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SOME NEW CONSTRUCTIONS OF THE ALMOST DIFFERENCE SET PAIRS BASED ON WHITEMAN GENERALIZED CYCLOTOMIC

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Abstract. *Almost difference set pairs have intersting applications in digital communications and coding theory. In this paper, we give a conclusion about the cyclotomic numbers of Whiteman generalized cyclotomy and construct new famillies of almost difference set pairs with period $N = pq$ based on Whiteman generalized cyclotomic classes and Chinese Remainder Theorem, where $gcd(p, q) = e$.*
Keywords: Almost difference set pairs, Whiteman generalized cyclotomy, Perfect discrete signal

1. **Introduction.** Sequences pairs have a wide range of applications such as Code Division Multiple Access (CDMA) systems, radar, signal processing and source coding in spectrum communication systems [1]. Almost Difference Set Pairs (ADSPs) are a mathematical tool to construct sequence pairs, which are presented in [2] for the first time. It is proven that an ADSP is equivalent to a binary sequence pair with three-level autocorrelation. However, the set H of the ADSPs in [2] only contains two elements, and they failed with the construction of ADSPs.

The known construction of ADSPs with perfect performance are the following.

1) Several kinds of ADSPs with period $N \equiv 0 \,(\mathrm{mod}\, 4)$ are constructed in [3] using the ideal two-level correlation binary sequence pairs.

2) The ADSPs on two times prime v of residual class ring Z_{2v} are constructed in [4] based on cyclotomic classes and Chinese Remainder Theorem.

3) Classes of ADSPs of period $N = 4f + 1$ and $N = 6f + 1$ are constructed in [5] by means of cyclotomy.

So far only a small number of classes of ADSPs has been discovered and it is far from meeting the needs of practical applications. In this paper, we will give new famillies of almost difference set pairs with period $N = pq$, where $gcd(p, q) = e$ using Whiteman generalized cyclotomy [6]. In addition, this discovery expands the space for the existence of almost difference set pairs, and provides a method for finding the ideal sequence pairs.

2. **Preliminaries.**

Definition 2.1. [2] *Let $Z_v = \{0, 1, \ldots, v-1\}$ be the mod v residual class ring, and U, V be two subsets of Z_v, $|U| = k_1$, $|V| = k_2$, $e = |U \cap V|$; if t nonzero element a are in Z_v let the equation: $x - y \equiv a(\mathrm{mod}\, v)$ have exactly λ ways, where $\lambda < t$, $(x, y) \in (U, V)$, and other $v - 1 - t$ nonzero elements have exactly $\lambda + 1$ ways; then (U, V) is called an $(v, k_1, k_2, e, \lambda, t)$ almost difference set pairs, denoted by $ADSP(v, k_1, k_2, e, \lambda, t)$.*

Definition 2.2. [7] *Let $v = pq$ where p and q are distinct odd primes with $gcd(p, q) = e$. By the Chinese Ramainder Theorem, there exists a common primitive root g of both p and q, such that*

$$ord_v(g) = lcm\{ord_p(g), ord_q(g)\} = lcm\{p-1, q-1\} = (p-1)(q-1)/d$$

Let x be an integer satisfying $x \equiv g \,(\mathrm{mod}\, p)$ and $x \equiv 1 \,(\mathrm{mod}\, q)$. Whiteman defined the generalized cyclotomic classes of order d over Z_{pq} as follows:

$$D_i = \{g^s x^i \,(\mathrm{mod}\, v) : s = 0, 1, \ldots, d-1, i = 0, 1, \ldots, e-1\}$$

Clearly the cosets D_i also depend on $0 \le i \ne j \le e-1$ and $D_i \bigcap D_j = 0$. Concurrently $d = (p-1)(q-1)/e$, $d_1 = (p-1)/e$, $d_2 = (q-1)/e$, $P = \{p, 2p, \ldots, (q-1)p\}$, $Q = \{q, 2q, \ldots, (p-1)q\}$, $R = \{0\}$.

For example: Let $N = 39$, $p = 3$, $q = 13$, $e = 2$, and then $D_0 = \{1, 2, 4, 8, 16, 32, 25, 11, 22, 5, 10, 20\}$, $D_1 = \{14, 28, 17, 34, 29, 19, 38, 37, 35, 31, 23, 7\}$, $P = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36\}$, $Q = \{13, 26\}$, $R = \{0\}$. In addition, $d = (p-1)(q-1)/e = 12$, $d_1 = 1$, $d_2 = 6$.

Lemma 2.1. [8] *For P and $\bigcup_{i=0}^{e-1} D_i$,*

$$\left|(P+\omega) \bigcap \bigcup_{i=0}^{e-1} D_i\right| = \begin{cases} q-2, & \omega \in \bigcup_{i=0}^{e-1} D_i \\ 0, & \omega \in P \\ q-1, & \omega \in Q \end{cases}$$

Proof: In order to prove this lemma, we calculated the number of solutions of the congruence $x - y = \omega \,(\mathrm{mod}\, v)$ with $x \in P$, $y \in \bigcup_{i=0}^{e-1} D_i$.

Case 1: let ω be a fixed integer not divisible by p, and then the number of solutions of the congruence equation is equal to 0.

Case 2: let ω be a fixed integer not divisible by q, and then the number of solutions of the congruence equation is equal to $q-1$.

Case 3: let ω be a fixed integer relatively prime by q, and then the number of solutions of the congruence equation is equal to $q-2$.

Combination of the above results yields the conclusion of the lemma.

According to the above Lemma 2.1, for P, Q, $\bigcup_{i=0}^{e-1} D_i$, the possible values for the cyclotomic numbers of Whiteman generalized cyclotomy are given in Table 1.

TABLE 1. The cyclotomic numbers of Whiteman generalized cyclotomy

	$\omega \in P$	$\omega \in Q$	$\omega \in \bigcup_{i=0}^{e-1} D_i$
$P \bigcap P$	$q-2$	0	0
$P \bigcap Q$	0	0	1
$P \bigcap R$	1	0	0
$Q \bigcap Q$	0	$p-2$	0
$Q \bigcap R$	0	1	0
$P \bigcap \bigcup_{i=0}^{e-1} D_i$	0	$q-1$	$q-2$
$Q \bigcap \bigcup_{i=0}^{e-1} D_i$	$p-1$	0	$p-2$
$\bigcup_{i=0}^{e-1} D_i \bigcap \bigcup_{i=0}^{e-1} D_i$	$(p-1)(q-2)$	$(p-2)(q-1)$	$(p-2)(q-2)$

Lemma 2.2. *For P and D_i,*

$$\text{when } d/e \text{ is odd, } |(D_i+\omega) \bigcap P| = \begin{cases} d_2 - 1, & \omega \in D_{i \,\mathrm{mod}\, e} \\ 0, & \omega \in P \\ d_2, & \omega \in else \end{cases}$$

$$\text{when } d/e \text{ is even, } |(D_i+\omega) \bigcap P| = \begin{cases} d_2 - 1, & \omega \in D_{(i+e/2) \,\mathrm{mod}\, e} \\ 0, & \omega \in P \\ d_2, & \omega \in else \end{cases}$$

Proof: This is similarly proven as Lemma 2.1.

According to the above Lemma 2.2, when d/e is odd, the cyclotomic numbers of Whiteman generalized cyclotomy of cosets P, Q, D_i are given in Table 2. When d/e is even, the cyclotomic numbers of Whiteman generalized cyclotomy of cosets P, Q, D_i are given in Table 3.

TABLE 2. The cyclotomic numbers if d/e is odd

	$\omega \in P$	$\omega \in Q$	$\omega \in D_{i \bmod e}$	$\omega \in$ else
$D_i \bigcap P$	0	d_2	$d_2 - 1$	d_2
$D_i \bigcap Q$	d_1	0	$d_1 - 1$	d_1
$D_i \bigcap R$	0	0	1	0

TABLE 3. The cyclotomic numbers if d/e is even

	$\omega \in P$	$\omega \in Q$	$\omega \in D_{(i+e/2) \bmod e}$	$\omega \in$ else
$D_i \bigcap P$	0	d_2	$d_2 - 1$	d_2
$D_i \bigcap Q$	d_1	0	$d_1 - 1$	d_1
$D_i \bigcap R$	0	0	1	0

3. **New Construction of Almost Difference Set Pairs.** In the section, we will construct some kinds of almost difference set pairs using the Whiteman generalized cyclotomic classes.

Theorem 3.1. *Let* $U = D_i \bigcup P \bigcup R$, $V = D_i \bigcup P$; (U, V) *is an* $ADSP(pq, d+q.d+q-1, d+q-1, (p^2+2p+1)/4, p-1+d)$ *if and only if* $q = p+2$.

Proof: For $i = 0, 1, \ldots, e-1$, D_i is a sampling transformation of D_0. Therefore, we only need to consider the case $i = 0$ to prove the above proposition. It is obvious that $|U| = d+q$, $|V| = d+q-1$, $e = |U \bigcap V| = d+q-1$. By Definition 2.1, we just need to prove that there are $p-1+d$ nonzero elements satisfying $\Delta = (U+\omega) \bigcap V = (p^2+2p+1)/4$.

For every nonzero element $\omega \in Z_N^*$, let $\Delta = (U+\omega) \bigcap V$, and then

$$\begin{aligned}\Delta &= \left(D_0 \bigcup P \bigcup R + \omega\right) \bigcap \left(D_0 \bigcup P\right) \\ &= (D_0+\omega) \bigcap D_0 + (D_0+\omega) \bigcap P + (P+\omega) \bigcap D_0 \\ &\quad + (P+\omega) \bigcap P + \omega \bigcap D_0 + \omega \bigcap P \\ &= d - (D_0+\omega) \bigcap Q - (D_0+\omega) \bigcap D_1 \\ &\quad + (P+\omega) \bigcap D_0 + (P+\omega) \bigcap P + \omega \bigcap P\end{aligned}$$

According to Tables 1, 2, 3, we discussed four cases of nonzero element ω as follows when d/e is odd.

a) when $\omega \in P$,

$$\begin{aligned}\Delta_1 &= d - (D_0+\omega) \bigcap Q - (D_0+\omega) \bigcap D_1 + (P+\omega) \bigcap D_0 + (P+\omega) \bigcap P + \omega \bigcap P \\ &= d - (p-1)/2 - (p-1)(q-1)/4 + 0 + q - 2 + 1 = (pq - 3p + 3q - 1)/4\end{aligned}$$

b) when $\omega \in Q$,

$$\begin{aligned}\Delta_2 &= d - (D_0+\omega) \bigcap Q - (D_0+\omega) \bigcap D_1 + (P+\omega) \bigcap D_0 + (P+\omega) \bigcap P + \omega \bigcap P \\ &= d - 0 - (p-1)(q-1)/4 + d_2 + 0 + 0 = (pq - p + q - 1)/4\end{aligned}$$

c) when $\omega \in D_0$,

$$\Delta_3 = d - (D_0+\omega)\bigcap Q - (D_0+\omega)\bigcap D_1 + (P+\omega)\bigcap D_0 + (P+\omega)\bigcap P + \omega\bigcap P$$
$$= d - d_1 + 1 - ((p-1)(q-1)-3)/4 + d_2 - 1 + 0 + 0 = (pq - 2p + 2q + 1)/4$$

d) when $\omega \in D_1$,

$$\Delta_4 = d - (D_0+\omega)\bigcap Q - (D_0+\omega)\bigcap D_1 + (P+\omega)\bigcap D_0 + (P+\omega)\bigcap P + \omega\bigcap P$$
$$= d - d_1 - ((p-1)(q-1)+1)/4 + d_2 + 0 + 0 = (pq - 2p + 2q - 3)/4$$

We can obtain $\Delta_1 = \Delta_2 + 1 = \Delta_3 = \Delta_4 + 1$, so $p = q + 2$, $\Delta = (p^2 + 2p + 1)/4$.

Hence, we have $\Delta = (U+\omega)\bigcap V = (p^2 + 2p + 1)/4$ for every nonzero element $\omega \in Q\bigcup D_1$.

Similar to the previous case, we can obtain $\Delta = (U+\omega)\bigcap V = (p^2 + 2p + 1)/4$ for every nonzero element $\omega \in Q\bigcup D_1$ when d/e is even.

In summary, (U, V) constructed an $ADSP(pq, d+q.d+q-1, d+q-1, (p^2+2p+1)/4, p-1+d)$ if and only if $q = p + 2$.

Example 3.1. *When $p = 5$, $q = 7$,*

$$U = \{1, 17, 9, 13, 11, 12, 29, 3, 16, 27, 4, 33, 5, 10, 15, 20, 25, 30, 0\}$$
$$V = \{1, 17, 9, 13, 11, 12, 29, 3, 16, 27, 4, 33, 5, 10, 15, 20, 25, 30\}$$

(U, V) constructs an $ADSP(35, 19, 18, 18, 9, 16)$.

Theorem 3.2. *Let $U = D_i\bigcup R$, $V = Q$; (U, V) is an $ADSP(pq, d+1, p-1, 0, 1, p-1+d)$ if and only if $d_1 = 2$.*

Proof: Similar to Theorem 3.1, we only consider the case $i = 0$. It is obvious that $|U| = d + 1$, $|V| = p - 1$, $e = |U\bigcap V| = 0$. By Definition 2.1, we just need to prove that there are $p - 1 + d$ nonzero elements satisfying $\Delta = (U+\omega)\bigcap V = 1$. For every nonzero element $\omega \in Z_N^*$, let $\Delta = (U+\omega)\bigcap V$, and then

$$\Delta = (D_0\bigcup R+\omega)\bigcap Q = (D_0+\omega)\bigcap Q + \omega\bigcap Q$$

According to Tables 2, 3, we discussed four cases of nonzero element ω as follows when d/e is odd.

a) when $\omega \in P$,
$\Delta_1 = (D_0\bigcup R+\omega)\bigcap Q = (D_0+\omega)\bigcap Q + \omega\bigcap Q = d_1 + 0 = d_1$

b) when $\omega \in Q$,
$\Delta_2 = (D_0\bigcup R+\omega)\bigcap Q = (D_0+\omega)\bigcap Q + \omega\bigcap Q = 0 + 1 = 1$

c) when $\omega \in D_0$,
$\Delta_3 = (D_0\bigcup R+\omega)\bigcap Q = (D_0+\omega)\bigcap Q + \omega\bigcap Q = d_1 - 1 + 0 = d_1 - 1$

d) when $\omega \in \bigcup_{i=0}^{e-1} D_i - D_0$,
$\Delta_4 = (D_0\bigcup R+\omega)\bigcap Q = (D_0+\omega)\bigcap Q + \omega\bigcap Q = d_1 + 0 = d_1$

We can obtain $\Delta_2 = \Delta_3$, so $d_1 = 2$, $\Delta = 1$.

Hence, we have $\Delta = (U+\omega)\bigcap V = 1$ for every nonzero element $\omega \in Q\bigcup D_0$.

Similar to the previous case, we can obtain $\Delta = (U+\omega)\bigcap V = 1$ for every nonzero element $\omega \in Q\bigcup D_0$ when d/e is even.

From the above, (U, V) constructed an $ADSP(pq, d+1, p-1, 0, 1, p-1+d)$ if and only if $d_1 = 2$.

Example 3.2. *When $p = 5$, $q = 7$,*

$$U = \{1, 17, 9, 13, 11, 12, 29, 3, 16, 27, 4, 33, 0\}$$
$$V = \{7, 14, 21, 28\}$$

(U, V) constructs an $ADSP(35, 13, 4, 0, 1, 16)$.

Theorem 3.3. *Let* $U = D_i \bigcup Q \bigcup R$, $V = D_i \bigcup Q$; (U, V) *is an* $ADSP(pq, d+q.d+q-1, d+q-1, (p^2+2p-7)/4, p-1+d)$ *if and only if* $q = p+2$.

Proof: Similar to Theorem 3.1, we only consider the case $i = 0$. It is obvious that $|U| = d+q$, $|V| = d+q-1$, $e = |U \bigcap V| = d+q-1$. By Definition 2.1, we just need to prove that there are $p-1+d$ nonzero elements satisfying $\Delta = (U+\omega) \bigcap V = (p^2+2p-7)/4$. For every nonzero element $\omega \in Z_N^*$, let $\Delta = (U+\omega) \bigcap V$, and then

$$\begin{aligned}\Delta &= \Big(D_0 \bigcup Q \bigcup R + \omega\Big) \bigcap \Big(D_0 \bigcup Q\Big) \\ &= (D_0+\omega) \bigcap D_0 + (D_0+\omega) \bigcap Q + (Q+\omega) \bigcap D_0 \\ &\quad + (Q+\omega) \bigcap Q + \omega \bigcap D_0 + \omega \bigcap Q \\ &= d - (D_0+\omega) \bigcap P - (D_0+\omega) \bigcap D_1 \\ &\quad + (Q+\omega) \bigcap D_0 + (Q+\omega) \bigcap Q + \omega \bigcap Q\end{aligned}$$

According to Tables 1, 2, 3, we discussed four cases of nonzero element ω as follows when d/e is odd.

a) when $\omega \in P$,

$$\begin{aligned}\Delta_1 &= d - (D_0+\omega) \bigcap P - (D_0+\omega) \bigcap D_1 + (Q+\omega) \bigcap D_0 + (Q+\omega) \bigcap Q + \omega \bigcap Q \\ &= d - 0 - (p-1)(q-1)/4 + (p-1)/2 + 0 + 0 = (pq+p-q-1)/4\end{aligned}$$

b) when $\omega \in Q$,

$$\begin{aligned}\Delta_2 &= d - (D_0+\omega) \bigcap P - (D_0+\omega) \bigcap D_1 + (Q+\omega) \bigcap D_0 + (Q+\omega) \bigcap Q + \omega \bigcap Q \\ &= d - d_2 - (p-1)(q-1)/4 + 0 + p - 2 + 1 = (pq+3p-3q-1)/4\end{aligned}$$

c) when $\omega \in D_0$,

$$\begin{aligned}\Delta_3 &= d - (D_0+\omega) \bigcap P - (D_0+\omega) \bigcap D_1 + (Q+\omega) \bigcap D_0 + (Q+\omega) \bigcap Q + \omega \bigcap Q \\ &= d - d_2 + 1 - ((p-1)(q-1)-3)/4 + d_1 - 1 + 0 + 0 = (pq+2p-2q+1)/4\end{aligned}$$

d) when $\omega \in D_1$,

$$\begin{aligned}\Delta_4 &= d - (D_0+\omega) \bigcap P - (D_0+\omega) \bigcap D_1 + (Q+\omega) \bigcap D_0 + (Q+\omega) \bigcap Q + \omega \bigcap Q \\ &= d - d_2 - ((p-1)(q-1)+1)/4 + d_2 + 0 + 0 = (pq+2p-2q-3)/4\end{aligned}$$

We can obtain $\Delta_1 = \Delta_2 + 1 = \Delta_3 = \Delta_4 + 1$, so $p = q+2$, $\Delta = (p^2+2p-7)/4$.

Hence, we have $\Delta = (U+\omega) \bigcap V = (p^2+2p-7)/4$ for every nonzero element $\omega \in Q \bigcup D_1$.

Similar to the previous case, we can obtain $\Delta = (U+\omega) \bigcap V = (p^2+2p-7)/4$ for every nonzero element $\omega \in Q \bigcup D_1$ when d/e is even.

In summary, (U, V) constructed an $ADSP(pq, d+q.d+q-1, d+q-1, (p^2+2p-7)/4, p-1+d)$ if and only if $q = p+2$.

Example 3.3. *When* $p = 5$, $q = 7$,

$$U = \{1, 17, 9, 13, 11, 12, 29, 3, 16, 27, 4, 33, 7, 14, 21, 28, 0\}$$
$$V = \{1, 17, 9, 13, 11, 12, 29, 3, 16, 27, 4, 33, 7, 14, 21, 28\}$$

(U, V) *constructs an* $ADSP(35, 17, 16, 16, 7, 16)$.

Theorem 3.4. *Let* $U = \bigcup_{i=0}^{e-1} D_i \bigcup P$, $V = \bigcup_{i=0}^{e-1} D_i \bigcup Q \bigcup R$; (U, V) *is an* $ADSP(pq, pq-p, pq-q+1, pq-p-q+1, pq-p-q+1, p-1)$.

Proof: Similar to Theorem 3.1, we only consider the case $i = 0$. Obviously, $|U| = pq-p$, $|V| = pq - p - 1$, $e = |U \bigcap V| = pq - p - q + 1$. By Definition 2.1, we just need to prove that there are $p - 1$ nonzero elements satisfying $\Delta = (U + \omega) \bigcap V = pq - p - q + 1$. For every nonzero element $\omega \in Z_N^*$, let $\Delta = (U + \omega) \bigcap V$, and then

$$\begin{aligned}\Delta = &\left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap \bigcup_{i=0}^{e-1} D_i + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap Q + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap R \\ &+ (P + \omega) \bigcap \bigcup_{i=0}^{e-1} D_i + (P + \omega) \bigcap Q + (P + \omega) \bigcap R\end{aligned}$$

According to Table 1, we discuss three cases of nonzero element ω as follows.

a) when $\omega \in P$,

$$\begin{aligned}\Delta_1 = &\left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap \bigcup_{i=0}^{e-1} D_i + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap Q + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap R \\ &+ (P + \omega) \bigcap Q + (P + \omega) \bigcap \bigcup_{i=0}^{e-1} D_i + (P + \omega) \bigcap R \\ = &(p - 1)(q - 1) + p - 1 + 0 + 0 + 0 + 1 = pq - p - q + 2\end{aligned}$$

b) when $\omega \in Q$,

$$\begin{aligned}\Delta_2 = &\left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap \bigcup_{i=0}^{e-1} D_i + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap Q + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap R \\ &+ (P + \omega) \bigcap Q + (P + \omega) \bigcap \bigcup_{i=0}^{e-1} D_i + (P + \omega) \bigcap R \\ = &(p - 2)(q - 1) + 0 + 0 + q - 1 + 0 + 0 = pq - p - q + 1\end{aligned}$$

c) when $\omega \in \bigcup_{i=0}^{e-1} D_i$,

$$\begin{aligned}\Delta_3 = &\left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap \bigcup_{i=0}^{e-1} D_i + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap Q + \left(\bigcup_{i=0}^{e-1} D_i + \omega\right) \bigcap R \\ &+ (P + \omega) \bigcap Q + (P + \omega) \bigcap \bigcup_{i=0}^{e-1} D_i + (P + \omega) \bigcap R \\ = &(p - 2)(q - 2) + p - 2 + 1 + q - 2 + 1 + 0 = pq - p - q + 2\end{aligned}$$

We can obtain $\Delta_1 = \Delta_2 + 1 = \Delta_3$, so $\Delta = pq - p - q + 1$ when every nonzero element $\omega \in Q$.

From the above, (U, V) constructed an $ADSP(pq, pq - p, pq - q + 1, pq - p - q + 1, pq - p - q + 1, p - 1)$.

Example 3.4. *When $p = 3$, $q = 11$,*

$$\begin{aligned}U = \{&1, 2, 4, 8, 16, 32, 31, 29, 25, 17, 23, 13, 26, 19, 5, 10, 20, 7, 14, 28, 3, 6, 9, \\ &12, 15, 18, 21, 24, 27, 30\} \\ V = \{&1, 2, 4, 8, 16, 32, 31, 29, 25, 17, 23, 13, 26, 19, 5, 10, 20, 7, 14, 28, 11, 22, 0\}\end{aligned}$$

(U, V) constructs an $ADSP(33, 30, 23, 20, 20, 2)$.

4. **Conclusions.** In this paper, we construct some almost difference set pairs with Whiteman generalized cyclotomy based on cyclotomic classes and Chinese Remainder Theorem. We get a large number of almost difference set pairs that satisfy practical engineering. Simultaneously, we can also construct lots of binary sequence pairs with three-level autocorrelation by almost difference sets pairs.

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REFERENCES

[1] K. T. Arasu, D. Arya and D. Kedia, Constructions of punctured difference set pairs and their corresponding punctured binary array pairs, *IEEE Trans. Information Theory*, vol.61, no.4, pp.2191-2199, 2015.

[2] J. Z. Li and P. H. Ke, Study on the almost difference set pairs and almost perfect autocorrelation binary sequence pairs, *J. Wuyi Univ.*, vol.27, no.2, pp.10-14, 2008.

[3] X. P. Peng, C. Q. Xu and K. T. Arasu, New families of binary sequence pairs with two-level and three-level correlation, *IEEE Trans. Information Theory*, vol.58, no.11, pp.6968-6978, 2012.

[4] Y.-G. Jia and Y. Shen, Some new constructions of the almost difference set pairs based on generalized cyclotomic classes, *International Journal of Innovative Computing, Information and Control*, vol.12, no.2, pp.467-475, 2016.

[5] L. L. Zheng, L. Y. Pu and S. Y. Zhang, Constructions of almost difference set pairs by cyclotomy, *Journal of Mathematics*, vol.34, no.1, pp.116-122, 2014.

[6] A. L. Whiteman, A family of differencesets, *Illinois J. Math.*, vol.40, no.6, pp.107-121, 1962.

[7] C. Fan and G. Ge, A unified approach to whiteman's and Ding-Helleseth's generalized cyclotomy over residue class rings, *IEEE Journals & Magazines*, vol.60, no.2, pp.1326-1336, 2014.

[8] C. Ding, Autocorrelation values of generalized cyclotomic seqenences of order two, *IEEE Trans. Information Theory*, vol.44, no.4, pp.1999-1702, 1998.