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# SYNCHRONIZATION OF UNCERTAIN T AND LÜ CHAOTIC SYSTEMS: AN ADAPTIVE NONLINEAR CONTROL METHOD

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Abstract. *In this research study, chaos synchronization problem between T and Lü, two chaotic systems, is addressed. The parameters of the drive T chaotic system are considered unknown. An adaptive nonlinear feedback control law and a parameter estimation law are introduced based on the Lyapunov stability theorem and the adaptive control theory. The validity of the proposed method is proved by the Lyapunov stability theorem. Furthermore, some numerical simulations are given to show the effectiveness of the theorical discussions and the proposed method.*
**Keywords:** T chaotic system, Lü chaotic system, Parameter identification, Lyapunov stability theorem

1. **Introduction.** Sensitivity to the initial conditions is the main property of the chaotic systems. Any small differences in the initial trajectories of the system state variables will make exponentially huge differences in the output of the system state variables. This feature makes the control and the synchronization of the chaotic systems a challenging subject and hence many applications would arise, due to these features, such as: electrical engineering, physics, chemistry and secure communication. Generally, chaos is an undesirable phenomenon in some situations in view of the fact that it generates some irregular oscillations. Therefore, chaos control and synchronization of chaotic systems have attracted more attention from scientists and researchers. To this end, many types of synchronization methods have been introduced to explain the complex behavior of the chaotic systems. Active method [1,2], Adaptive method [3-5], backstepping method [6,7], generalized method [8], phase method [9,10], sliding method [11-13], projective method [14-17] and modified projective synchronization [20-23] are some of them. Among these methods, adaptive method is a common control method, which plays an important role in many other synchronization methods. In this paper, adaptive synchronization problem of the T and Lü chaotic systems was firstly addressed. Chaos synchronization is carried out by introducing a new adaptive nonlinear feedback control law.

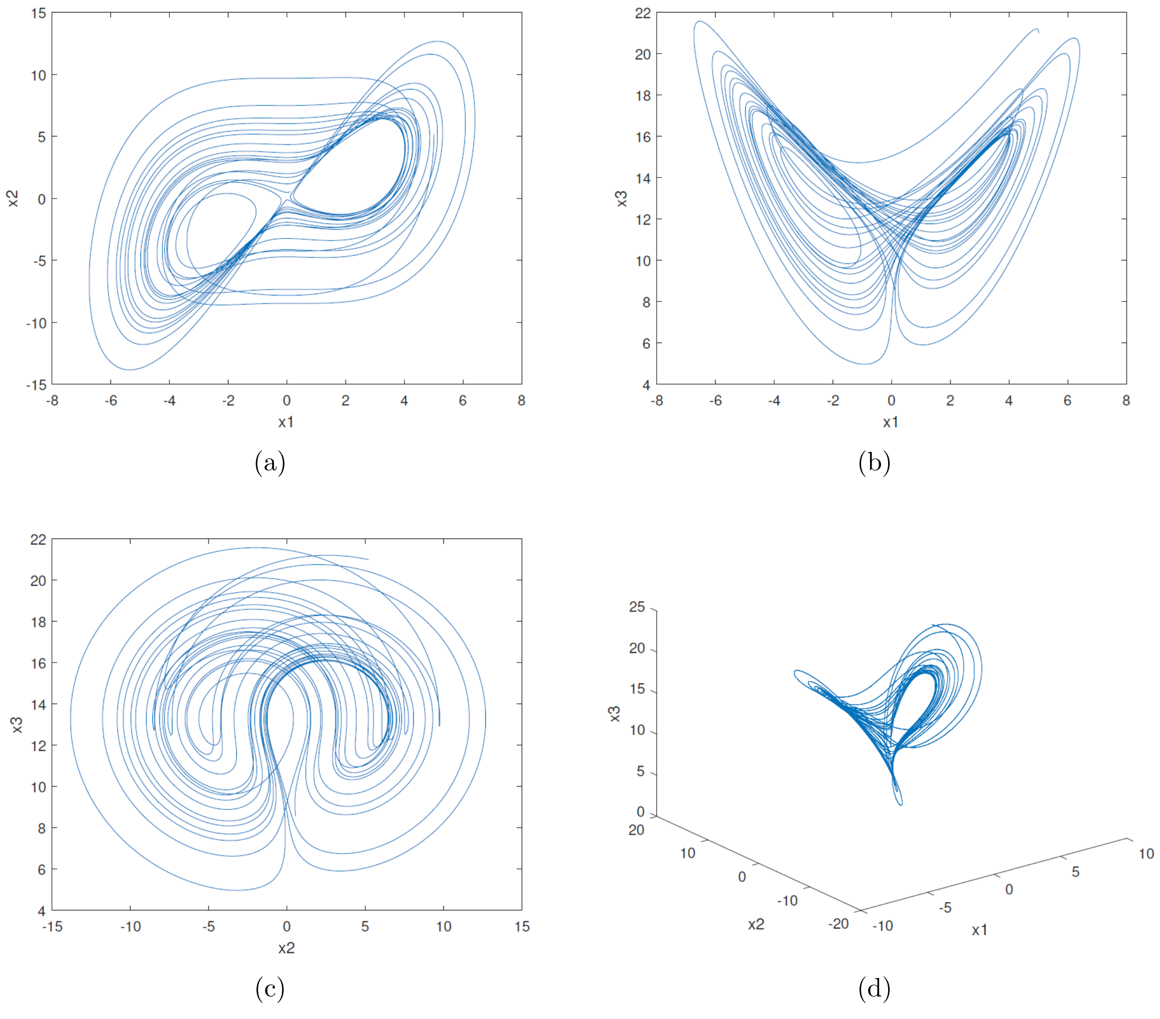
Since the system parameters are usually uncertain or unknown, adaptive synchronization methods have to be utilized instead of the active ones. Therefore, this paper concentrates on adaptive control procedure. During adaptive control implementation, the unknown parameters of the chaotic system will be estimated, in most of the published papers of synchronization between chaotic systems.

In this paper, some results on adaptive synchronization of T chaotic system and the Lü chaotic system are derived. Section 2 gives some preliminaries and mathematical modeling. Then, the chaos synchronization between T chaotic system as the leader chaotic system and the Lü chaotic system as the response system is addressed in Section 3. An adaptive control law and a parameter estimation law are obtained based on the Lyapunov stability theorem and the adaptive control. Numerical simulations are presented for each section of 2 and 3 in order to verify the theorical analysis in Section 4. Finally, concluding remark is given in Section 5.

**2. Mathematical Modeling and Preliminaries.** In this section, some mathematical modeling of the T and Lü chaotic systems are investigated. T chaotic system was recently introduced in [18], which is constructed based on the three-dimensional dynamical system. T system can be presented as follows:

$$\begin{aligned}\dot{x}_1 &= \alpha_1(x_2 - x_1)\\ \dot{x}_2 &= (\alpha_2 - \alpha_1)x_1 - \alpha_1 x_1 x_3\\ \dot{x}_3 &= x_1 x_2 - \alpha_3 x_3\end{aligned} \tag{1}$$

where $\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$ represent the time derivatives of the system state variables $x_1$, $x_2$ and $x_3$, respectively, and $a$, $b$ and $c$ stand for parameter of the system. When $a = 2.1$, $b = 30$ and $c = 0.6$, the behavior of the T system (1) is chaotic. The chaotic behavior of the T system (1) is shown in Figure 1 for initial state variables $x_1 = 4.3$, $x_2 = 7.2$ and $x_3 = 5.8$.




(a) (b)

(c) (d)

FIGURE 1. Time portrait of the T chaotic system

Lü chaotic system is another chaotic system firstly presented in [19] with three-dimension dynamical system as follows:

$$\begin{aligned}\dot{y}_1 &= \beta_1(y_2 - y_1)\\ \dot{y}_2 &= \beta_2 y_2 - y_1 y_3\end{aligned} \tag{2}$$

$$\dot{y}_3 = y_1 y_2 - \beta_3 y_3$$

where $y_1$, $y_2$ and $y_3$ stand for the state variables of system, and $\beta_1$, $\beta_2$ and $\beta_3$ are the parameters of system. The chaotic behavior of the Lü chaotic system (2) is shown in Figure 2, with system parameters: $\beta_1 = 36$, $\beta_2 = 30$ and $\beta_3 = 20$, and the initial values for the system state variables as: $y_1 = 5$, $y_2 = 2$ and $y_3 = 30$.

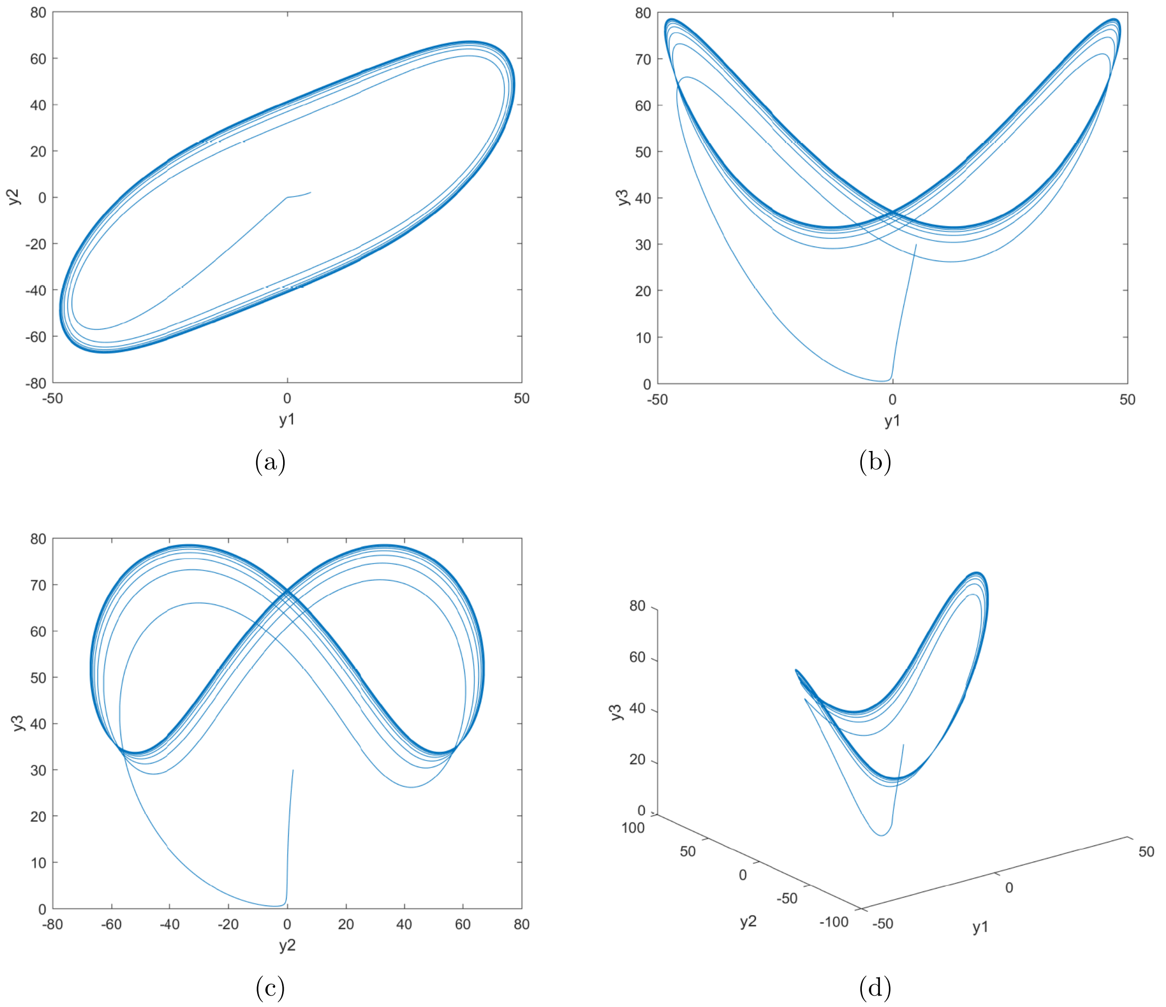



FIGURE 2. Time portrait of the Lü chaotic system

3. **Adaptive Synchronization.** Consider the T chaotic system presented in (1), as the drive chaotic system. Then, the follower chaotic system can be presented using the Lü chaotic system (2), as follows:

$$\begin{aligned} \dot{y}_1 &= (\alpha_1 + \Delta\alpha_1)(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (\alpha_2 + \Delta\alpha_2)y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= y_1 y_2 - (\alpha_3 + \Delta\alpha_3)y_3 + u_3 \end{aligned} \quad (3)$$

where $u_1$, $u_2$ and $u_3$ indicate the three adaptive feedback controllers, which have to be designed. $\Delta\alpha_1$, $\Delta\alpha_2$ and $\Delta\alpha_3$ denote the disparity amount of unknown system parameters $\alpha_1$, $\alpha_2$ and $\alpha_3$ in the leader chaotic system (1), respectively.

Assume the synchronization error between the leader and the follower chaotic systems (1) and (2) as follows:

$$e_1 = y_1 - x_1$$

$$\begin{aligned} e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{4}$$

The dynamical representation of system errors (4) can be described as:

$$\begin{aligned} \dot{e}_1 &= \dot{y}_1 - \dot{x}_1 \\ \dot{e}_2 &= \dot{y}_2 - \dot{x}_2 \\ \dot{e}_3 &= \dot{y}_3 - \dot{x}_3 \end{aligned} \tag{5}$$

**Definition 3.1.** *For the drive T system (1) and the response Lü system (3), it is said that if an appropriate feedback controller law and a parameter estimation law are achieved, then the chaos synchronization would occur and synchronization errors would be zero as time tends to infinity, namely:*

$$\lim_{t\to\infty} |y_i - x_i| = 0 \qquad \forall i = 1, 2, 3 \tag{6}$$

An appropriate control law and a parameter estimation law are designed in the following theorem to provide the leader and the follower synchronization practice.

**Theorem 3.1.** *The T chaotic system (1) with the system state variables $x_1$, $x_2$ and $x_3$ with system parameters $a$, $b$ and $c$ would be synchronized with the response Lü chaotic system (3), and assuming the adaptive synchronization errors defined in (4) and the control law and the parameter estimation law defined as follows:*

$$\begin{aligned} u_1(t) &= -(\alpha_1 + \Delta\alpha_1)(y_2 - y_1) + (\alpha_1 + \Delta\alpha_1)(x_2 - x_1) - k_1 e_1 \\ u_2(t) &= -(\alpha_2 + \Delta\alpha_2)y_2 + y_1 y_3 + ((\alpha_2 + \Delta\alpha_2) - (\alpha_1 + \Delta\alpha_1))x_1 x_3 - k_2 e_2 \\ &\quad -(\alpha_1 + \Delta\alpha_1) \\ u_3(t) &= -y_1 y_2 + (\alpha_3 + \Delta\alpha_3)y_3 + x_1 x_2 - (\alpha_3 + \Delta\alpha_3)x_3 - k_3 e_3 \end{aligned} \tag{7}$$

*and*

$$\begin{aligned} \Delta'\alpha_1 &= -e_1(x_2 - x_1) + e_2 x_1 x_3 - \phi_1 \Delta\alpha_1 \\ \Delta'\alpha_2 &= -e_2 x_1 - \phi_2 \Delta\alpha_2 \\ \Delta'\alpha_3 &= +e_3 x_3 - \phi_3 \Delta\alpha_3 \end{aligned} \tag{8}$$

**Proof:** Proving the system errors stability provides a sufficient condition for synchronization task between the drive T chaotic system (1) and the response Lü chaotic system (3). To this end, a typical Lyapunov candidate function is given, which uses the system state variables errors and the errors of the system parameter.

Let us consider the Lyapunov candidate function as follows:

$$V(t) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + (\Delta\alpha_1)^2 + (\Delta\alpha_2)^2 + (\Delta\alpha_3)^2\right) \tag{9}$$

It is clear that $V$ is positive definite. The time derivative of the Lyapunov candidate function (9) can be described as follows:

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + \Delta\alpha_1\Delta'\alpha_1 + \Delta\alpha_2\Delta'\alpha_2 + \Delta\alpha_3\Delta'\alpha_3 \tag{10}$$

With considering the dynamical representations of drive and response systems (1) and (3), the dynamical error system (5), the proposed controller (7) and the parameter estimation (8), the dynamical Equation (10) can be simplified as follows:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - \phi_1(\Delta\alpha_1)^2 - \phi_2(\Delta\alpha_2)^2 - \phi_3(\Delta\alpha_3)^2 \tag{11}$$

When $k_i$ $(i = 1, 2, 3)$ and $\phi_i$ $(i = 1, 2, 3)$ are positive constants, then, it is clear that the dynamical representation of the Lyapunov candidate function (10) is negative definite. This means that the anticipated synchronization between the leader T chaotic system (1) and the response Lü chaotic system (3) will be achieved, based on the Lyapunov stability theory and adaptive control theory. Therefore, the theorem is proved, namely, $\lim |e_i(t)| \rightarrow 0$ as time tends to infinity.

4. **Numerical Simulations.** In this section, some numerical results related to the synchronization of drive T chaotic system (1) and the response Lü chaotic system (3) are presented to clarify the effectiveness of the theorical discussions given at the previous section. A Matlab implementation is carried out to solve the drive-response system synchronization with the time step of the size $10^{-7}$.

Runge-Kutta is used as an iterative method for solving the synchronization problem with the T chaotic system (1), the Lü chaotic system (3) with the adaptive control (7) and the estimated parameter calculated at (8).

The unknown parameters of the T chaotic system, as the drive system are initially chosen as: $a = 2$, $b = 5$ and $c = 1$, and the initial values for the drive chaotic system (1) are taken as $x_1(0) = 3$, $x_2(0) = 2$, and $x_3(0) = 7$. In addition, the initial values of the

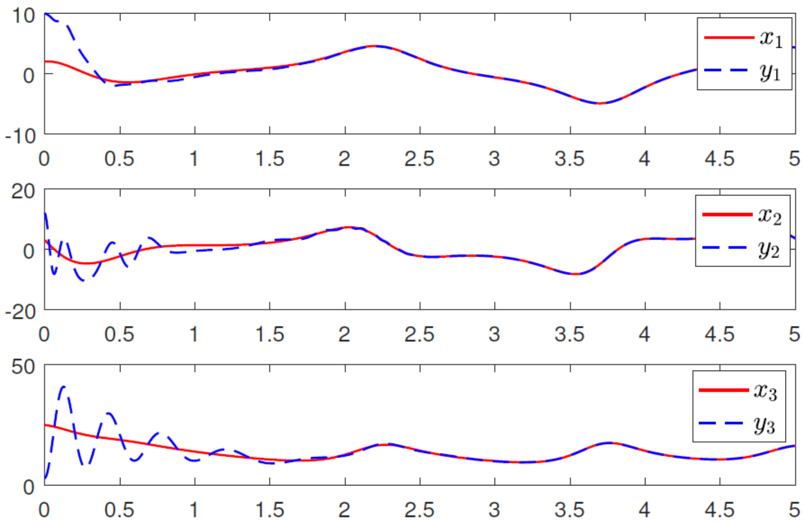



FIGURE 3. Motion trajectories of the state variables along the time

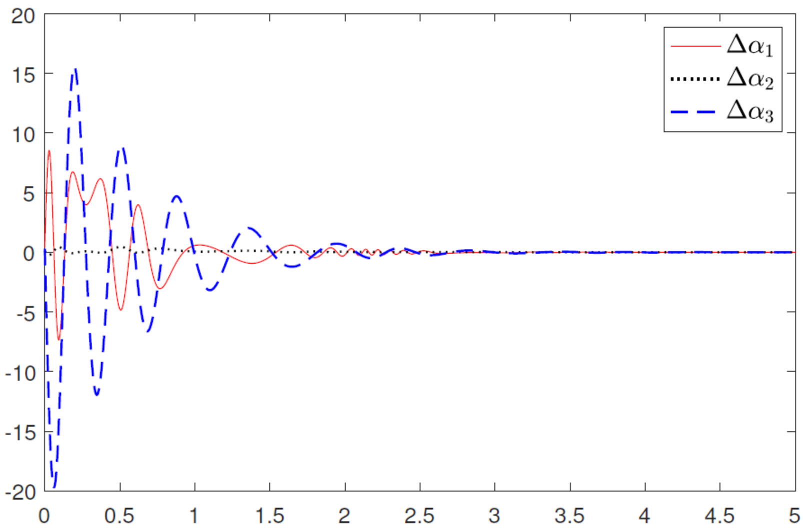



FIGURE 4. The disparity amount of the system parameters estimations

response Lü system (3) are selected as: $y_1(0) = 4$, $y_2(0) = 1$ and $y_3(0) = 8$. The gain constants are set as $k_1 = 1.5$, $k_2 = 1.5$, $k_3 = 1.5$ and also $\phi_1 = 2$, $\phi_2 = 2$ and $\phi_3 = 2$. Finally we have supposed the initial values of the estimation parameters as: $\Delta\alpha_1 = 0.5$, $\Delta\alpha_2 = 0.7$ and $\Delta\alpha_3 = 1.2$.

The effectiveness of the proposed control method for synchronization of the T chaotic system (1) and the Lü chaotic system (3) with unknown drive system parameters is shown in Figures 3 and 4. Figure 3 shows that the state variables of the system (1) converge to zero. In addition, Figure 4 exhibits that the distance between drive unknown parameters and its estimation values converges to zero.

5. **Conclusion.** In this paper, an adaptive method for synchronization of T chaotic system as the drive system and the Lü chaotic system as the response system is studied. The parameters of the drive chaotic system are considered unknown. An appropriate feedback control law and a parameter estimation law are derived based on the Lyapunov stability theorem and the adaptive control theorem. Then, numerical simulations are carried out to verify the effectiveness method. As it can be seen from the simulated results, the anticipated drive-response synchronization is achieved and the synchronization errors of the system parameters and also errors from the disparity amount of system parameters tend to zero as time goes to the infinity.

## REFERENCES


[1] H. S. Nik, J. Saberi-Nadjafi, S. Effati and R. A. Van Gorder, Hybrid projective synchronization and control of the Baier-Sahle hyperchaotic flow in arbitrary dimensions with unknown parameters, *Applied Mathematics and Computation*, vol.248, pp.55-69, 2014.

[2] H. Richter, Controlling chaotic systems with multiple strange attractors, *Physics Letters A*, vol.300, no.2, pp.182-188, 2002.

[3] H. Adloo and M. Roopaei, Review article on adaptive synchronization of chaotic systems with unknown parameters, *Nonlinear Dynamics*, vol.65, nos.1-2, pp.141-159, 2011.

[4] K.-S. Hong et al., Adaptive synchronization of two coupled chaotic Hindmarsh-Rose neurons by controlling the membrane potential of a slave neuron, *Applied Mathematical Modelling*, vol.37, no.4, pp.2460-2468, 2013.

[5] T. Ma, J. Zhang, Y. Zhou and H. Wang, Adaptive hybrid projective synchronization of two coupled fractional-order complex networks with different sizes, *Neurocomputing*, vol.164, pp.182-189, 2015.

[6] W. Wang and Y. Fan, Synchronization of Arneodo chaotic system via backstepping fuzzy adaptive control, *Optik – International Journal for Light and Electron Optics*, vol.126, no.20, pp.2679-2683, 2015.

[7] Y. Yu and S. Zhang, Adaptive backstepping synchronization of uncertain chaotic system, *Chaos, Solitons & Fractals*, vol.21, no.3, pp.643-649, 2004.

[8] J.-S. Lin and J.-J. Yan, Adaptive synchronization for two identical generalized Lorenz chaotic systems via a single controller, *Nonlinear Analysis: Real World Applications*, vol.10, no.2, pp.1151-1159, 2009.

[9] M. G. Rosenblum, A. S. Pikovsky and J. Kurths, From phase to lag synchronization in coupled chaotic oscillators, *Physical Review Letters*, vol.78, no.22, p.4193, 1997.

[10] B. Blasius, A. Huppert and L. Stone, Complex dynamics and phase synchronization in spatially extended ecological systems, *Nature*, vol.399, pp.354-359, 1999.

[11] M. S. Tavazoei and M. Haeri, Synchronization of chaotic fractional-order systems via active sliding mode controller, *Physica A: Statistical Mechanics and Its Applications*, vol.387, no.1, pp.57-70, 2008.

[12] M. P. Aghababa, S. Khanmohammadi and G. Alizadeh, Finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique, *Applied Mathematical Modelling*, vol.35, no.6, pp.3080-3091, 2011.

[13] H.-T. Yau, Chaos synchronization of two uncertain chaotic nonlinear gyros using fuzzy sliding mode control, *Mechanical Systems and Signal Processing*, vol.22, no.2, pp.408-418, 2008.

[14] J. H. Park, Adaptive control for modified projective synchronization of a four-dimensional chaotic system with uncertain parameters, *Journal of Computational and Applied Mathematics*, vol.213, no.1, pp.288-293, 2008.

[15] Y. Tang and J. Fang, General methods for modified projective synchronization of hyperchaotic systems with known or unknown parameters, *Physics Letters A*, vol.372, no.11, pp.1816-1826, 2008.

[16] P. Sarasu and V. Sundarapandian, The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control, *Int. J. Soft Comput.*, vol.6, no.5, pp.216-223, 2011.
[17] R. Mainieri and J. Rehacek, Projective synchronization in three-dimensional chaotic systems, *Physical Review Letters*, vol.82, no.15, p.3042, 1999.
[18] G. Tigan and D. Opris, Analysis of a 3D chaotic system, *Chaos Solitons Fractals*, vol.36, no.5, pp.1315-1319, 2008.
[19] G. Chen and J. Lv, A new chaotic attractor coined, *Int. J. Bifurcat. Chaos*, vol.12, no.3, pp.659-661, 2002.
[20] H. Tirandaz and A. Hajipour, Adaptive synchronization and anti-synchronization of TSUCS and Lü unified chaotic systems with unknown parameters, *Optik – International Journal for Light and Electron Optics*, vol.130, pp.543-549, 2017.
[21] M. Hu and Z. Xu, Adaptive feedback controller for projective synchronization, *Nonlinear Analysis: Real World Applications*, vol.9, no.3, pp.1253-1260, 2008.
[22] L. Guo, Z. Xu and M. Hu, Adaptive projective synchronization with different scaling factors in networks, *Chinese Physics B*, vol.17, no.11, p.4067, 2008.
[23] Y. Niu, X. Wang and B. Pei, Adaptive projective synchronization of different chaotic systems with nonlinearity inputs, *Chinese Physics B*, vol.21, no.3, p.030503, 2012.