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STOCHASTIC STABILITY ANALYSIS OF PERIODIC UNCERTAIN MARKOV JUMP SYSTEMS WITH PARTIALLY KNOWN INFORMATION

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Abstract. *In this paper, the issue of stochastic stability and stabilization of periodic Markov jump systems with uncertain transition probabilities and parameters is investigated. The partially unknown transition probability matrix is described as a polytope with uncertain vertices. By constructing a Lyapunov function, sufficient conditions are established under which the system is stochastically stable. Numerical example is given to show the effectiveness of the method proposed.*
Keywords: Markov jump systems, Periodic systems, Partly unknown transition probabilities, Stochastic stability

1. **Introduction.** In the past decades, as its widely practical application, Markov jump systems (MJSs) have been extensively studied. In reality, there are many complex systems whose structures or parameters may have abrupt changes once external environment changes suddenly, such as communication network systems, manufacturing systems, and chemical systems. As a special kind of hybrid system, MJSs have great potential for describing such systems, and plenty of results have been proposed, such as stability and stabilization [1,2], filtering [3,4], control [5,6] and fault detection [7]. On the other hand, the parameters of complex systems are difficult to reach precisely, which leads to the study on uncertain MJSs [8-11].

On another research front line, many periodic systems exist in real world, such as satellites that move around the earth periodically, computer applications which run with clock cycles of CPU and some economic systems. Thus, periodic MJSs have also caught much attention in recent years. Periodic MJSs are reasonable to describe many systems whose structure and parameters have abrupt change in a cycle. Economic system is a typical example, and it is well-known that the development of global economy is periodic. In a cycle, the model of normal economic development has fixed structure and parameters. However, when some emergencies happen, such as wars, and national policies, the structure and parameters will be changed abruptly. Recently, a few results about periodic MJSs have been proposed. In 2012, Aberkane and Dragan studied the problem of H_∞ filtering for periodic MJSs under constraints [12]. In 2015, Hou and Ma designed an $H_2 - H_\infty$ controller for periodic MJSs [13]. However, these results presented are all under the assumption that system models are precisely known, which leads to the study of uncertain periodic MJSs.

It is worth mentioning that most results about MJSs assume that transition probabilities are totally known, which may not be realistic. Some new results about MJSs with partially unknown transition probabilities are presented. In 2009, Zhang and Boukas investigated stability and stabilization problem of discrete and continuous MJSs with partially unknown transition probabilities [14]; later, Zhang et al. conducted the problem of stabilization of MJSs with uncertain transition probabilities in [15] and more recent result for uncertain systems is obtained [16]. Nevertheless, to the best of our knowledge, the problem of stability and stabilization of periodic MJSs with uncertain transition probabilities has not been investigated. Here, we do study on stability analysis of uncertain periodic MJSs with partially unknown transition probabilities, and a state feed-back controller is designed. The main contributions of this paper are concluded in the following. Firstly, the periodic MJSs with partially unknown transition probabilities are studied for the first time. Secondly, the uncertain parameters are considered.

The rest of this paper is organized as follows. In Section 2, problem statement and preliminaries are given. Section 3 proposes stochastic stability and stabilization conditions. A numerical example is provided to ensure the effectiveness in Section 4 and Section 5 concludes this paper.

Notation: R^n stands for an n-dimensional Euclidean space; A^{T} denotes the transpose of the matrix A; $E\{\cdot\}$ denotes the mathematical statistical expectation; a positive-definite matrix is denoted by $P > 0$; $diag\{\dots\}$ stands for a block-diagonal matrix; I is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix. The matrix sequences $\{U(k)\}_{k\geq 0}$ are said to be q-periodic once $U(k+q) = U(k)$.

2. **Statement and Preliminaries.**

We consider the following discrete-time q-periodic MJSs:

$$x(k+1) = A_{r_k}(k)x(k) + B_{r_k}(k)u(k) + g(x_k, r_k), \tag{1}$$

where $x(k) \in R^n$ is system state vector, $u(k) \in R^p$ is control input vector, and $g(\cdot)$ is time-dependent and norm-bounded uncertainty function. $\{r_k, k \geq 0\}$ is the concerned discrete-time Markov chain, which is defined in a finite set $\Lambda = \{1, 2, 3, \dots, N\}$. The q-periodic transition probability is denoted as $\pi_{ij}(k) = P\{r_{k+1} = j | r_k = i\}$, which satisfies $\pi_{ij}(k) \geq 0$, $\sum_{j=1}^{N} \pi_{ij}(k) = 1$ and $\pi_{ij}(k+q) = \pi_{ij}(k)$. The transition probability matrix is defined as $\Pi(k) = \{\pi_{ij}(k)\}$, $i, j \in \Lambda$. For simplicity, let $r_k = i$, and $A_{r_k}(k)$ and $B_{r_k}(k)$ are denoted by $A_i(k)$, $B_i(k)$, respectively. The matrices sequences $\{A_i(k)\}_{k\geq 0}$, $\{B_i(k)\}_{k\geq 0}$ are q-periodic matrix sequences, which means $A_i(k+q) = A_i(k)$ and $B_i(k+q) = B_i(k)$. The time varying q-periodic transition probability matrix is described as a polytope below:

$$\Pi(k) = \sum_{l=1}^{w} \xi_l(k) \Pi^l,$$

where $0 \leq \xi_l(k) \leq 1$, $\sum_{l=1}^{w} \xi_l(k) = 1$, $\xi_l(k+q) = \xi_l(k)$ and $\Pi(k+q) = \Pi(k)$. In this paper, the vertices of polytope are partially unknown. Then, we denote

$$\Pi^i(k) = \Pi^i_K(k) + \Pi^i_{UK}(k),$$

and

$$\Pi^i_K(k) \triangleq \{j : \pi_{ij}(k) \text{ is known}\}, \quad \Pi^i_{UK}(k) \triangleq \{j : \pi_{ij}(k) \text{ is unknown}\}.$$

Moreover, $\Pi^i_K(k)$ is described as $\Pi^i_K(k) = \left(K^i_1(k), \dots, K^i_m(k)\right)$, $\forall 1 \leq m \leq N$, where $\Pi^i(k)$ represents the ith row of matrix $\Pi(k)$, and $K^i_m(k)$ represents the mth known element in the ith row of matrix $\Pi(k)$. To proceed the study, some concepts are given below.

Assumption 2.1. *The norm-bounded uncertainty* $g(x_k, r_k)$ *is assumed to satisfy*

$$g(x_k, r_k) = \Delta A_i(k)x(k),$$

and

$$\Delta A_i(k) = M_i(k) * \Upsilon_i(k) * N_i(k),$$

where $M_i(k)$ *and* $N_i(k)$ *are q-periodic constant matrices, and* $\Upsilon_i(k)$ *is an unknown q-periodic matrix with Lebesque measurable elements satisfying* $\Upsilon_i^{\mathrm{T}}(k)\Upsilon_i(k) \leq 1$.

Thus, q-periodic system (1) can be written as

$$x(k+1) = (A_i(k) + \Delta A_i(k))\, x(k) + B_i(k)u(k). \tag{2}$$

Definition 2.1. *Given any initial mode* r_0*, and an initial state* x_0*, system (1) is said to be stochastically stable if it holds:*

$$\lim_{m\to\infty} E\left\{\sum_{k=0}^{m} x^{\mathrm{T}}(k)x(k)|x_0, r_0\right\} < \infty. \tag{3}$$

Lemma 2.1. [17] Let W, S, V be real matrices with appropriate dimensions, and $S^{\mathrm{T}}S \leq I$. There exists a scalar $\alpha > 0$ such that it holds:

$$WSV + V^{\mathrm{T}}S^{\mathrm{T}}W^{\mathrm{T}} \leq \alpha^{-1}WW^{\mathrm{T}} + \alpha V^{\mathrm{T}}V. \tag{4}$$

3. Controller Design.

Lemma 3.1. *Let* $u(k) = 0$*, q-periodic system (2) is stochastically stable, if there exist positive symmetric matrices* P_i^l*,* $l \in \{1, 2, \ldots, w\}$*, such that it holds*

$$\hat{A}_i^{\mathrm{T}}(k)\left(\sum_{j=1}^{N}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)\right)\hat{A}_i(k) - P_i(k) < 0, \tag{5}$$

where $P_i(k) = \sum_{l=1}^{w}\xi_l(k)P_i^l$*,* $\hat{A}_i(k) = A_i(k) + \Delta A_i(k)$.

Proof: Construct a Lyapunov function:

$$V_i(k) = x^{\mathrm{T}}(k)P_i(k)x(k).$$

We have

$$\begin{aligned}\Delta V_i(k) &= E\left\{x^{\mathrm{T}}(k+1)P_j(k+1)x(k+1)\right\} - x^{\mathrm{T}}(k)P_i(k)x(k)\\ &= x^{\mathrm{T}}(k)\left[\hat{A}_i^{\mathrm{T}}(k)\sum_{j=1}^{N}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)\hat{A}_i(k) - P_i(k)\right]x(k)\\ &= x^{\mathrm{T}}(k)\Omega_i(k)x(k),\end{aligned}$$

where $\Omega_i(k) = \hat{A}_i^{\mathrm{T}}(k)\sum_{j=1}^{N}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)\hat{A}_i(k) - P_i(k)$. Then, inequality (5) implies

$$\Delta V_i(k) < 0.$$

Let

$$\rho_i = \min_k\left\{\lambda_{\min}(-\Omega_i(k))\right\},$$

where $\lambda_{\min}(-\Omega_i(k))$ is the minimal eigenvalue of $-\Omega_i(k)$. Thus,

$$\Delta V_i(k) \leq -\rho_i x^{\mathrm{T}}(k)x(k) \leq -\rho x^{\mathrm{T}}(k)x(k),$$

where $\rho = \min_{i\in\Lambda}\rho_i$. We have

$$E\left\{V_j(k+1)\right\} - V_0 \leq -\rho E\left\{\sum_{k=0}^{\infty}\|x(k)\|^2\right\}.$$

Then,

$$E\left\{\sum_{k=0}^{\infty}\|x(k)\|^2\right\} \leq \frac{1}{\rho}V_0 - \frac{1}{\rho}E\left\{V_j(k+1)\right\} \leq \frac{1}{\rho}V_0 < \infty.$$

Therefore, inequality (5) ensures that system (2) is stochastically stable. The proof is completed.

Theorem 3.1. *Periodic system (2) is stochastically stable under condition $u(k) = 0$, if there exist positive symmetric matrices, P_i^l, $l \in \{1, 2, \ldots, w\}$ such that it holds*

$$\hat{A}_i^{\mathrm{T}}(k)P_K^i(k)\hat{A}_i(k) - \pi_K^i(k)P_i(k) < 0, \quad j \in \Pi_K^i(k), \tag{6}$$

$$\hat{A}_i^{\mathrm{T}}(k)P_j(k+1)\hat{A}_i(k) - P_i(k) < 0, \quad j \in \Pi_{UK}^i(k), \tag{7}$$

where $P_K^i(k) = \sum_{j\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)$, $\pi_K^i(k) = \sum_{j\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l$, $P_i(k) = \sum_{l=1}^{w}\xi_l(k)P_i^l$.

Proof: According to Lemma 3.1, if condition (5) holds, then, system (2) is stochastically stable. Let

$$\Psi_i = \hat{A}_i^{\mathrm{T}}(k)\left(\sum_{j=1}^{N}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)\right)\hat{A}_i(k) - P_i(k).$$

Since $\sum_{j=1}^{N}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l = 1$, we have

$$\begin{aligned}\Psi_i &= \hat{A}_i^{\mathrm{T}}(k)\left(\sum_{j=1}^{N}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)\right)\hat{A}_i(k) - \left(\sum_{j=1}^{N}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l\right)P_i(k+1)\\
&= \hat{A}_i^{\mathrm{T}}(k)\left(\sum_{k\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)\right)\hat{A}_i(k) - \left(\sum_{k\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l\right)P_i(k+1)\\
&\quad + \hat{A}_i^{\mathrm{T}}(k)\left(\sum_{k\in\Lambda_{UK}}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)\right)\hat{A}_i(k) - \left(\sum_{k\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l\right)P_i(k+1)\\
&= \hat{A}_i^{\mathrm{T}}(k)P_K^i(k)\hat{A}_i(k) - \pi_K^i(k)P_i(k) + \left(\sum_{j\in\Lambda_{UK}}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l\right)\left(\hat{A}_i^{\mathrm{T}}(k)P_j(k)\hat{A}_i(k) - P_i(k)\right),\end{aligned}$$

where $\pi_{ij}^l > 0$. Therefore, conditions (6) and (7) confirm condition (5). The proof is completed.

Theorem 3.2. *Periodic system (2) is stochastically stable if there exist q-periodic gain matrix $K_i(k)$, positive symmetric matrices X_i^l, $l = 1, 2, \ldots, w$, and matrices Y_i^l, $l = 1, 2, \ldots, w$ such that it holds*

$$\begin{bmatrix} -\mathrm{X}_K^i(k+1) & \Phi_K^i\left(\hat{A}_i(k)X_i(k) + B_i(k)Y_i(k)\right) \\ * & -\pi_K^i(k)X_i(k)\end{bmatrix} < 0, \quad j \in \Pi_K^i(k), \tag{8}$$

$$\begin{bmatrix} -X_j(k+1) & \hat{A}_i(k)X_i(k) + B_i(k)Y_i(k) \\ * & -X_i(k)\end{bmatrix} < 0, \quad j \in \Pi_{UK}^i(k), \tag{9}$$

where

$$X_i(k) = \sum_{l=1}^{w}\xi_l(k)X_i^l, \quad Y_i(k) = \sum_{l=1}^{w}\xi_l(k)Y_i^l, \quad \Phi_n = \sqrt{\sum_{l=1}^{w}\xi_l(k)\pi_{iK_n^i}^l}, \quad n = \{1, 2, \ldots, m\},$$

$$\mathrm{X}_K^i(k) = diag\left\{X_{K_i^i}(k), X_{K_2^i}(k), \ldots, X_{K_m^i}(k)\right\}, \quad \Phi_K^i = \left[\Phi_1 I \ \ldots \ \Phi_m I\right]^{\mathrm{T}}.$$

Furthermore, the controller gain matrix is $K_i(k) = Y_i(k)X_i^{-1}(k)$.

Proof: Let $u(k) = K_i(k)x(k)$, and then system (2) is transformed into

$$x(k+1) = \left(\hat{A}_i(k) + B_i(k)K_i(k)\right) x(k).$$

For $j \in \Pi_K^i(k)$, from condition (6), we have

$$\left(\hat{A}_i(k) + B_i(k)K_i(k)\right)^{\mathrm{T}} \sqrt{\sum_{j\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l P_j(k+1)}$$
$$*\sqrt{\sum_{j\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l}\left(\hat{A}_i(k) + B_i(k)K_i(k)\right) - \pi_K^i(k)P_i(k) < 0.$$

By Schur complement, and let $X_i(k) = P_i^{-1}(k)$, $\Phi_n = \sqrt{\sum_{l=1}^{w}\xi_l(k)\pi_{iK_n^i}^l}$, $n = \{1, 2, \ldots, m\}$, we have

$$\begin{bmatrix} -X_{K_1^i}(k+1) & 0 & \ldots & 0 & \Phi_1\left(\hat{A}_i(k) + B_i(k)K_i(k)\right) \\ * & -X_{K_2^i}(k+1) & \ldots & 0 & \Phi_2\left(\hat{A}_i(k) + B_i(k)K_i(k)\right) \\ * & * & \ldots & \vdots & \vdots \\ * & * & \ldots & -X_{K_m^i}(k+1) & \Phi_m\left(\hat{A}_i(k) + B_i(k)K_i(k)\right) \\ * & * & \ldots & * & -\sum_{j\in\Lambda_K}\sum_{l=1}^{w}\xi_l(k)\pi_{ij}^l X_i^{-1}(k) \end{bmatrix} < 0.$$

Let $\mathrm{X}_K^i(k) = diag\left\{X_{K_i^i}(k), X_{K_2^i}(k), \ldots, X_{K_m^i}(k)\right\}$, $\Phi_K^i = [\Phi_1 I \ \ldots \ \Phi_m I]^{\mathrm{T}}$, and we have

$$\begin{bmatrix} -\mathrm{X}_K^i(k) & \Phi_K^i\left(\hat{A}_i(k) + B_i(k)K_i(k)\right) \\ * & -\pi_K^i(k)X_i^{-1}(k) \end{bmatrix} < 0.$$

Thus,

$$\begin{bmatrix} -\mathrm{X}_K^i(k) & \Phi_K^i\left(\hat{A}_i(k)X_i(k) + B_i(k)Y_i(k)\right) \\ * & -\pi_K^i(k)X_i(k) \end{bmatrix} < 0,$$

where $Y_i(k) = K_i(k)X_i(k)$. For $j \in \Pi_{UK}^i(k)$, the proof is similar to the above part. Therefore, conditions (8) and (9) imply conditions (6) and (7). The proof is completed.

Theorem 3.3. *Periodic system (1) is stochastically stable if there exist q-periodic scalar* $K_i(k)$, *scalar* β *and q-periodic matrices* $M_i(k)$, $N_i(k)$, *positive symmetric matrices* X_i^l, $l \in \{1, 2, \ldots, w\}$, *matrices* Y_i^l, $l \in \{1, 2, \ldots, w\}$, *such that it holds*

$$\begin{bmatrix} \Xi_K^i(k) & \zeta_i^{\mathrm{T}}(k) \\ * & -\beta I \end{bmatrix} < 0, \quad j \in \Pi_K^i(k), \tag{10}$$

$$\begin{bmatrix} \Xi_{UK}^i(k) & \zeta_i^{\mathrm{T}}(k) \\ * & -\beta I \end{bmatrix} < 0, \quad j \in \Pi_{UK}^i(k), \tag{11}$$

where

$$\Xi_K^i(k) = \begin{bmatrix} \beta\Phi_K^i M_i(k)M_i^{\mathrm{T}}(k){\Phi_K^i}^{\mathrm{T}} - \mathrm{X}_K^i(k+1) & \Phi_K^i\left(A_i(k)X_i(k) + B_i(k)Y_i(k)\right) \\ * & -\pi_K^i(k)X_i(k) \end{bmatrix},$$

$$\Xi^i_{UK}(k) = \begin{bmatrix} \beta M_i(k)M_i^{\mathrm{T}}(k) - X_j(k+1) & A_i(k)X_i(k) + B_i(k)Y_i(k) \\ * & -X_i(k) \end{bmatrix},$$

$$\Xi^i_K(q-1) = \begin{bmatrix} \beta\Phi^i_K M_i(q-1)M_i^{\mathrm{T}}(q-1){\Phi^i_K}^{\mathrm{T}} - \mathrm{X}^i_K(0) & \Phi^i_K\left(A_i(q-1)X_i(q-1) + B_i(q-1)Y_i(q-1)\right) \\ * & -\pi^i_K(q-1)X_i(q-1) \end{bmatrix},$$

$$\Xi^i_{UK}(q-1) = \begin{bmatrix} \beta M_i(q-1)M_i^{\mathrm{T}}(q-1) - X_j(0) & A_i(q-1)X_i(q-1) + B_i(q-1)Y_i(q-1) \\ * & -X_i(q-1) \end{bmatrix},$$

$$\zeta_i^{\mathrm{T}}(k) = \begin{bmatrix} 0 \\ X_i^{\mathrm{T}}(k)N_i^{\mathrm{T}}(k) \end{bmatrix}, \quad \mathrm{X}^i_K(k) = diag\left\{X_{K^i_1}(k), X_{K^i_2}(k), \ldots, X_{K^i_m}(k)\right\},$$

$$\Phi^i_K = [\Phi_1 I \ \ldots \ \Phi_m I]^{\mathrm{T}}, \quad \Phi_n = \sqrt{\sum_{l=1}^{w} \xi_l(k)\pi^l_{iK^i_n}}, \quad n = \{1, 2, \ldots, m\},$$

$$X_i(k) = \sum_{l=1}^{w} \xi_l(k)X_i^l, \quad Y_i(k) = \sum_{l=1}^{w} \xi_l(k)Y_i^l.$$

Proof: According to Assumption 2.1, condition (8) is transformed into

$$\begin{bmatrix} 0 & \Phi^i_K \Delta A_i(k)X_i(k) \\ * & 0 \end{bmatrix} = \begin{bmatrix} \Phi^i_K M_i(k) \\ 0 \end{bmatrix} \Upsilon_i(k) \left[0 \quad N_i(k)X_i(k)\right] + \begin{bmatrix} 0 \\ X_i^{\mathrm{T}}(k)N_i^{\mathrm{T}}(k) \end{bmatrix} \Upsilon_i^{\mathrm{T}}(k) \left[M_i^{\mathrm{T}}(k){\Phi^i_K}^{\mathrm{T}} \quad 0\right].$$

By Lemma 2.1, the following inequality is obtained

$$\begin{aligned} &\begin{bmatrix} -\mathrm{X}^i_K(k+1) & \Phi^i_K(A_i(k)X_i(k) + B_i(k)Y_i(k)) \\ * & -\pi^i_K(k)X_i(k) \end{bmatrix} + \begin{bmatrix} 0 & \Phi^i_K \Delta A_i(k)X_i(k) \\ * & 0 \end{bmatrix} \\ \leq &\begin{bmatrix} -\mathrm{X}^i_K(k+1) & \Phi^i_K(A_i(k)X_i(k) + B_i(k)Y_i(k)) \\ * & -\pi^i_K(k)X_i(k) \end{bmatrix} \\ &+ \alpha^{-1} \begin{bmatrix} \Phi^i_K M_i(k) \\ 0 \end{bmatrix} \left[M_i^{\mathrm{T}}(k){\Phi^i_K}^{\mathrm{T}} \quad 0\right] + \alpha \begin{bmatrix} 0 \\ X_i^{\mathrm{T}}(k)N_i^{\mathrm{T}}(k) \end{bmatrix} \left[0 \quad N_i(k)X_i(k)\right]. \end{aligned}$$

Thus, by Schur complement, we have condition (10). By the same way, it is obtained that condition (9) implies condition (11). The proof is completed.

4. **Numerical Example.** We consider 2-periodic discrete-time MJSs with 4 jumping modes, where

$$A_1(1) = \begin{bmatrix} 0 & -0.45 \\ 1.9 & 0.9 \end{bmatrix}, \ A_1(2) = \begin{bmatrix} 0 & -0.29 \\ 1.9 & 1.26 \end{bmatrix}, \ A_2(1) = \begin{bmatrix} 0 & -0.5 \\ 1.9 & 0.9 \end{bmatrix},$$

$$A_2(2) = \begin{bmatrix} 0 & -0.3 \\ 1.9 & 1.23 \end{bmatrix}, A_3(1) = \begin{bmatrix} 0 & -0.4 \\ 1.9 & 0.9 \end{bmatrix}, \ A_3(2) = \begin{bmatrix} 0 & -0.52 \\ 1.9 & 1.2 \end{bmatrix},$$

$$A_4(1) = \begin{bmatrix} 0 & -0.35 \\ 1.9 & 0.9 \end{bmatrix}, \ A_4(2) = \begin{bmatrix} 0 & -0.42 \\ 1.9 & 1.3 \end{bmatrix},$$

$$B_1(1) = \begin{bmatrix} 0.5 \\ 1.1 \end{bmatrix}, \ B_1(2) = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}, \ B_2(1) = \begin{bmatrix} 0.6 \\ 1.2 \end{bmatrix}, \ B_2(2) = \begin{bmatrix} 0.7 \\ 1.3 \end{bmatrix},$$

$$B_3(1) = \begin{bmatrix} 0.4 \\ 1.5 \end{bmatrix}, \ B_3(2) = \begin{bmatrix} 0.6 \\ 1.3 \end{bmatrix}, \ B_4(1) = \begin{bmatrix} 0.5 \\ 1.2 \end{bmatrix}, \ B_4(2) = \begin{bmatrix} 0.7 \\ 1.1 \end{bmatrix},$$

$$M_1(1) = M_2(1) = M_3(1) = M_4(1) = M_1(2) = M_2(2) = M_3(2) = M_4(2) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$N_1(1) = N_2(1) = N_3(1) = N_4(1) = N_1(2) = N_2(2) = N_3(2) = N_4(2) = [0.1 \quad 0.1].$$

The uncertain vertices of transition probability matrices are given as below:

$$\Pi^1 = \begin{bmatrix} 0.3 & ? & 0.1 & ? \\ ? & ? & 0.3 & 0.2 \\ ? & 0.1 & ? & 0.3 \\ 0.2 & ? & ? & ? \end{bmatrix}, \quad \Pi^2 = \begin{bmatrix} 0.1 & ? & 0.4 & ? \\ ? & ? & 0.1 & 0.4 \\ ? & 0.5 & ? & 0.3 \\ 0.3 & ? & ? & ? \end{bmatrix},$$

where (?) means the unknown element. For given initial state vector $x(0) = \begin{bmatrix} -0.5 \\ 0.4 \end{bmatrix}$, and two uncertain polytope vertices matrices Π^1, Π^2, we construct a 2-periodic partly unknown transition probability matrix as below:

$$\Pi(k) = \begin{cases} \begin{bmatrix} 0.3 & ? & 0.1 & ? \\ ? & ? & 0.3 & 0.2 \\ ? & 0.1 & ? & 0.3 \\ 0.2 & ? & ? & ? \end{bmatrix}, & k = 2t, \ t = 0, 1, 2, \ldots \\ \begin{bmatrix} 0.1 & ? & 0.4 & ? \\ ? & ? & 0.1 & 0.4 \\ ? & 0.5 & ? & 0.3 \\ 0.3 & ? & ? & ? \end{bmatrix}, & k = 2t+1, \ t = 0, 1, 2, \ldots \end{cases}$$

By solving linear matrix inequalities (LMIs) (10) and (11), we obtain the controller gain as follows

$$\begin{aligned} K_1(1) &= [-1.6027 \quad -0.5240], & K_1(2) &= [-1.2759 \quad -0.6853], \\ K_2(1) &= [-1.4333 \quad -0.4333], & K_2(2) &= [-1.2936 \quad -0.6372], \\ K_3(1) &= [-1.2656 \quad -0.4938], & K_3(2) &= [-1.3512 \quad -0.6088], \\ K_4(1) &= [-1.4970 \quad -0.5355], & K_4(2) &= [-1.4353 \quad -0.6682]. \end{aligned}$$

The system states excluding controller are shown in Figure 1. After applying controller, the system states are shown in Figure 2.

Figure 1 shows the system (1) is almost unstable without controller. However, from Figure 2, it is observed that system (1) turns out stochastically stable under controller, which means the controller designed in this paper is effective.

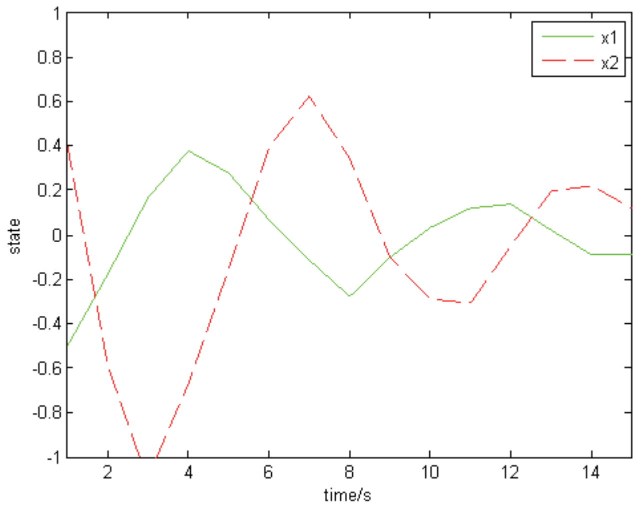

FIGURE 1. System states excluding controller

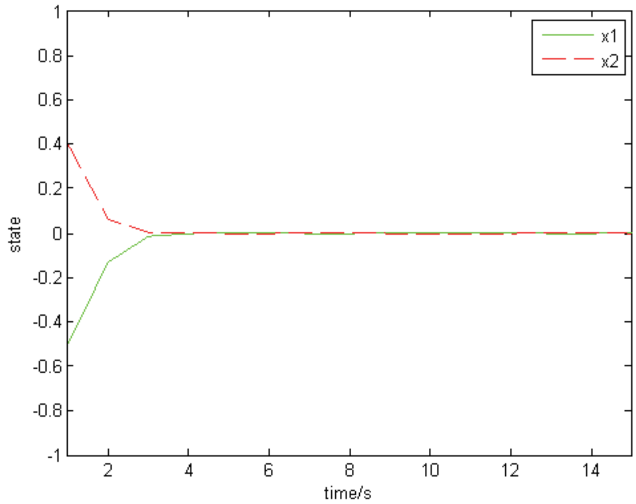

Figure 2. System states under controller

5. **Conclusions.** In this paper, a periodic controller is designed for periodic MJSs with uncertainties and partially unknown transition probabilities. The partially unknown transition probabilities are described as a polytope with uncertain vertices. The controller designed makes the underlying MJSs stochastically stable. A numerical example is given to show that the method is effective. In the future, to have a better description of realistic system, nonlinear phenomenon will be considered.

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