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A MULTIPLE PARAMETER ESTIMATION METHOD FOR DISTRIBUTED MIMO-OFDM SYSTEM

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Abstract. *An efficient method of multiple carrier frequency offsets (MCFOs) and multiple channel responses (MCRs) estimation algorithm was proposed for distributed multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) system, which utilizes the polynomial root finding method and the iterative interference cancellation procedure. The scheme transforms the multi-dimensional frequency searching function into multi one-dimensional searching functions by using the optimum training sequences, which is specially designed, and turns these one-dimensional functions into closed-form estimators by Taylor series expanding. MCFOs are obtained by rooting the polynomial and MCRs are obtained through solving the likelihood function. In order to avoid the problem that the estimation performance is not accurate because the length of pilot is short, the proposed algorithm utilizes the iterative interference cancellation method to get more data to improve the performance. Simulation results show that the proposed algorithm can achieve* $1\mathrm{e}-4$ *MCFOs estimation performance and* $1\mathrm{e}-3$ *MCRs estimation performance when the normalized CFO is 0.1 and signal to noise (SNR) is 5dB.*
Keywords: Distributed MIMO-OFDM, Multiple carrier frequency offsets, Multiple channel responses, Polynomial rooting, Iterative inference cancellation

1. **Introduction.** The transmitting antennas are placed in distributed geographical location, connected to the same signal processing center by fibers or cables; such multiple input multiple output (MIMO) system is called distributed MIMO system [1]. Distributed MIMO system can overcome the path loss caused by the "near far effect" and "shadow effect", and solve the problem of cell communication blind zone. Orthogonal frequency division multiplexing (OFDM) technology has been successfully applied in the third generation wireless communication system, which is a key technology in the next generation wireless communication system. The combination of OFDM and distributed MIMO system can improve the system capacity, the spectrum efficiency, and the reliability of system. Different from the traditional MIMO-OFDM system, the receiving ends of distributed MIMO system receive the sum of different transmitting signals, multiple carrier frequency offsets (MCFOs) and multiple channel responses (MCRs) that exist in the system [2]. Therefore, compared with the traditional single CFO estimation problem, the MCFOs and MCRs estimation problem seems more challenging.

Existing References [3-6] estimate MCFOs and MCRs by using the maximum likelihood (ML) algorithm. However, ML algorithm needs to jointly optimize the multi-dimensional likelihood function of MFCOs and MCRs, in which the algorithm complexity is increased as the number of parameters increases. [7,8] utilizes the orthogonal pilots to simplify the multi-dimensional likelihood function to multiple one-dimensional likelihood functions, and analyzes the influence of pilots on the MCRs estimation performance. However, it still needs one-dimensional search on the carrier frequency offset (CFO) likelihood function. [9] introduces the Taylor series expansion to get the closed polynomial of CFO likelihood function, and gets MCFOs by rooting the polynomial. [10] utilizes the importance sampling method to transform the integration part in CFO likelihood function to the sum form, then gets MCFOs. However, the performance of [9,10] still needs to be improved.

In this paper, a low complexity and high accuracy MCFOs and MCRs estimation algorithm is proposed. This algorithm utilizes the orthogonal pilots to transform the multi-dimensional likelihood function of MCFOs and MCRs to multiple one-dimensional likelihood functions, then takes Taylor series expansion to transform the one-dimensional likelihood function to the closed polynomial. MCFOs are obtained by rooting the polynomial, and MCRs are obtained by solving the likelihood function. Also, iterative interference cancellation method is used to get more data to improve the estimation performance. In this paper, Section 2 gives the signal model of distributed MIMO-OFDM system, and then Section 3 gives the proposed algorithm. Section 4 gives the complexity analysis. Finally, Sections 5 and 6 are the simulation result and conclusion respectively.

2. **Signal Model.** Suppose there are N_t transmitting base stations (BSs) in the intercept mode of distributed MIMO system, each BS transmits its own OFDM signal, and the number of subcarriers is N. There are N_r receiving BSs, and each one has one receiving antenna. The transmitted OFDM signal in frequency domain of the p-th BS is $\boldsymbol{x}_p = [x_p(0), x_p(1), \ldots, x_p(N-1)]^{\mathrm{T}}$, after inverse fast Fourier transform (IFFT) and adding the cyclic prefix (CP), the receiving signal of the q-th receiving BS is,

$$y_q(i) = \sum_{p=1}^{N_t} e^{j2\pi f_{q,p} i/N} \sum_{l=0}^{L-1} h_{q,p}(l) x_p(i-l-\mu_{q,p}) + w_q(i), \quad i = 0, 1, \ldots, N+N_{cp}-1 \tag{1}$$

where $f_{q,p}$ is the normalized CFO of the q-th receiving BS to the p-th transmitting BS (normalized to the subcarrier space), $h_{q,p}(l)$ is the channel response (CR), L is the number of multipath. $\mu_{q,p}$ is the integer timing error, and $w_q(i)$ is the complex additive Gaussian white noise whose mean and variance are zero and σ_w^2 respectively. Let L and $\mu_{q,p}$ limit in the same range, that is $0 < L + \mu_{q,p} \leq \Delta$, where Δ is the maximum delay spread of multipath channel. Because OFDM contains CP, it can be thought that time delay estimation is contained in the CR estimation, the time delay estimation is not discussed here, then $0 < L \leq \Delta$. Inserting CP into OFDM to suppress the inter symbol interference (ISI) which is caused by the multipath delay, the maximum delay spread should be smaller than N_{cp}, that is $\Delta < N_{cp}$ and $L < N_{cp}$. After getting rid of CP, (1) can be rewritten in matrix form,

$$\boldsymbol{y}_q = \sum_{p=1}^{N_t} \boldsymbol{C}_{q,p} \boldsymbol{X}_p \boldsymbol{h}_{q,p} + \boldsymbol{w}_q = \tilde{\boldsymbol{X}}_q \boldsymbol{h}_q + \boldsymbol{w}_q \tag{2}$$

where CFO vector is,

$$\boldsymbol{C}_{q,p} = diag\left(e^{j2\pi f_{q,p}\cdot 0/N}, e^{j2\pi f_{q,p}\cdot 1/N}, \ldots, e^{j2\pi f_{q,p}\cdot (N-1)/N}\right) \tag{3}$$

Signal matrix is,

$$\boldsymbol{X}_p = \begin{bmatrix} x_p(0) & x_p(N-1) & \cdots & x_p(N-L+1) \\ x_p(1) & x_p(0) & \cdots & x_p(N-L+2) \\ \vdots & \vdots & \ddots & \vdots \\ x_p(N-1) & x_p(N-2) & \cdots & x_p(N-L) \end{bmatrix} \tag{4}$$

$\boldsymbol{h}_{q,p} = [h_{q,p}(0), h_{q,p}(1), \ldots, h_{q,p}(L-1)]^{\mathrm{T}}$ is CR vector, $\boldsymbol{w}_q = [w_q(0), w_q(1), \ldots, w_q(N-1)]^{\mathrm{T}}$ is noise vector, and

$$\begin{aligned} \tilde{\boldsymbol{X}}_q &= [\boldsymbol{C}_{q,1}\boldsymbol{X}_1, \boldsymbol{C}_{q,2}\boldsymbol{X}_2, \ldots, \boldsymbol{C}_{q,N_t}\boldsymbol{X}_{N_t}] \\ \boldsymbol{h}_q &= \left[\boldsymbol{h}_{q,1}^{\mathrm{T}}, \boldsymbol{h}_{q,2}^{\mathrm{T}}, \ldots, \boldsymbol{h}_{q,N_t}^{\mathrm{T}}\right]^{\mathrm{T}} \end{aligned} \tag{5}$$

3. **The Proposed Joint Estimation Algorithm of MCFOs and MCRs.** ML algorithms estimate MCFOs and MCRs by maximizing the joint likelihood function; if given $\boldsymbol{f}_q$ and $\boldsymbol{h}_q$, the conditional probability density function of $\boldsymbol{y}_q$ is,

$$\begin{aligned} \ln p\left(\boldsymbol{y}_q | \boldsymbol{f}_q, \boldsymbol{h}_q\right) &= -\frac{N}{2}\ln(2\pi\sigma_w^2) - \frac{1}{2\sigma_w^2}\left[\boldsymbol{y}_q - \tilde{\boldsymbol{X}}_q\boldsymbol{h}_q\right]^{\mathrm{H}}\left[\boldsymbol{y}_q - \tilde{\boldsymbol{X}}_q\boldsymbol{h}_q\right] \\ &= C_0 - C_1\left[\boldsymbol{y}_q - \tilde{\boldsymbol{X}}_q\boldsymbol{h}_q\right]^{\mathrm{H}}\left[\boldsymbol{y}_q - \tilde{\boldsymbol{X}}_q\boldsymbol{h}_q\right] \end{aligned} \tag{6}$$

where $C_0 = -\frac{N}{2}\ln\left(2\pi\sigma_w^2\right)$, $C_1 = \frac{1}{2\sigma_w^2}$, $(\cdot)^{\mathrm{H}}$ represents conjugate transpose. Suppose $\boldsymbol{f}_q$ is known, deviating (6) and setting the result zero, the ML estimator of $\boldsymbol{h}_q$ is,

$$\hat{\boldsymbol{h}}_q = \left[\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\right]^{-1}\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{y}_q \tag{7}$$

And the ML estimator of $\boldsymbol{f}_q$ is,

$$\hat{\boldsymbol{f}}_q = \arg\max_{\boldsymbol{f}_q}\left\{\boldsymbol{y}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\left[\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\right]^{-1}\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{y}_q\right\} \tag{8}$$

It can be seen from (8) that, $\hat{\boldsymbol{f}}_q$ needs multi-dimensional search. As the antennas of transmitting end and receiving end increase, the complexity of computing $\left[\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\right]^{-1}$ increases. The proposed algorithm utilizes orthogonal pilots to transform multi-dimensional likelihood function of MCFOs and MCRs to multiple one-dimensional likelihood functions. By using the optimum pilot [7], $\left[\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\right]^{-1}$ can be turned into real value diagonal matrix, that is $\left[\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\right]^{-1} \approx N\boldsymbol{I}$, where $\boldsymbol{I}$ is unit matrix, then (8) can be rewritten as,

$$\begin{aligned} \boldsymbol{y}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\left[\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\right]^{-1}\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{y}_q &= \boldsymbol{y}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{y}_q \\ &= \sum_{p=1}^{N_t}\sum_{q=1}^{N_r}\boldsymbol{y}_q^{\mathrm{H}}\boldsymbol{C}_{q,p}\tilde{\boldsymbol{X}}_q\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{C}_{q,p}^{\mathrm{H}}\boldsymbol{y}_q \\ &= \sum_{p=1}^{N_t}\sum_{q=1}^{N_r}Q_p^{-1}\left|\sum_{i=0}^{N-1}y_q(i)x_p^*(i)e^{-j2\pi i f_{q,p}/N}\right|^2 \end{aligned} \tag{9}$$

where Q_p represents each BS's power. From (9), it can be seen that, N_t dimensional likelihood function is transformed into N_t one-dimensional likelihood functions, and each item in summation can be solved by FFT. However, it still needs searching processes on the frequency domain, and the estimation precision is influenced by frequency resolution and search scope. In order to achieve high estimation precision, large data and high signal-to-noise ratio (SNR) are needed. To solve the above problems, a kind of estimation algorithm without searching is given below.

3.1. **The MCFOs estimation algorithm of rooting the polynomial.** The proposed algorithm takes Taylor series expansion of $\boldsymbol{C}_{q,p}$ in the p-th BS [9],

$$\boldsymbol{C}_{q,p} = \sum_{m=0}^{\infty}\left.\frac{\partial^m\left(\boldsymbol{C}_{q,p}\right)}{\partial(f_{q,p})^m}\right|_{f_{q,p}=f_{q,p,0}} \cdot \frac{\left(f_{q,p} - f_{q,p,0}\right)^m}{m!} = \sum_{m=0}^{\infty}\Lambda^m(f_{q,p,0}) \cdot \frac{\left(f_{q,p} - f_{q,p,0}\right)^m}{m!} \tag{10}$$

$$\begin{aligned}\Lambda^m(f_{q,p,0}) = & \left(j\frac{2\pi}{N}\right)^m \cdot diag\left[0^m, 1, \ldots, (N-1)^m\right] \\ & \odot diag\left(e^{j2\pi f_{q,p,0}\cdot 0/N}, e^{j2\pi f_{q,p,0}\cdot 1/N}, \ldots, e^{j2\pi f_{q,p,0}\cdot (N-1)/N}\right)\end{aligned} \tag{11}$$

where $\odot$ represents multiplication of corresponding elements of matrixes. Let the p-th function related to CFO be $J(f_{q,p})$,

$$\begin{aligned}J(f_{q,p}) &= \sum_{q=1}^{N_r} \boldsymbol{y}_q^{\mathrm{H}} \boldsymbol{C}_{q,p} \tilde{\boldsymbol{X}}_q \tilde{\boldsymbol{X}}_q^{\mathrm{H}} \boldsymbol{C}_{q,p}^{\mathrm{H}} \boldsymbol{y}_q \\ &= \sum_{m=0}^{M'} a_m^{(p)} (f_{q,p} - f_{q,p,0})^m + \sum_{m=M'+1}^{\infty} a_m^{(p)} (f_{q,p} - f_{q,p,0})^m\end{aligned} \tag{12}$$

where the first term on the right side of (12) is the valid term $J_{PRS}(f_{q,p})$, which approximates the CFO function as much as possible, the second item is the remainder term. M' represents the order of $J_{PRS}(f_{q,p})$, $a_m^{(p)}$ is the coefficient of polynomial. Taking the first derivative of $J_{PRS}(f_{q,p})$, and setting the result zero, it has

$$\frac{\partial J_{PRS}(f_{q,p})}{\partial f_{q,p}} = \sum_{m=0}^{M'-1} (f_{q,p} - f_{q,p,0})^m c_m^{(p)} = 0 \tag{13}$$

where $c_m^{(p)} = (m+1) a_{m+1}^{(p)}$, and

$$c_m^{(p)} = \frac{1}{m!} \sum_{i=1}^{m+1} \binom{m+1}{i} R\left\{\sum_{q=1}^{N_r} \boldsymbol{y}_q^{\mathrm{H}} \Lambda^i(f_{q,p,0}) \tilde{\boldsymbol{X}}_p \left(\tilde{\boldsymbol{X}}_p\right)^{\mathrm{H}} (\Lambda^{m+1-i}(f_{q,p,0}))^{\mathrm{H}} \boldsymbol{y}_q\right\} \tag{14}$$

Let $\xi_{q,p}$ be the real value root of (14) when $\left.\frac{\partial^2 J_{PRS}(f_{q,p})}{\partial f_{q,p}^2}\right|_{f_{q,p}=\xi_{q,p}} < 0$, MCOs can be got by rooting the $M'-1$ order polynomial, then the CFO of the p-th BS is,

$$\hat{f}_{q,p} = f_{q,p,0} + \xi_{q,p}, \quad p = 1, 2, \ldots, N_t \tag{15}$$

Taking $\hat{f}_{q,p}$ into $\boldsymbol{C}_{q,p}$, it has,

$$\hat{\boldsymbol{C}}_{q,p} = \sum_{m=0}^{M'-1} \Lambda^m(f_{q,p,0}) \cdot \frac{\left(\hat{f}_{q,p} - \hat{f}_{q,p,0}\right)^m}{m!} \tag{16}$$

From (7), CRs are

$$\hat{\boldsymbol{h}}_q = \hat{\boldsymbol{X}}_q^{\mathrm{H}} \boldsymbol{y}_q \tag{17}$$

3.2. **The iterative interference cancellation algorithm.** Although the training sequence described before can be used to cancel the interferences between signals, as a result of short training sequences, the data used to estimate MCFOs and MCRs are little, estimation error is large. If the data are increased, the estimation performance can be improved. The iterative interference cancellation algorithm is taken to get more data. The proposed algorithm takes the estimated value of MCFOs and MCRs as known parameters, then cancels other antennas' interferences from the receiving signals, and estimates the determined antenna's parameters. Suppose $\hat{f}_{q,p}$ and $\hat{\boldsymbol{h}}_p$ have been got, and MCFOs are included into MCRs and signals, (1) can be rewritten as,

$$y_q(i) = \sum_{p=1}^{N_t} \sum_{l=0}^{L_p-1} \tilde{h}_{q,p}(l) \tilde{x}_p(i-l) + w_q(i), \quad i = 0, 1, \ldots, N-1 \tag{18}$$

where $\tilde{h}_{q,p}(l) = e^{j2\pi \hat{f}_{q,p} l/N} \hat{h}_{q,p}(l)$, $\tilde{x}_p(i) = e^{j2\pi \hat{f}_{q,p} i/N} x_p(i)$. Transforming (18) into matrix form,

$$\boldsymbol{y}_q = \tilde{\boldsymbol{H}}_q \tilde{\boldsymbol{x}} + \boldsymbol{w}_q \tag{19}$$

where

$$\begin{aligned} &\boldsymbol{y}_q = [y_q(0), y_q(1), \ldots, y_q(N-1)]^{\mathrm{T}} \\ &\tilde{\boldsymbol{h}}_q(l) = \left[\tilde{h}_{q,1}(l), \tilde{h}_{q,2}(l), \ldots, \tilde{h}_{q,N_t}(l)\right] \\ &\boldsymbol{w}_q = [w_q(0), w_q(1), \ldots, w_q(N-1)]^{\mathrm{T}} \\ &\tilde{\boldsymbol{x}} = [\tilde{x}_1(-(L-1)), \ldots, \tilde{x}_{N_t}(-(L-1)), \ldots, \tilde{x}_1(N-1), \ldots, \tilde{x}_{N_t}(N-1)]^{\mathrm{T}} \end{aligned} \tag{20}$$

$$\tilde{\boldsymbol{H}}_q = \begin{bmatrix} \tilde{\boldsymbol{h}}_q(L-1) & \cdots & \tilde{\boldsymbol{h}}_q(0) & & & \\ & \tilde{\boldsymbol{h}}_q(L-1) & \cdots & \tilde{\boldsymbol{h}}_q(0) & & \\ & \ddots & & & \ddots & \\ & & & \tilde{\boldsymbol{h}}_q(L-1) & \cdots & \tilde{\boldsymbol{h}}_q(0) \end{bmatrix} \tag{21}$$

Using least square (LS) algorithm to equalize (19),

$$\hat{\tilde{\boldsymbol{x}}} = \left(\hat{\tilde{\boldsymbol{H}}}_q^{\dagger}\hat{\tilde{\boldsymbol{H}}}_q\right)^{-1}\hat{\tilde{\boldsymbol{H}}}_q^{\dagger}\boldsymbol{y}_q \tag{22}$$

where $(\cdot)^{\dagger}$ means conjugate transpose. Utilizing (22), (2) and the p-th signal's estimated CFO and CR, the p-th signal's non-training sequence is,

$$\hat{\boldsymbol{y}}_{q,p}^k = \hat{\boldsymbol{C}}_{q,p}^k \hat{\tilde{\boldsymbol{x}}}_p^k \hat{\boldsymbol{h}}_{q,p}^k, \quad 1 \le p \le N_t \tag{23}$$

where

$$\hat{\boldsymbol{C}}_{q,p}^k = diag\left(e^{j2\pi\hat{f}_{q,p}\cdot 0/N}, e^{j2\pi\hat{f}_{q,p}\cdot 1/N}, \ldots, e^{j2\pi\hat{f}_{q,p}\cdot(N-1)/N}\right),$$

$$\hat{\tilde{\boldsymbol{x}}}_p^k = \left[\hat{\tilde{x}}_p(-(L-1)), \ldots, \hat{\tilde{x}}_p(N-1)\right]^{\mathrm{T}},$$

k means the k-th iterative. In order to reduce the interference between signals, subtract other signals from receiving signals,

$$\tilde{\boldsymbol{y}}_{q,p}^k = \boldsymbol{y}_q - \sum_{m=1,m\neq p}^{N_t} \hat{\boldsymbol{C}}_{q,m}^k \hat{\tilde{\boldsymbol{x}}}_m^k \hat{\boldsymbol{h}}_{q,m}^k \tag{24}$$

$\tilde{\boldsymbol{y}}_{q,p}^k$ is the p-th non-training sequence, then the new CFO $\hat{f}_{q,p}^{k+1}$ and CR $\hat{\boldsymbol{h}}_{q,p}^{k+1} = \hat{\tilde{\boldsymbol{x}}}_p^{\mathrm{H}} \hat{\boldsymbol{C}}_{q,p}^{(k+1)\mathrm{H}} \tilde{\boldsymbol{y}}_{q,p}^k \big/ N$ can be obtained by using the proposed estimation algorithm. Repeat above steps, MCFOs and MCRs of all receiving antennas can be got. Taking the obtained parameters into (23) to equalize the receiving signals, then more non-training sequences and their MCFOs and MCRs can be obtained. The above algorithm can increase the length of data and improve the estimation performance.

4. **Complexity Analysis.** First, the complexity of ML algorithm is given. From (8), it can be seen that, $\boldsymbol{y}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{y}_q$ can be got by calculating $\boldsymbol{\Psi} = \tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{y}_q$ and $\boldsymbol{\Psi}^{\mathrm{H}}\boldsymbol{\Psi}$. Because $\tilde{\boldsymbol{X}}_q = [\boldsymbol{C}_{q,1}\boldsymbol{X}_1, \boldsymbol{C}_{q,2}\boldsymbol{X}_2, \ldots, \boldsymbol{C}_{q,N_t}\boldsymbol{X}_{N_t}]$, and $\boldsymbol{C}_{q,p}$ is a diagonal matrix, the complexity of $\boldsymbol{C}_{q,p}\boldsymbol{X}_p$ is $\boldsymbol{N}$, the complexity of $\boldsymbol{\Psi}$ is $\mathrm{O}(N_tN)$ and $\boldsymbol{\Psi}^{\mathrm{H}}\boldsymbol{\Psi}$ is $\mathrm{O}(N_tN)$. A searching process of the maximum value is needed on $\hat{\boldsymbol{f}}_q = \arg\max_{\boldsymbol{f}_q}\left\{\boldsymbol{y}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\left[\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\tilde{\boldsymbol{X}}_q\right]^{-1}\tilde{\boldsymbol{X}}_q^{\mathrm{H}}\boldsymbol{y}_q\right\}$, thus a search step needs to be set. Suppose the frequency resolution is f_{search}, then the overall complexity of the ML algorithm is $\mathrm{O}(N_tNf_{search})$.

In the proposed algorithm, the polynomial coefficient $c_m^{(p)}$ occupies most of computation. From (14), it can be got that the complexity of $\Lambda^i(f_{q,p,0})$ is N, then the complexity of $R\left\{\sum_{q=1}^{N_r}\boldsymbol{y}_q^{\mathrm{H}}\Lambda^i(f_{q,p,0})\tilde{\boldsymbol{X}}_p(\tilde{\boldsymbol{X}}_p)^{\mathrm{H}}\left(\Lambda^{m-i}(f_{q,p,0})\right)^{\mathrm{H}}\boldsymbol{y}_q\right\}$ is $\mathrm{O}(N_rN)$, $c_m^{(p)}$ is $\mathrm{O}(M'N_rN)$. Compared with the ML algorithm, the proposed algorithm has lower complexity.

5. **Simulation Results and Analysis.** In order to verify the correction of the theoretical analysis and the accuracy of the algorithm, this paper will conduct three experiments, and simulation experiments are all carried out under the MATLAB environment. The number of transmitting BS N_t is 2, each BS transmits its own OFDM signal, and the inner modulation is QPSK. Taking one OFDM symbol for example, the signal length is N, the length of CP is $N/4$. The path number of multipath channel L is 9, channel coefficients are random Gauss variables of zero mean and unit variance. The channel is tapped delay line model with Rayleigh fading coefficients and power delay distribution, the typical model is IEEE 802.11a [9]. The normalized CFO is 0.1 and -0.1. The pilots are $\boldsymbol{T}_1 = \boldsymbol{Z}$, $\boldsymbol{T}_2 = \boldsymbol{Z}\langle D_2 \rangle$, the prime M in pilot is 3, the length of cyclic shift D_2 is 43. SNR is $[0, 25]$dB, and the Monte Carlo simulation number is 100. Use average mean square error (AMSE) as the evaluation standard.

Experiment 1. Considering MCFOs and MCRs estimation performance under different SNRs and N between the proposed algorithm and Reference [9]. N is 128, 256, 512, 1024 respectively, and the number of receiving antennas N_r is 3. Reference [9] is based on polynomial algorithm without optimum pilots and iterative interference cancellation.

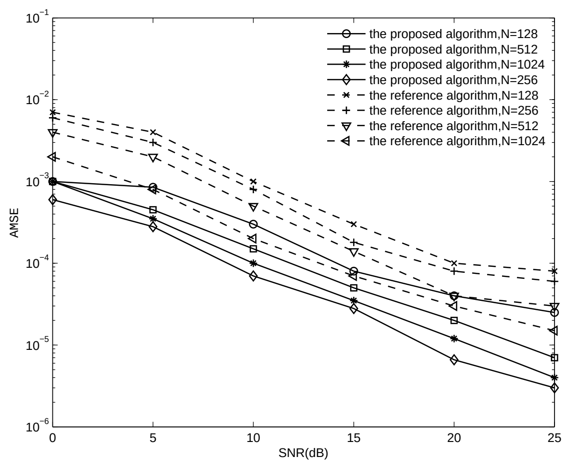


Figure 1. The performance of MCFOs estimation

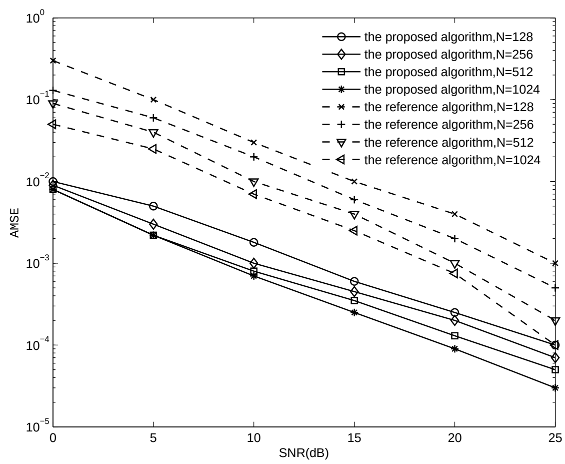


Figure 2. The performance of MCRs estimation

From Figure 1 and Figure 2, it can be seen that, as N increases, the performances of MCFOs and MCRs of the proposed algorithm and reference are both improved, and the performance of the proposed algorithm is better than that of the reference. When N increases to some value, like 1024, the improvement of the performance is not obvious. Considering the performance and the complexity of algorithms, N is chosen as 512.

Experiment 2. Considering MCFOs and MCRs estimation performance under different SNRs and N_r of the proposed algorithm and Reference [5]. N is 512, N_r is 2, 3, 5 respectively. Add asymptotic CRB (asCRB) [5] as the comparison standard.

From Figure 3 and Figure 4, it can be seen that as N_r increases, the performance of MCFOs and MCRs estimation are both improved, and can reach asCRB, which means the increasement of N_r can enhance the diversity effect, and plays a certain role in improving the performance. The proposed algorithm can achieve $1e-4$ CFO performance and $1e-3$ channel responses performance when SNR is 5dB.

Experiment 3. Considering the effect of M' on MCFOs estimation performance of the proposed algorithm when the normalized CFOs are 0.1, -0.1, 0.3, -0.3 respectively. M' is 2, 3, 4, 5. When M' is 5, decompose it into $M_1 = 3$, $M_2 = 2$ to avoid the uncertainty of rooting the polynomial. N_r is 3, and N is 512.

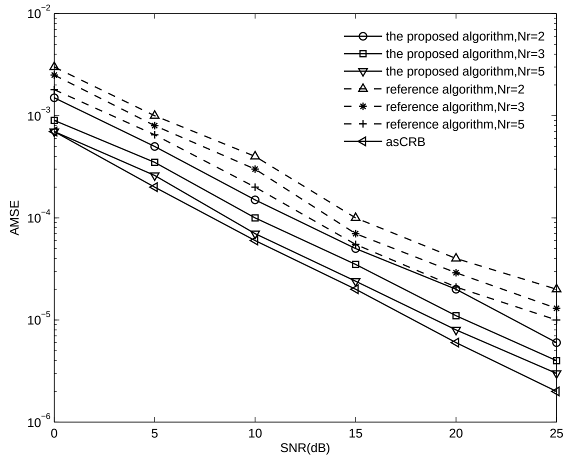


FIGURE 3. The performance of MCFOs estimation

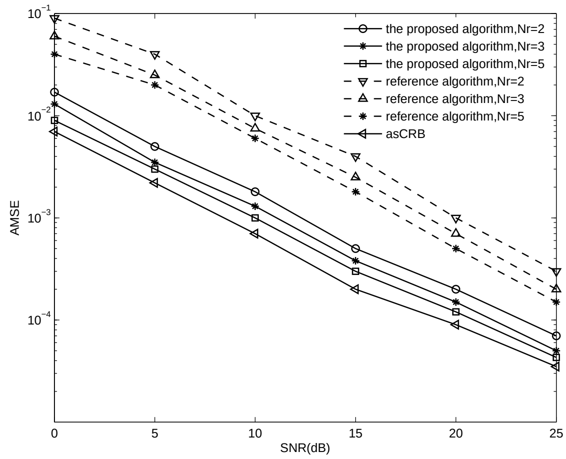
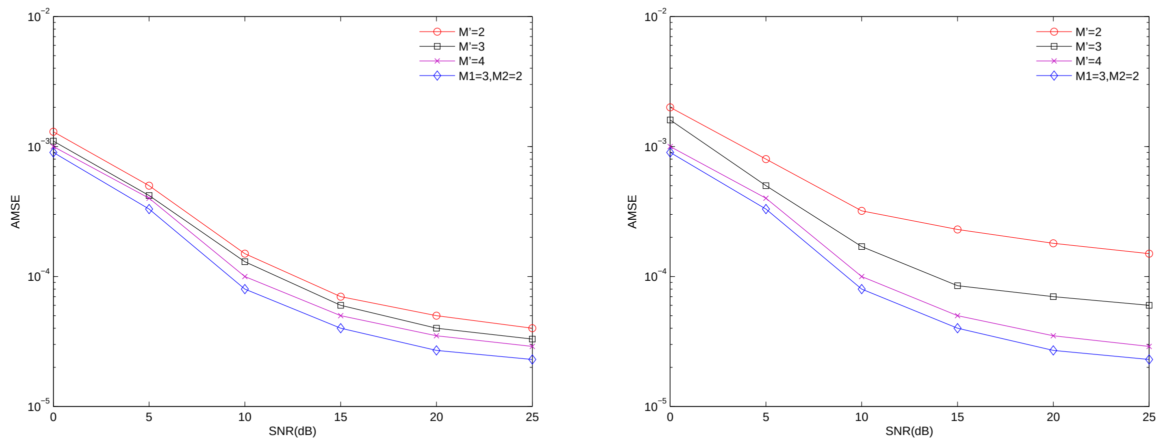


FIGURE 4. The performance of MCRs estimation



(a) The normalized CFOs are 0.1, -0.1. (b) The normalized CFOs are 0.3, -0.3.

FIGURE 5. The effect of M' for MCFOs estimation performance

When the normalized CFOs are 0.1 and -0.1, M' is 2, the performance of MCFOs estimation is good. When the normalized CFOs are 0.3 and -0.3, M' is 2 and 3, the performance is poor. When CFOs are large, and M' is 4 and 5, the performance became good. The simulation result explores that, when the normalized CFOs are larger, small polynomial order cannot be used to approximate the CFO function, larger polynomial order is needed.

6. **Conclusion.** Existing joint MCFOs and MCRs estimation algorithms based on pilots have a problem that the complexity of algorithms is high. To solve this, a kind of algorithm based on Taylor series expansion and polynomial rooting is proposed in this paper. This algorithm introduced the pilots to transform the multi-dimensional joint MCFOs and MCRs likelihood function into multi one-dimensional likelihood functions. Then Taylor series expansion is proposed to simplify the one-dimensional CFO search function, and CFO is obtained by rooting the polynomial, CR is got by solving the one-dimensional likelihood function. In order to avoid the poor performance because of short pilots, this algorithm utilizes the LS algorithm and iterative interference cancellation to equalize the transmitting data, the length of data is increased, and the performances of MCFOs and

MCRs estimation are improved. Simulation results show that, compared with existing algorithms, the proposed algorithm has better performance.

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