag{12}$$

In Equation (12), η is the residual threshold factor, which is determined by noise level. Obviously, Equation (12) is a convex problem, which can be solved by classical sparse reconstruction algorism or convex package of software.

We must point out that, in theory, when the number of signal sampled at the rate of Fs in time domain is T, the frequency resolution is Fs/T with the fast Fourier transform method, which cannot separate two signals whose frequency interval is smaller than Fs/T. However, through sparse reconstruction algorithm to estimate frequency, the frequency resolution is Fs/Q $(Q > T)$. Therefore, the frequency resolution in the above method is higher than the one in traditional fast Fourier transform method.

3.2. Solution of multiple signals direction finding. As shown in Figure 1, 1# element is the reference antenna, the other antennas consist of three baselines with the reference antenna, and the lengths of the three baselines are d_1, d_2, d_3, separately. Firstly, it uses the method above to estimate the frequencies of two incident signals, which denote as f_1, f_2. And then, we can get the phase differences $\{\varphi_{f_1,1}, \varphi_{f_1,2}, \varphi_{f_1,3}\}$ and $\{\varphi_{f_2,1}, \varphi_{f_2,2}, \varphi_{f_2,3}\}$ of the three baselines. Denote the unambiguous phase differences as $\{\phi_{f_1,1}, \phi_{f_1,2}, \phi_{f_1,3}\}$ and $\{\phi_{f_2,1}, \phi_{f_2,2}, \phi_{f_2,3}\}$. Without loss of generality, we will give the method to estimate the DOA of the signal whose frequency is in the following.

Step 1. Choose the baseline whose length is d_1 as the base to reverse the ambiguity of other baselines. So we can get $\phi_{f_1,1} = \varphi_{f_1,1}$.

Step 2. Define $d_m = k_m c/f_1$, $d_m = r_{m,1} d_1$, and then we can get

$$-k_m \times 2\pi \leq \phi_{f_1,m} \leq k_m \times 2\pi \tag{13}$$

$$\hat{\phi}_{f_1,m} = \varphi_{f_1,m} + 2N_m\pi,\ -k_m \le N_m \le k_m,\ m = 2, 3 \tag{14}$$

Step 3. Based on the unambiguity phase difference $\varphi_{f_1,1}$ to get estimation $\phi_{f_2,m}$, i.e.,

$$\hat{\phi}'_{f_1,m} = r_{m,1}\varphi_{f_1,1} \tag{15}$$

Step 4. Search for the optimum solution of $\hat{N}_m$ by Equation (16),

$$\hat{N}_m = \arg\min_{N_m} \left|\hat{\phi}_{f_1,m} - \hat{\phi}'_{f_1,m}\right| \quad \text{s.t. } N_m \in \{-k_m, -k_m+1, \ldots, k_m-1, k_m\} \tag{16}$$

Step 5. According to $\hat{N}_m$, we can get $\phi_{f_1,m}$ as follows,

$$\phi_{f_1,m} = \varphi_{f_1,m} + 2\hat{N}_m\pi \tag{17}$$

Step 6. Based on $\phi_{f_1,m}$ to estimate the DOA of signal,

$$\theta_1 = \arcsin\left(\frac{\frac{c}{f_1}\sum_{m=2}^{3}\phi_{f_1,m}}{2\pi\sum_{m=2}^{3}d_m}\right) \tag{18}$$

In Equation (18), constant c is the speed of light. Similarly, we can get the DOA estimation of another signal whose frequency is f_2 with the above method.

4. Simulation.

4.1. **Simulation 1.** In this simulation, consider there are two signals incident into array. The center frequencies of them are 100MHz and 100.7MHz, separately. The bandwidths of both are all 100kHz. The modulated types of their baseband signal are BPSKs. During the 6.4μs signal duration, we can get 128 sampling data at the sampling rate of 20MHz.

Under ideal case, the spectrum of the baseband is shown at the upper left subplot of Figure 2, where the line shown with ':' belongs to the signal whose center frequency is 100MHz and the line shown with '—' belongs to the signal whose center frequency is 100.7MHz.

As shown at the upper right subplot of Figure 2, it is the spectrum estimation result of the 128 samples under the circumstance of the signal to noise radio (SNR) being 15dB. In theory, when the number of signal data sampled at the rate of 20MHz in time domain is 128, the frequency resolution is 156.3kHz. Therefore, when the center frequency interval is 700kHz, they can be separated.

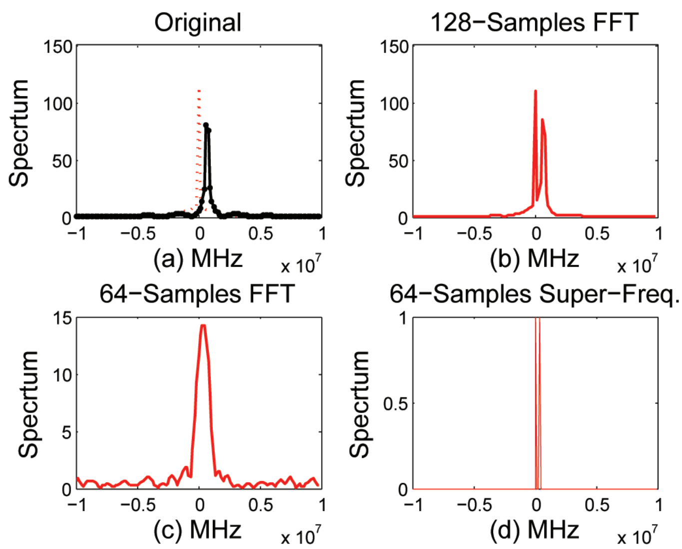

FIGURE 2. The baseband spectrum of signals

As shown at the left lower subplot of Figure 2, it is the spectrum estimation result of the 64 samples using the Fourier transform method under the circumstance of SNR being 15dB. In theory, when the number of signal data sampled at the rate of 20MHz in time domain is 64, the frequency resolution is 312.5kHz. Although the center frequency interval of both signals is larger than 312.5kHz, using the Fourier transform method cannot separate the two signals because of the effect of noise.

As shown at the right lower subplot of Figure 2, it is the spectrum estimation result of the 64 samples using super-resolution frequency estimation method under the circumstance of SNR being 15dB. The two signals can be separated. In summary, with the proposed method, it can increase the spectral resolution.

4.2. **Simulation 2.** In this simulation, consider there are two signals incident into array. The 50 DOAs at 50 different incident times of the two signals are shown in the above of Figure 3 separately. During the 3.2μs signal duration, we can get 64 sampling data at the sampling rate of 20MHz. Firstly, we can get the frequencies estimation of the two signals using the method proposed in this paper. And then, we can get the two DOA estimations according to the frequencies estimation. As shown at the bottom subplot of Figure 3, they are the root mean squared error (RMSE) curves of DOA estimation at 50 different DOAs. At each incident time, the RMSE is obtained by 100 Monte Carlo experiments. Through the simulation results above, we can conclude that the DOAs of multiple signals can be estimated right with classical interferometer direction finding with the proposed method in this paper. The angle RMSE is within 2°.

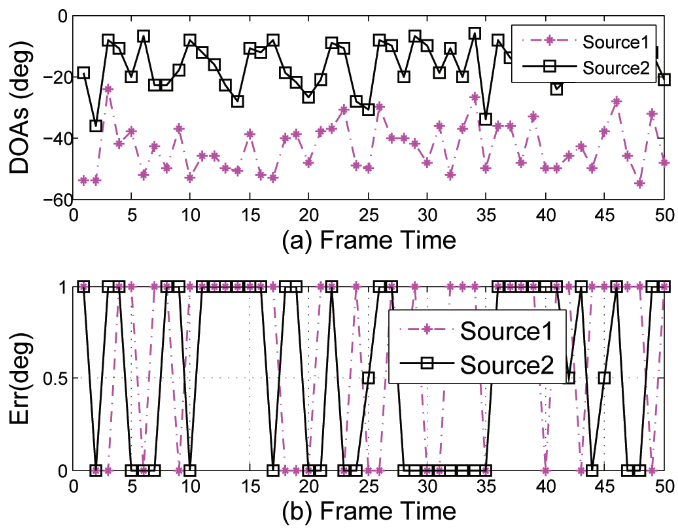

Figure 3. DOA error versus time

4.3. **Simulation 3.** In order to compare the advantages of our method with the existing version proposed in the reference [14]. In this experiment, we also consider the case that there are two signals incident into array. The 50 DOAs at 50 different incident times of the two signals are shown in the above of Figure 4 separately. All parameters in simulation are the same as in experiment 2. The simulation results with 100 Monte Carlo trials are shown in Figure 4. As it cannot separate multiple signals being with same frequency with existing method, the DOA estimation error is increased dramatically. According to the results of simulation 2 and simulation 3, it shows the advantages of the proposed method than the traditional version in multiple sources scenario.

5. **Conclusion.** In the paper, focusing on the drawback that the existing interferometer direction finding method cannot estimate the angle rightly when the incident signals are

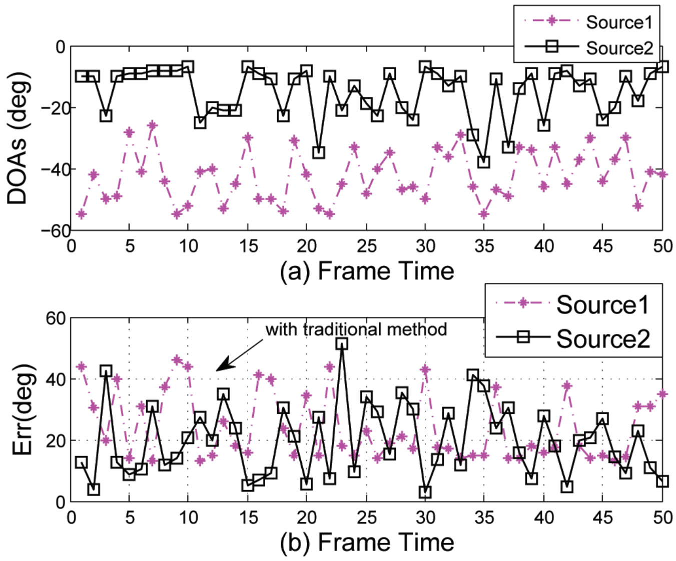

FIGURE 4. DOA error versus time

more than one, we propose an interferometer DF method with limited sampling data aiming at the angle estimations when multiple signals impinge on the array at the same time. It can improve the frequency resolution of interferometer multiple signals DF method. And it applies to the multiple signals DF system with limited sampling data especially. The validity of conclusions are verified by simulations and experiments. The method proposed in this paper has a good engineering application value. In future work, we plan to use this method in the practical engineering.

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