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# BLIND SYMBOL-RATE ESTIMATION BASED ON COMPRESSIVE CYCLIC STATISTICS

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Abstract. *In non-cooperative communication fields, modulation parameter estimation is the key step of subsequent processes, such as modulation classification and signal demodulation. As a powerful tool for frequency estimation, cyclic statistics are widely used for symbol-rate estimation. In this work, inspired by the development of compressive sensing (CS), we show that the cyclic statistics are compressible in the Fourier domain, and hence, symbol-rate can be estimated from nonuniform low-rate samples of incoming signal. As a result, the proposed compressive estimator can significantly relieve the sampling and storage burdens on the implementation of an intercept receiver. We also evaluate the performance of the proposed approach, and show that it is more effective compared with the traditional estimator.*
**Keywords:** Cyclic statistics, Compressive sensing (CS), Symbol-rate estimation

1. **Introduction.** Blind modulation parameter estimation is a technique to infer the modulation parameters of the received signal without any prior knowledge. It plays a crucial role in various applications such as software defined and cognitive radios, and spectrum surveillance and management. In this paper we investigate the estimation of symbol-rate of received signal, which is not only necessary for signal demodulating but also a key parameter for modulation recognition. In recent years, blind symbol-rate estimation based on cyclic statistics is adopted as a simple and efficient approach. The idea behind is that, for linearly modulated signals, known as the cyclostationary signal, the second-order cyclic spectrum exhibits a local maximum at the symbol-rate [1-3].

However, in this method, the higher-than-Nyquist rate sampling is often required to induce the cyclostationarity and the reliable estimates of cyclic statistics, which imposes the heavy computational and storage burdens on hardware implementation, especially when the received signal has high bandwidth. To overcome these barriers, we appeal to the recent emerging theory of compressive sensing (CS), which claims that a signal having a sparse representation in some basis can be reconstructed from a small set of samples collected via random linear projections. By exploiting the sparsity of cyclic statistics in the Fourier domain and applying a rough CS recovery algorithm, we argue that the symbol-rate can also be estimated with the second-order compressive cyclic spectrum derived from a significantly small amount of nonuniform samples. Moreover, the nonuniform samples can be directly acquired at a sensor, and a large reduction in the sampling and computation costs can be achieved.

The rest of the paper is organized as follows. Section 2 provides the signal model. In Section 3, we explain the sparsity of cyclic statistics, which validates the parameter estimation in the framework of CS. Section 4 provides a weighting method for peak detection applied in symbol-rate estimation. Simulation and performance analysis are presented in Section 5. Finally, the conclusion is summarized in Section 6.

2. **Signal Model and Problem Statement.** In practice, the received signal can be expressed as a continuous-time signal:

$$x(t) = ae^{j2\pi f_c t+\theta} \sum_k s_k p(t - kT - t_0) + n(t) \tag{1}$$

where $a$ is the signal amplitude, $f_c$ denotes the carrier frequency and is treated as the intermediate frequency in this work, $\theta$ is the carrier phase, $p(t)$ is the raised root cosine (RRC) pulse, $T$ represents the symbol period, which implies symbol-rate $f_b = 1/T$. $t_0$ is the symbol timing offset, and $n(t)$ is the additive Gaussian noise. $s_k$ is the $k$th complex data symbol, and is assumed independent and identically distributed with zero mean and unit variance ($E\{|s_k|^2\} = 1$).

Then, through a sampling process with the oversampling rate $\rho$, the corresponding discrete-time signal is yielded as

$$x[l] = ae^{j\theta} e^{j\frac{2\pi}{\rho} f_c T l} \sum_k s_k p[l - k\rho - l_0] + n[l] \tag{2}$$

where $l_0 = t_0\rho/T$, which is not necessarily an integer, and $n[l]$ is the wide-sense stationary complex noise sequence.

Our concerns can be described as follows: given $\{x[l]\}_{l=0}^{N-1}$, estimate symbol-rate of the received signals without the parameters $\{a, \theta, s_k, l_0\}$.

3. **Analysis of Compressive Cyclic Statistics.** For the purpose of making this paper self-contained, we first provide some definitions of involved statistical functions.

The $n$th-order/$q$-conjugate temporal moment (TM) is defined by

$$r_x(t, \boldsymbol{\tau})_{n,q} = E\left\{ \prod_{i=1}^{n} x^{(*)_i}(t + \tau_i) \right\} \tag{3}$$

where $E\{\cdot\}$ is the expectation operation, and $(*)$ represents the one of $q$ conjugations. To illustrate the underlying periodicities, the temporal cyclic moment (TCM) is given by

$$r_x^{\alpha}(\boldsymbol{\tau})_{n,q} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} r_x(t, \boldsymbol{\tau})_{n,q} e^{-j2\pi\alpha t} dt, \ r_x^{\alpha}(\boldsymbol{\tau})_{n,q} \neq 0 \tag{4}$$

where $\alpha$ is the impure $n$th-order cycle frequency (CF), and $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_n]$ is the delay-vector. It is obvious that the set of values of $\alpha$ for which $r_x^{\alpha}(\boldsymbol{\tau}) \neq 0$ is denumerable.

Correspondingly, when coping with finite length of samples, the TCM can be estimated as

$$\widetilde{r}_x^{\alpha}[\boldsymbol{\sigma}]_{n,q} = \frac{1}{N} \sum_{l=0}^{N-1} p_x[l, \boldsymbol{\sigma}]_{n,q} e^{-j2\pi\alpha l}, \ \widetilde{r}_x^{\alpha}[\boldsymbol{\sigma}]_{n,q} \neq 0 \tag{5}$$

where $\sigma_i = \tau_i\rho/T$, and $p_x[l, \boldsymbol{\sigma}]$ denotes the $n$th-order lag product of the input $x[l]$, i.e., $p_x[l, \boldsymbol{\sigma}]_{n,q} = \prod_{i=1}^{n} x^{(*)_i}[l + \sigma_i]$.

By representing quantities $\{p_x[l, \boldsymbol{\sigma}]_{n,q}\}_{l=0}^{N-1}$ by vector $\boldsymbol{p_x}$, we have

$$\boldsymbol{\eta} = \boldsymbol{\Phi} \boldsymbol{p_x} \tag{6}$$

equivalently,

$$\boldsymbol{p_x} = \boldsymbol{\Phi}^{\mathrm{H}} \boldsymbol{\eta} \tag{7}$$

where $\boldsymbol{\Phi}$ is the $N$-point discrete Fourier transform (DFT) matrix, $\boldsymbol{\eta}$ is the corresponding Fourier coefficient vector, and $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose operator. Evidently, we can infer that the non-zero elements of $\boldsymbol{\eta}$ are actually the TCMs, and their index implies the CFs. However, in practice, the non-zero elements of $\boldsymbol{\eta}$, also appear at other frequencies besides the CFs, due to the nonlinear transformation and the finite number of samples. Fortunately, TCMs are usually extremely larger than the others; thus, we can still claim that TCMs are compressible in Fourier domain [4].

Thus, we consider applying the CS approach to the field of cyclic statistics. Assume that received signal $x(t)$ undergoes a compressive sampling system, and we obtain the nonuniform sample vector $\boldsymbol{x_c}$, where $\boldsymbol{x_c} = \{x_c[0], \ldots, x_c[M-1]\}$. The sampling process is actually implemented by a matrix multiplication: $\boldsymbol{x_c} = \boldsymbol{\Psi x}$, where $\boldsymbol{\Psi} \in \{0,1\}^{M\times N}$, $M \ll N$, is the measurement matrix, and $\boldsymbol{x} = \{x[0], \ldots, x[N-1]\}$ is the uniform sample vector. It should be noted that $\boldsymbol{\Psi}$ is a binary matrix with only one single 1 in a random position of each row; moreover, in each column, there exists at most one 1. Clearly, we deduce $\boldsymbol{p_c} = \boldsymbol{\Psi p_x}$, where $p_c[m, \boldsymbol{\sigma}]_{n,q} = \prod_{i=1}^{n} x_c^{(*)_i}[m+\sigma_i]$. From (7), it holds that $\boldsymbol{p_c} = \boldsymbol{\Psi}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{\eta}$. As a consequence, the problem is formulated as a typical CS problem

$$\text{minimize}\|\hat{\boldsymbol{\eta}}\|_0 \text{ subject to } \boldsymbol{p_c} = \mathbf{A}\hat{\boldsymbol{\eta}} \tag{8}$$

where $\mathbf{A} = \boldsymbol{\Psi}\boldsymbol{\Phi}^{\mathrm{H}}$.

Nevertheless, to achieve the recovery of $\boldsymbol{\eta}$ with high probability, the matrix $\mathbf{A}$ need satisfy the so-called restricted isometry property (RIP). By examining the product $\mathbf{A} = \boldsymbol{\Psi}\boldsymbol{\Phi}^{\mathrm{H}}$, it is easy to see that $\mathbf{A}$ is actually a submatrix consisting of $M$ random rows of the inverse DFT matrix $\boldsymbol{\Phi}^{\mathrm{H}}$. When the number of measurements $M$ is larger than a fixed bound provided by [5], which is dependent of the length and sparsity of $\boldsymbol{\eta}$, the RIP is met and the exactly recovery of $\boldsymbol{\eta}$ can be achieved by solving a convex optimization problem or by using a variety of greedy iterative algorithms. Besides, the ratio $M/N$ is defined as the compression factor denoted by $\gamma$.

Since our goal is equivalent to estimating the frequencies at which certain cyclic statistics locate, it seems not necessary to recover the vector $\boldsymbol{\eta}$ accurately. By borrowing the idea of compressive signal processing (CSP) [6], which is applied to fields of detection, classification and estimation problems without reconstructing the full-scale signal, we define the simple estimate of $\boldsymbol{\eta}$ [7]

$$\hat{\boldsymbol{\eta}} = \mathbf{A}^{\mathrm{H}}\boldsymbol{p_c} = \boldsymbol{\Phi}\boldsymbol{\Psi}^{\mathrm{H}}\boldsymbol{p_c} \tag{9}$$

which means that $\hat{\boldsymbol{\eta}}$ can be computed by zero-padding $\boldsymbol{p_c}$ (creating a length-$N$ vector containing the $M$ entries of $\boldsymbol{p_c}$ at the nonuniform sample locations) and taking the FFT of this zero-padded vector.

Result in [8] guarantees the accuracy of this estimate. Although $\boldsymbol{\eta}$ cannot be exactly sparse in practice, our experimental results confirm that $\hat{\boldsymbol{\eta}}$ still provides a suitable, efficient estimate of $\boldsymbol{\eta}$ in the proposed method. For example, with the compression factor $\gamma = 0.1$, the curve in Figure 1(b) shows $|\hat{\boldsymbol{\eta}}|$, while the curve in Figure 1(a) shows $|\boldsymbol{\eta}|$. We see that positions of peaks in the right-hand curve, although weaker, coincide with the positions of the peaks in the left-hand curve respectively. It is noted that in this context, we assign the value of 0-frequency component of spectrum to 0 in each plot for the sake of clarity.

4. **Estimation of Symbol-Rate.** As revealed in previous studies, we apply the second-order cyclic spectrum to symbol-rate estimation, dominant peaks of which are actually the second-order TCMs with $\boldsymbol{\tau} = \mathbf{0}$ and $q = 1$. The second-order cyclic spectrum of received sequence $x[l]$ is estimated by

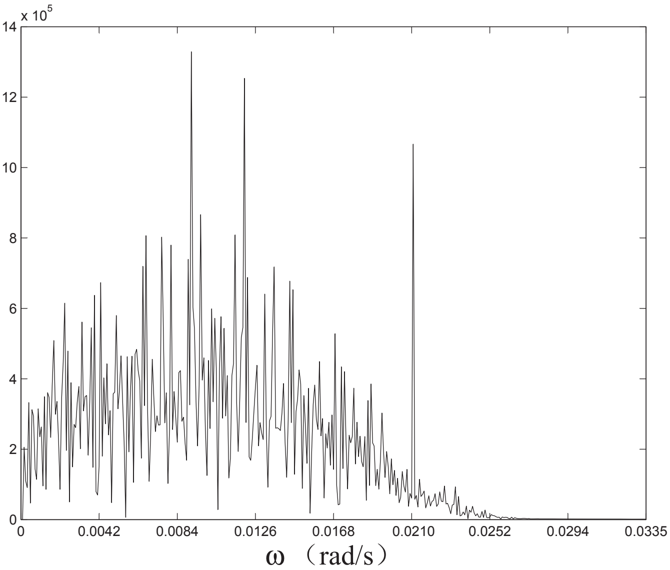
$$\widetilde{r}_x^{\alpha}[\mathbf{0}]_{2,1} = \frac{1}{N}\sum_{l=0}^{N-1} x[l]x^*[l]e^{-j2\pi l\alpha} \tag{10}$$

Substituting (2) into (10), the $i$th frequency component of cyclic spectrum can be expressed as

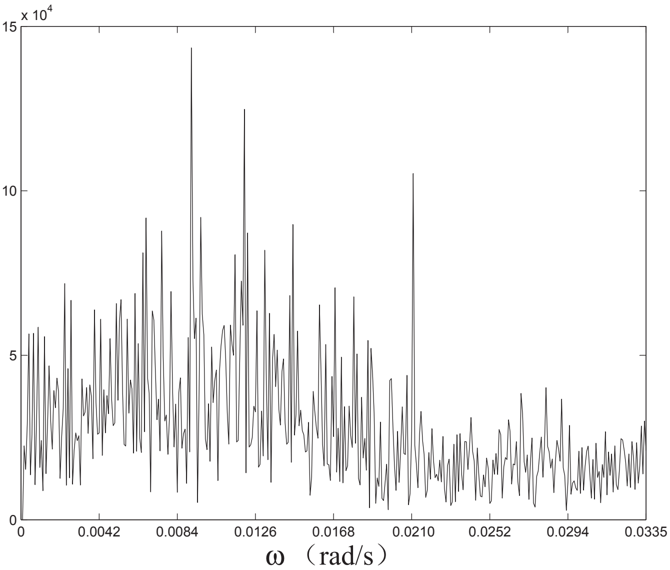
$$\widetilde{r}_x^{\alpha_i}[\mathbf{0}]_{2,1} = \frac{a^2}{\rho}\sum_{k} P[k/\rho]e^{-j2\pi k l_0/\rho}\delta\left[\alpha_i - \frac{k}{\rho}\right] + \sigma^2\delta[\alpha_i] \tag{11}$$

where $\delta[\cdot]$ denotes the Kronecker deltas, $P[\cdot]$ is the DFT of $p[\cdot]$, and $\sigma^2 = E\{|n[l]|^2\}$. It is apparent that if and only if $\alpha_i = k/\rho$, $\widetilde{r}_x^{\alpha_i}[\mathbf{0}]_{2,1} \neq 0$, which means that the prominent peak of cyclic spectrum appears at $\alpha_i = k/\rho$. Further, in the scenario of using bandlimited filter, i.e., $\widetilde{r}_x^{\alpha}[\mathbf{0}]_{2,1} \approx 0$ for $k > 1$, $\widetilde{r}_x^{\alpha}[\mathbf{0}]_{2,1}$ should have prominent peaks at $-1/\rho, 0, 1/\rho$. Since $1/\rho$ is actually the normalized symbol-rate, our problem can be considered as detecting the non-zero peak located at the positive axis.

However, as shown in Figure 1, whether we estimate the second-order cyclic moments from uniform or nonuniform samples, due to the nonlinearly processed signal, simply choosing the peak with the highest level on the right side of DC component would lead to a false estimate of the symbol-rate, since $\widetilde{r}_x^{\alpha}[\mathbf{0}]_{2,1}$ at frequencies near 0 may take rather large values that can be compared with $\widetilde{r}_x^{1/\rho}[\mathbf{0}]_{2,1}$. Even so, $\widetilde{r}_x^{1/\rho}[\mathbf{0}]_{2,1}$ is a local maximum at least.




(a) Scenario of uniform samples




(b) Scenario of nonuniform samples

FIGURE 1. FFT of second-order moment and its estimate obtained from nonuniform samples

In [9], the desired peak was detected in a narrowed search interval, which was determined by a roughly estimated 3 dB bandwidth. In [10], the cyclic spectrum was weighted by a simple nonlinear operation, as a result, the local maximum was converted into a global maximum; however, an interval decided by 3 dB bandwidth was still introduced to reduce search complexity. In this work, we attempt to design a novel weighting approach to make the desired peak a global maximum without 3 dB bandwidth estimation.

It is worth noting that the peak $\widetilde{r}_x^{1/\rho}[\mathbf{0}]_{2,1}$ is the superior over neighboring frequencies even for a low SNR and a small excess bandwidth signal, and meanwhile, the compared components tend to concentrate at the low frequencies far from $1/\rho$. Based on the analysis above, with the 0-frequency component omitted (assigned to be 0), we first divide the cyclic spectrum into $N_s$ segments, and the number of components in each segment should be appropriately chosen. For each segment, we define an indication function as

$$Q_v = \frac{\max_i |\widetilde{r}_x^{\alpha_i}[\mathbf{0}]_{2,1}|}{1/L_s \sum_{i=(v-1)L_s+1}^{vL_s} \widetilde{r}_x^{\alpha_i}[\mathbf{0}]_{2,1}} \tag{12}$$

where $v \in \{1, 2, \dots, N_s\}$, $L_s$ denotes the number of components in each segment, $i \in [(v-1)L_s + 1, vL_s]$ and we have $N_s = \lceil N_{FFT}/L_s \rceil$, where $N_{FFT}$ denotes the FFT-length, and $\lceil z \rceil$ denotes the smallest integer larger than $z$. It is also noted that the length of the last segment may equal to $W = N_{FFT} \mod L_s$ when $N_{FFT}/L_s$ is not an integer, and correspondingly, $Q_v = \frac{\max_i |\widetilde{r}_x^{\alpha_i}[\mathbf{0}]_{2,1}|}{1/W \sum_{i=(N_s-1)L_s+1}^{(N_s-1)L_s+W} \widetilde{r}_x^{\alpha_i}[\mathbf{0}]_{2,1}}$, $v = N_s$.

Subsequently, we replace the maximum component of $v$th segment by the value of $Q_v$ at the same location with the others assigned to zero. Hence, we can obtain the weighted spectrum. As analysed in Section 3, the aforementioned process can be performed with the compressive spectrum as well, and as shown in Figure 2, it is easy to observe that the specified local maximum of compressive spectrum becomes the global maximum of weighted spectrum, which indicates the symbol-rate.

## 5. Implementation and Simulation Results.

**5.1. Implementation.** Based on the discussions in previous sections, the whole process of symbol-rate estimation from nonuniform samples is illustrated in Figure 3. First, nonuniform samples are acquired by passing the IF signal through a low-rate nonuniform sample-and-hold (s/h) device which is driven by a common nonuniform clock (NU CLK) dictating the sample times. For more details of the s/h device, one can refer to [7]. Then, convert nonuniform samples into complex values through the Hilbert transform and compute the compressive cyclic spectrum by taking a zero-padding FFT. Subsequently, we obtain the weighted spectrum by the strategy of segment division, and the desired peak emerges, which indicates the symbol-rate.

**5.2. Simulation and performance evaluation.** In order to validate the proposed algorithm and evaluate its performance, we consider signals belonging to four different modulation formats, such as QPSK, 32PSK, 16QAM and 64QAM. In simulations, each IF signal is generated with oversampling factor $\rho = 300$. The carrier frequency and symbol-rate of each signal are 5 MHz and 122 KHz respectively. The roll-off factor of RRC filter is set to 0.35. The channel conditions are set with SNR in the range of 5 dB to 25 dB. The number of symbols is assumed to be 250. For each modulation, 100 Monte Carlo simulations are run and we adopt the normalized mean square error (NMSE) as a consistent performance metric, which reflects both the bias and the variance of a symbol-rate estimate. The NMSE is defined as $E\left\{\left(f_b - \hat{f}_b\right)^2 / f_b^2\right\}$ and it is converted into dB scale for the sake of intuitive clarity.

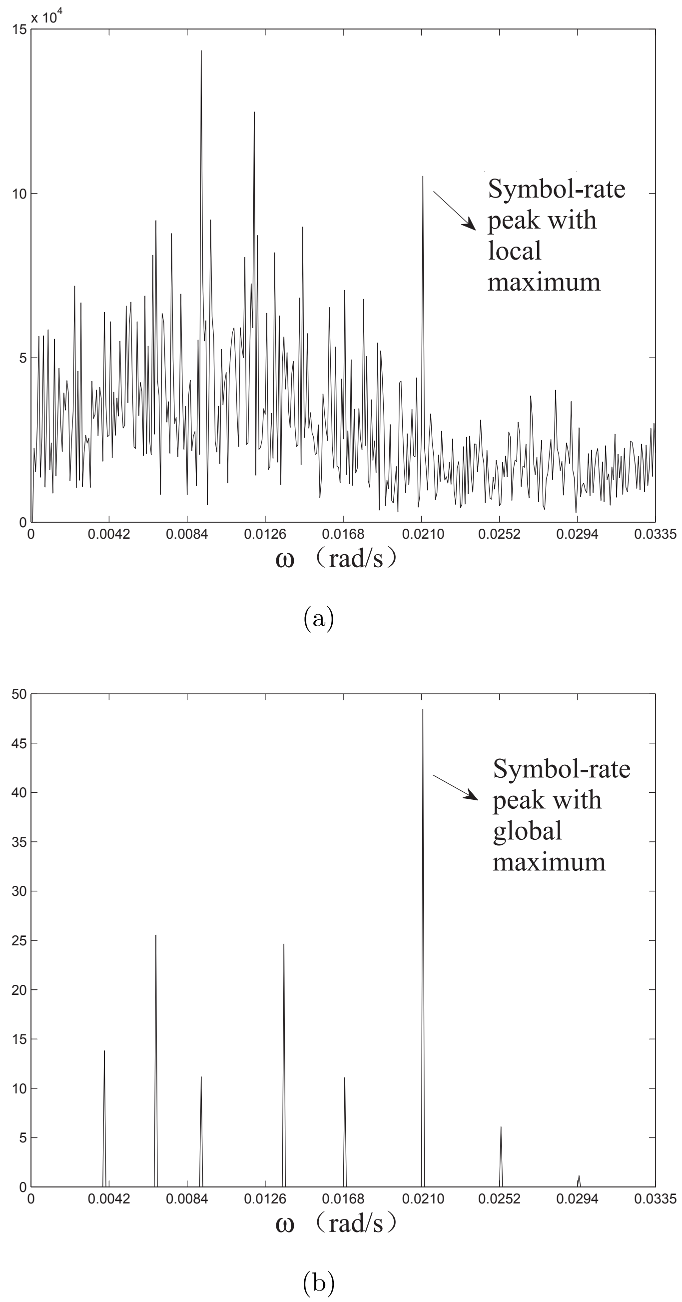



FIGURE 2. (a) Nonweighted compressive cyclic spectrum, where the symbol-rate peak is a local maximum. (b) Weighted compressive cyclic spectrum for the same environment as (a), where the symbol-rate peak is a global maximum.

In our simulations, for each modulation, the performance of classical method in the scenario of uniform samples and the proposed CS-based method with compression factor $\gamma = 0.1$ is compared. Figure 4 shows the performance of symbol rate estimation versus SNR. It is observed that the NMSE decreases as the SNR increases in the case of nonuniformly sampling. For QPSK and 16QAM signals, the NMSE of CS-based method declines sharply, when SNR achieves 10 dB, whereas the classical method always exhibits a robust performance over the whole range of SNR. It means that for QPSK and 16QAM signals, the CS-based method satisfies an acceptable performance only when the SNR is above 10 dB. However, for the other signals, the proposed method compares to the classical method when the SNR is above 5 dB. In general, the CS-based method can be recognized

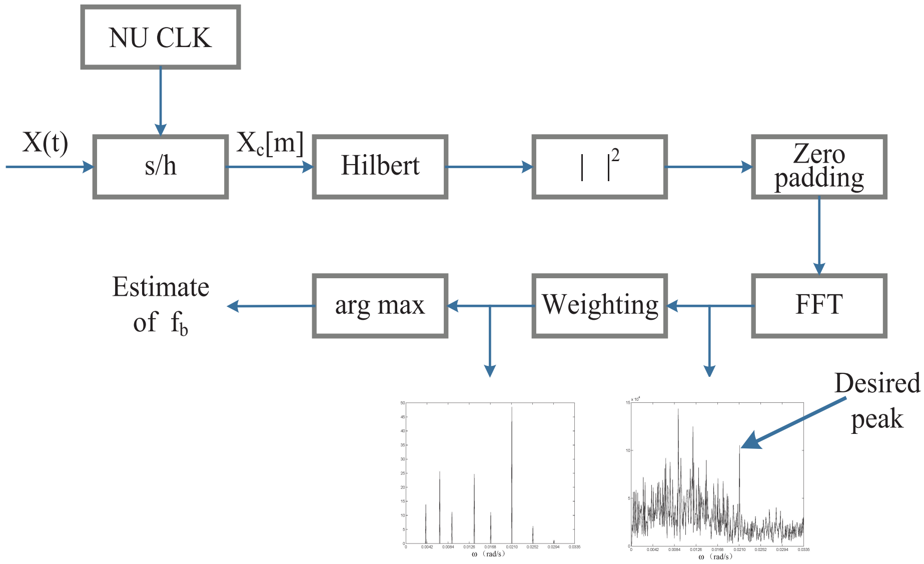



FIGURE 3. Flowchart of the symbol rate estimation

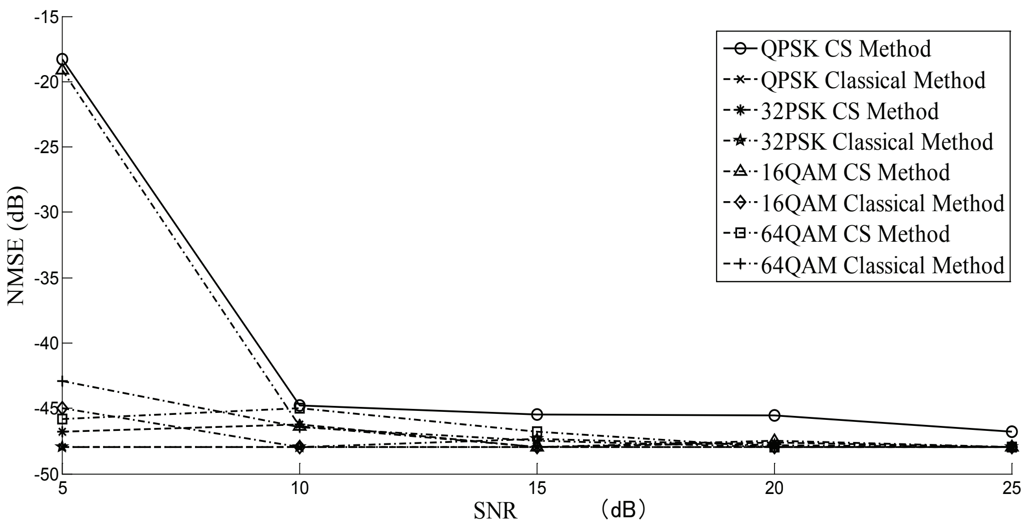



FIGURE 4. Performance of the symbol rate estimation

as an alternative way for symbol-rate estimation when the SNR is not too low, and, most of all, only 1% of samples are required in contrast to classical method.

6. **Conclusion.** Inspired by the development of CS theory, we propose a new method for the symbol-rate estimation of received signal from extremely small set of nonuniform samples, which significantly reduces the computation and storage burdens of hardware implementation. We first present a novel approach to generating compressive cyclic spectrum from nonuniform samples, and then provide a simple weighting scheme for identifying the required peak indicating the symbol-rate in the estimated spectrum. Our simulation results confirm that the proposed method is indeed effective and feasible, although it requires somewhat higher SNR compared to the classical method.

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