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MULTIDIMENSIONAL SCALING-BASED STATIONARY TARGET LOCALIZATION FROM TIME DIFFERENCE OF ARRIVALS MEASUREMENTS WITH BASE STATION POSITIONS ERRORS

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Abstract. *A novel weighted multidimensional scaling algorithm is proposed to estimate the position of the static target by considering time difference of arrival measurements and base station position uncertainties. The solution is accurate and closed form. The results show that the proposed algorithm has small bias and minimum square error compared with the two-stage weighted least squares algorithm (2WLS) and constraint total least square algorithm (CTLS). The performance of the proposed method achieves the Cramer-Rao lower bound (CRLB) level in low base stations noise power.*

Keywords: Multidimensional scaling, Localization, Time difference of arrivals, Base station position errors

1. **Introduction.** Finding the source position has attracted a lot of debates in recent years in various research fields such as radar, sonar and communication. Recently, many types of research have focused on this issue since the US Federal Commission has adopted a decision to develop the Emergency 911 (E-911) services [1].

The source position can be estimated by exploiting the time of arrival (TOA), the angle of arrival (AOA), time difference of arrival (TDOA), frequency difference of arrival (FDOA) and received signal strength (RSS). TOAs, TDOAs and RSSs provide distance measurements. FDOA provides the rate of distance measurements, the AOA provides the source direction according to sensors. On the other hand, the distances and bearing information are derived from the measurements and location for the known position sensors [2,3]. A set of non-linear equations is generated from these measurements. They can be solved by iterative algorithms such as Taylor method [4,5]. The iterative methods need a good initial guess close to the actual position coordinates to guarantee the convergence. In contrast, the closed form algorithms are efficient, and they do not require initial guesses and convert the nonlinear equations to the set of the linear equations; hence, it provides an accurate estimation for emitter location.

Nowadays, multidimensional scaling (MDS) [6,7] has become a powerful tool in exploratory data analysis. It has been used to find the position of mobile and static sources in the complex environment. The MDS is started by constructing a dis/similarity distance matrix among all pairs of sensors, and then, the double centring procedure is applied on it. Finally, the eigenvalue decomposition figures the coordinate positions out.

The performance of localization algorithm depends on the availability of precise base station locations. When the base station locations are accurately known, a good performance will be expected under certain signal to noise ratio conditions. Most of these algorithms could achieve the Cramer-Rao lower bound (CRLB). The CRLB is the minimum variance that can be obtained by any unbiased estimator. However, the base station

(BS) may have errors. When the BS positions are not precise, the performance of the estimator is degraded, if the covariance matrices of BS position error and the measurement noise satisfy some certain conditions, then the acceptable estimate could be achieved.

In this monograph, we proposed an MDS localization algorithm using TDOA measurements with the base stations positions uncertainties; it is closed form accurate estimator and it reaches the CRLB at the low BS noise power. Moreover, it overcomes the 2WLS and CTLS method is moderate and high level of BS noise power. The proposed estimator can be used for non-cooperative target and passive target.

This monograph is organized as the following: the second section includes the system model and CRLB analysis, the proposed solution of localization of BS positions uncertainty is introduced in the third section, the simulation configuration and result are presented in the forth section while the work is concluded in the fifth section.

2. **The Proposed Model and CRLB.** In 3-D coordinates system, it assumed that an emitting target $\mathbf{u}$ is located at an unknown position $\mathbf{u} = [x, y, z]^T$, and the BSs are located at $\mathbf{s}_m = [x_m, y_m, z_m]^T$, $m = 1, 2, \ldots, M$ where M is the total number of BS. The BSs receive the signal from a target at the different time slot τ_i. To get a unique solution for the target position, we assume that the receiving BSs are neither deploying on straight line nor a plane. The noisy positions of the BSs are expressed as

$$\mathbf{S} = \mathbf{S}^o + \Delta\mathbf{S} \tag{1}$$

where $\mathbf{S}^o = [\mathbf{s}_1^o, \mathbf{s}_2^o, \ldots, \mathbf{s}_M^o]^T$ and $\Delta\mathbf{S} = \left[\Delta\mathbf{s}_1^T, \Delta\mathbf{s}_2^T, \ldots, \Delta\mathbf{s}_M^T\right]^T$, $\Delta\mathbf{s}_m = [\Delta x_m, \Delta y_m, \Delta z_m]^T$ is the location error vector while $\mathbf{s}_m = [x_m, y_m, z_m]^T$ is the actual position vector for BSs, and the error vector is a zero mean Gaussian random vector with the covariance matrix $\mathbf{Q}_\beta = E\{\Delta\mathbf{S}\Delta\mathbf{S}^T\}$.

The first BS is selected as the reference one. Therefore, the range difference is obtained by subtracting the distances between all sensors and the reference sensor. The TDOA measurements are proportional to the range difference in a speed of signal c. The conventional range difference obtained from TDOA τ_{i1} is given as

$$d_{i1} = c.\tau_{i1} = d_i - d_1, \quad i = 2, 3, \ldots, M \tag{2}$$

$$d_i = \sqrt{(\mathbf{s}_i^o - \mathbf{u})^T (\mathbf{s}_i^o - \mathbf{u})}, \quad i = 1, 2, \ldots, M \tag{3}$$

From above equations, we have $d_{01} = -d_1$ and $d_{11} = 0$. In practical situations, the effect of noise should be considered, so define r_{i1} as the noisy range difference given by

$$r_{i1} = d_{i1} + n_{i1} \tag{4}$$

Then the matrix form of the previous equation is

$$\mathbf{r} = \mathbf{d} + \mathbf{n} \tag{5}$$

where $\mathbf{r} = [r_{21}, r_{31}, \ldots, r_{M1}]^T$, $\mathbf{d} = [d_{21}, d_{31}, \ldots, d_{M1}]^T$, $\mathbf{n} = [n_{21}, n_{31}, \ldots, n_{M1}]^T$, $\mathbf{n}$ is modelled as the Gaussian noise with zero mean and covariance $\mathbf{Q}_n = E\{\mathbf{n}\mathbf{n}^T\}$. The error of BS and the TDOA noise $\mathbf{n}$ are assumed to be uncorrelated.

The CRLB is the minimum variance that could be attained by any unbiased estimator; it is defined as the inverse of Fisher information matrix [8], and the Fisher information matrix of estimating the unknown source position using TDOA measurements with presence of BS position uncertainties can be obtained as the following

$$FIM = \mathbf{X} - \mathbf{Y}^T\mathbf{Z}^{-1}\mathbf{Y} \tag{6}$$

Applying the matrix inversion lemma on the above equation, the result is the CRLB and can be expressed as follows

$$CRLB(\mathbf{u}_o) = \mathbf{X}^{-1} - \mathbf{X}^{-1}\mathbf{Y}\left(\mathbf{Z} - \mathbf{Y}^T\mathbf{X}^{-1}\mathbf{Y}\right)^{-1}\mathbf{Y}^T\mathbf{X}^{-1} \tag{7}$$

where $\mathbf{Z} = (\partial \mathbf{r}/\partial \mathbf{s})^T \mathbf{Q}_n^{-1} (\partial \mathbf{r}/\partial \mathbf{s}) + \mathbf{Q}_\beta^{-1}$ is $3M \times 3M$ matrix, $\mathbf{Y} = (\partial \mathbf{r}/\partial \mathbf{u})^T \mathbf{Q}_n^{-1} (\partial \mathbf{r}/\partial \mathbf{s})$ is $3 \times 3M$ matrix, $\mathbf{X} = (\partial \mathbf{r}/\partial \mathbf{u})^T \mathbf{Q}_n^{-1} (\partial \mathbf{r}/\partial \mathbf{u})$ is 3×3 matrix, and the partial derivatives $(\partial \mathbf{r}/\partial \mathbf{s})$ and $(\partial \mathbf{r}/\partial \mathbf{u})$ are evaluated as follows

$$(\partial \mathbf{r}/\partial \mathbf{s}) = \begin{bmatrix} (\mathbf{s}_1 - \mathbf{u})^T/r_1 & -(\mathbf{s}_2 - \mathbf{u})^T/r_2 & \mathbf{0} & \cdots & \mathbf{0} \\ (\mathbf{s}_1 - \mathbf{u})^T/r_1 & \mathbf{0} & -(\mathbf{s}_3 - \mathbf{u})^T/r_3 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \mathbf{0} \\ (\mathbf{s}_1 - \mathbf{u})^T/r_1 & \mathbf{0} & \mathbf{0} & \cdots & -(\mathbf{s}_M - \mathbf{u})^T/r_M \end{bmatrix} \tag{8}$$

$$(\partial \mathbf{r}/\partial \mathbf{u}) = \begin{bmatrix} (\mathbf{s}_2 - \mathbf{u})^T/r_2 - (\mathbf{s}_1 - \mathbf{u})^T/r_1 \\ (\mathbf{s}_3 - \mathbf{u})^T/r_3 - (\mathbf{s}_1 - \mathbf{u})^T/r_1 \\ \vdots \\ (\mathbf{s}_M - \mathbf{u})^T/r_M - (\mathbf{s}_1 - \mathbf{u})^T/r_1 \end{bmatrix} \tag{9}$$

It is noteworthy that the first part of the right-hand side of Equation (7) is the CRLB of $\mathbf{u}$ when there is no base station position error. The trace of Equation (7) is the possible minimum square error that can be achieved by this estimator. We consider this CRLB as a benchmark to judge our proposed algorithm.

3. MDS Algorithm for TDOA Localization with Base Stations Uncertainty.

In the MDS algorithm for TDOA measurements, the relationship between the coordinates of BSs, the scalar product matrix $\mathbf{B}$, TDOA and the coordinate of the target is given by [9]

$$\mathbf{B}\mathbf{A}[1 \quad \mathbf{z}]^T = \mathbf{0}_M \tag{10}$$

where $\mathbf{z} = [\mathbf{u}^T \quad -d_1]$. The above equation is a linear equation w.r.t the unknown parameter, as we mentioned before the scalar product matrix $\mathbf{B}$ is $M \times M$ with elements given by

$$\mathbf{B} = 0.5\left[(d_{m1} - d_{n1})^2 - (s_m^o - s_n^o)^T (s_m^o - s_n^o)\right] \tag{11}$$

$$\mathbf{A} = \mathbf{P}^T \left(\mathbf{P}\mathbf{P}^T\right)^{-1} \tag{12}$$

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \mathbf{s}_1^o & \mathbf{s}_2^o & \cdots & \mathbf{s}_M^o \\ d_{11} & d_{21} & \cdots & d_{M1} \end{bmatrix} \tag{13}$$

By considering the noisy distance measurements instead of the actual one, Equation (10) is rewritten as

$$\boldsymbol{\varepsilon} = \hat{\mathbf{B}}\hat{\mathbf{A}}[1 \quad \mathbf{z}]^T \tag{14}$$

where $\hat{\mathbf{B}}$ and $\hat{\mathbf{A}}$ are erroneousness of $\mathbf{B}$ and $\mathbf{A}$ respectively. $\boldsymbol{\varepsilon}$ is the $M \times 1$ residual vector. If we reorganize Equation (14), we have the vector equation w.r.t the unknown vector $\mathbf{z}$ as

$$\boldsymbol{\varepsilon} = \hat{\mathbf{B}}\hat{\mathbf{A}}_2\mathbf{z} + \hat{\mathbf{B}}\hat{\mathbf{A}}_1 \tag{15}$$

where $\boldsymbol{\varepsilon}$ is the residual vector, $\hat{\mathbf{B}}$ is the noisy version of $\mathbf{B}$, $\hat{\mathbf{A}}_1$ is the first column of noisy $\mathbf{A}$, while the $\hat{\mathbf{A}}_2$ represents the rest of columns of the matrix $\hat{\mathbf{A}}$. Note that the matrix $\hat{\mathbf{A}}$ comprises only the BS positions error terms, while the matrix $\hat{\mathbf{B}}$ contains the TDOA measurements error and BS positions error terms, the erroneous matrix $\hat{\mathbf{B}}$ can be given as

$$\hat{\mathbf{B}} = \mathbf{B} + \Delta\mathbf{B} \tag{16}$$

The entries of $\Delta\mathbf{B}$ can be expressed as

$$[\Delta\mathbf{B}]_{mn} = (d_{m1} - d_{n1})(n_{m1} - n_{n1}) - (s_m^o - s_n^o)^T(\Delta s_m - \Delta s_n) \tag{17}$$

The first term of Equation (17) includes the TDOA measurements error while the second term contains the erroneous base station positions. The noisy $\hat{\mathbf{A}}$ and $\hat{\mathbf{P}}$ are expressed as

$$\hat{\mathbf{A}} = \mathbf{A} + \Delta\mathbf{A} \tag{18}$$

$$\hat{\mathbf{P}} = \mathbf{P} + \Delta\mathbf{P} \tag{19}$$

where

$$\Delta\mathbf{P} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \Delta s_1 & \Delta s_2 & \cdots & \Delta s_M \\ 0 & n_{21} & \cdots & n_{M1} \end{bmatrix} \tag{20}$$

Matrix $\mathbf{A}$ is a full rank matrix because $\mathbf{A} = \mathbf{P}^{-1}$ when the number of base stations is equal to five. Using this assumption we can extract the error matrix $\Delta\mathbf{A}$, and by exploiting Neumann series, the error matrix can be written as the following

$$\Delta\mathbf{A} = -\mathbf{A}^{-1}\Delta\mathbf{P}\mathbf{A}^{-1} \tag{21}$$

In general, the error term can be expressed when the matrix $\mathbf{A} = \mathbf{P}^{\dagger}$,

$$\Delta\mathbf{A} = -\mathbf{A}^{-1}\Delta\mathbf{P}\mathbf{A}^{-1} + (\mathbf{I} - \mathbf{A}\mathbf{P})\,\Delta\mathbf{P}^T\left(\mathbf{P}\mathbf{P}^T\right)^{-1} \tag{22}$$

To achieve CRLB performance, BS positions error should be considered, and by substituting Equation (22) into (15) we have

$$\boldsymbol{\varepsilon} = \left(\Delta\mathbf{B}\mathbf{A} - \mathbf{B}\mathbf{A}^{-1}\Delta\mathbf{P}\mathbf{A}^{-1} + (\mathbf{I} - \mathbf{A}\mathbf{P})\Delta\mathbf{P}^T\left(\mathbf{P}\mathbf{P}^T\right)^{-1}\right)\begin{bmatrix} 1 \\ \mathbf{z} \end{bmatrix} \tag{23}$$

Equation (23) can be written as

$$\boldsymbol{\varepsilon} = \mathbf{G}\mathbf{n} + \mathbf{G}_{\mathbf{s}}\Delta\mathbf{S} \tag{24}$$

The preceding expression in the residuals error vector with TDOA measurements $\mathbf{n}$ and the BS position errors $\Delta\mathbf{S}$, $\mathbf{G}$ and $\mathbf{G}_{\mathbf{s}}$ is defined as

$$\mathbf{G} = \mathbf{T}_1 - \mathbf{B}\mathbf{A}\mathbf{T}_2 + \mathbf{F} \tag{25}$$

and

$$\mathbf{G}_{\mathbf{s}} = \mathbf{H}_1 - \mathbf{B}\mathbf{A}\mathbf{H}_2 + \mathbf{F}_{\mathbf{s}} \tag{26}$$

where $\mathbf{T}_1$, $\mathbf{T}_2$, $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}$ and $\mathbf{F}_{\mathbf{s}}$ are defined in Appendix (A).

Equation (24) represents the residuals vector which has a linear relationship with TDOA measurements error as well as the BS positions error. If we neglect the BS position location errors, Equation (24) becomes as $\boldsymbol{\varepsilon} = \mathbf{G}\mathbf{n}$ which is the same as the expression in [9], and the solution is easy and can be obtained by least square (LS) and weighted least square (WLS). The LS provides an optimum performance only when the noise components in the linear equation are independent and identically distributed (i.i.d). The weighted least square is a straightforward and optimal solution; in addition, it overcomes the LS solution when the noise components are not i.i.d. Here we consider the WLS solution for our proposed problem when the information of TDOA measurements and sensor position errors are known.

Noting that the elements in the residual vector are correlated, the weighted least squares solution is proposed to enhance the correlation between the elements in $\boldsymbol{\varepsilon}$, and the solution is given by

$$\hat{\mathbf{z}} = -\left(\mathbf{H}^T\mathbf{W}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{W}\mathbf{h} \tag{27}$$

where $\mathbf{W}$ is the symmetric weighting matrix. $\mathbf{W}$ is equal to the inverse of the covariance of the residual vector $\boldsymbol{\varepsilon}$ and given by

$$\mathbf{W} = \left[E\left\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\right\}\right]^{-1} = \left(\mathbf{G}\mathbf{Q}_n\mathbf{G}^T + \mathbf{G}_{\mathbf{s}}\mathbf{Q}_{\beta}\mathbf{G}_{\mathbf{s}}^T\right)^{-1} \tag{28}$$

Applying the matrix inversion lemma, Equation (28) is given by

$$\mathbf{W} = \left(\mathbf{G}\mathbf{Q}_n\mathbf{G}^T\right)^{-1} - \left(\mathbf{G}\mathbf{Q}_n\mathbf{G}^T\right)^{-1}\mathbf{G}_{\mathbf{s}}^T\left(\mathbf{Q}_{\mathbf{s}}^T + \mathbf{G}_{\mathbf{s}}^T\left(\mathbf{G}_{\mathbf{s}}\mathbf{Q}_{\beta}\mathbf{G}_{\mathbf{s}}^T\right)\mathbf{G}_{\mathbf{s}}\right)\mathbf{G}_{\mathbf{s}}^T\left(\mathbf{G}\mathbf{Q}_n\mathbf{G}^T\right)^{-1} \tag{29}$$

In the end, the first three entries of $\hat{\mathbf{z}}$ in Equation (27) represent the estimation of the target position.

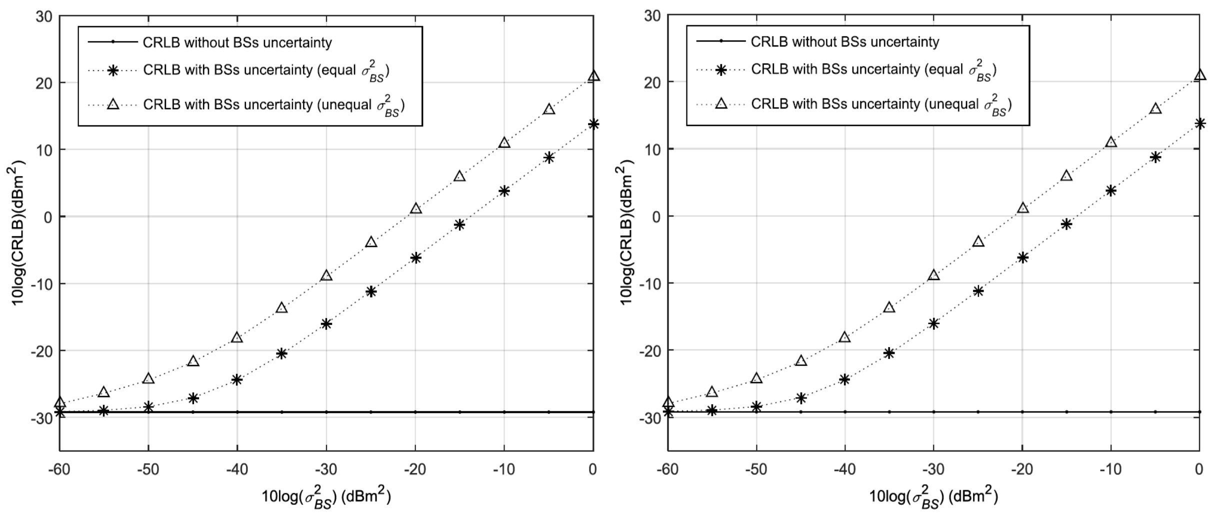
The summary of the proposed algorithm using the weighting matrix is given as the following:

1) obtain $\mathbf{H}$ and $\mathbf{h}$ using the noisy range difference $\mathbf{r}_{1m}$, $m = 1, 2, \ldots, M$,
2) find the initial estimate of $\hat{\mathbf{z}}$ using (27), using $\mathbf{W} = \mathbf{I}_{M+1}$,
3) update the following steps one or two times,
   a) use $\hat{\mathbf{z}}$ to obtain $\mathbf{G}$,
   b) update $\mathbf{H}$ and $\mathbf{h}$,
   c) obtain $\mathbf{W}$ using (28),
4) find $\hat{\mathbf{z}}$.

4. **Simulation and Results.** In the simulation, the proposed algorithm is compared with 2 stage weighted least squares [8,10], the constrained total least squares (CTLS) [11], and the CRLB, and the three-dimensional case will be considered in these experiments. The BSs are located at $(300, 100, 150)$, $(400, 150, 100)$, $(300, 500, 200)$, $(350, 200, 100)$, $(-100, -100, 100)$ and $(200, -300, -200)$. The estimation accuracies regarding the position estimation bias and minimum squares error are defined as $\sqrt{(E\left[\hat{\mathbf{u}}\right] - \mathbf{u})^T (E[\hat{\mathbf{u}}] - \mathbf{u})}$ and $E\left[(\hat{\mathbf{u}} - \mathbf{u})^T (\hat{\mathbf{u}} - \mathbf{u})\right]$ respectively, and they are applying to evaluating the performance of our proposed algorithm. All results are figured out by averaged 5000 times.

The TDOA measurements noise power σ_n^2 is set to 10^{-4}, the TDOA measurements are generated by adding white Gaussian noise with zero mean and covariance is equal to $\sigma_n^2 \Theta$ where Θ is diagonal matrix having a size $(M-1) \times (M-1)$ that it is diagonal elements are equal to one and the rest of elements are equal to 0.5. The notation σ_{BS}^2 is the BS position error power. Therefore, the noisy BS positions error is created in much the same way using covariance matrix $\mathbf{Q}_\beta = \sigma_{BS}^2 diag[1, 1, 1, 2, 2, 2, 10, 10, 10, 40, 40, 40, 20, 20, 20, 3, 3, 3]$ which indicates the different amount of noise power on the base stations.

Figure 1 shows the CRLB as a function of base station position error. The simulation demonstrates the comparison between CRLB with and without base stations position uncertainty. We assume the near-field target fixed at $(280, 320, 275)$ (Figure 1(a)) while the far-field target located at $(2000, 2500, 3000)$ (Figure 1(b)). As the noise power increases, the CRLB with the presence of base station positions uncertainties diverges further from

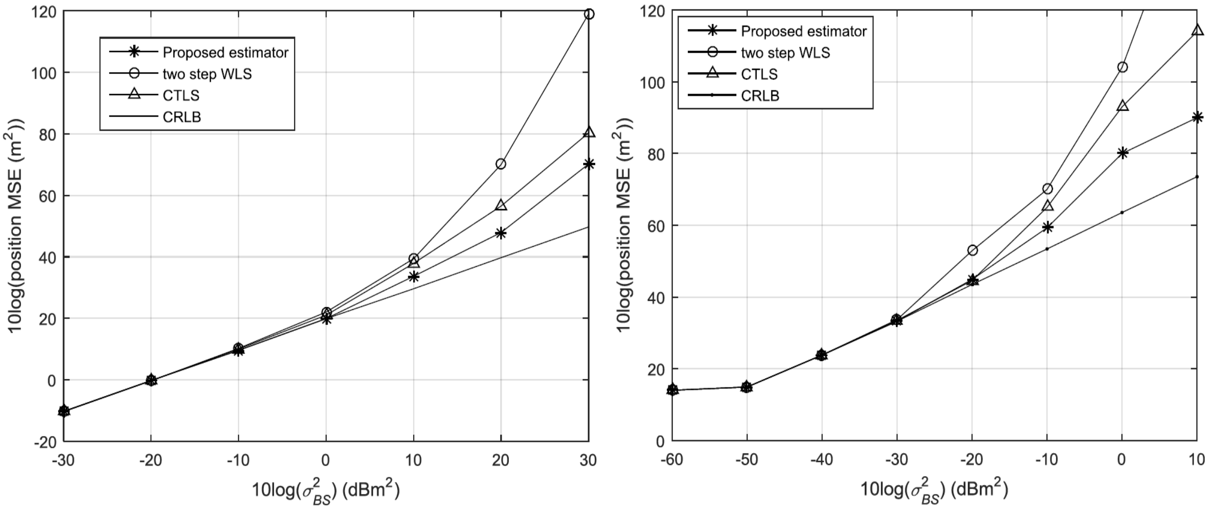


(a) (b)

FIGURE 1. Comparison of the CRLB with and without base station positions uncertainty for (a) near-field target and (b) far-field target

the variance accuracy without base station positions uncertainty. In these two figures, we show the CRLB for the case when the noise power is equal on each base station, and the case when the noise power is varying in the base stations. The figures show us a significant result for the CRLB with the base station position uncertainty when the noise power is varying among all base stations.

Figure 2 is concerned about the position MSE as a function of σ_{BS}^2. The MSE of the proposed estimator for near-field target and far-field target is compared with the two-step WLS, CTLS, and CRLB. The proposed solution reaches the CRLB level when the $\sigma_{BS}^2 < 0$dB and $\sigma_{BS}^2 < -20$dB for near-field Figure 2(a) and far-field target Figure 2(b), respectively. When $\sigma_{BS}^2 > 0$dB in Figure 2(a) and $\sigma_{BS}^2 > -20$dB in Figure 2(b), the proposed solution diverges from the CRLB level but it overcomes the 2WLS and CTLS, on the other hand, the proposed algorithm overcomes the other two methods in moderate and high BS noise power. As the noise power of base stations increases, the threshold affects the estimator; the effect is considered as a result of the nonlinear nature of the positioning problem.

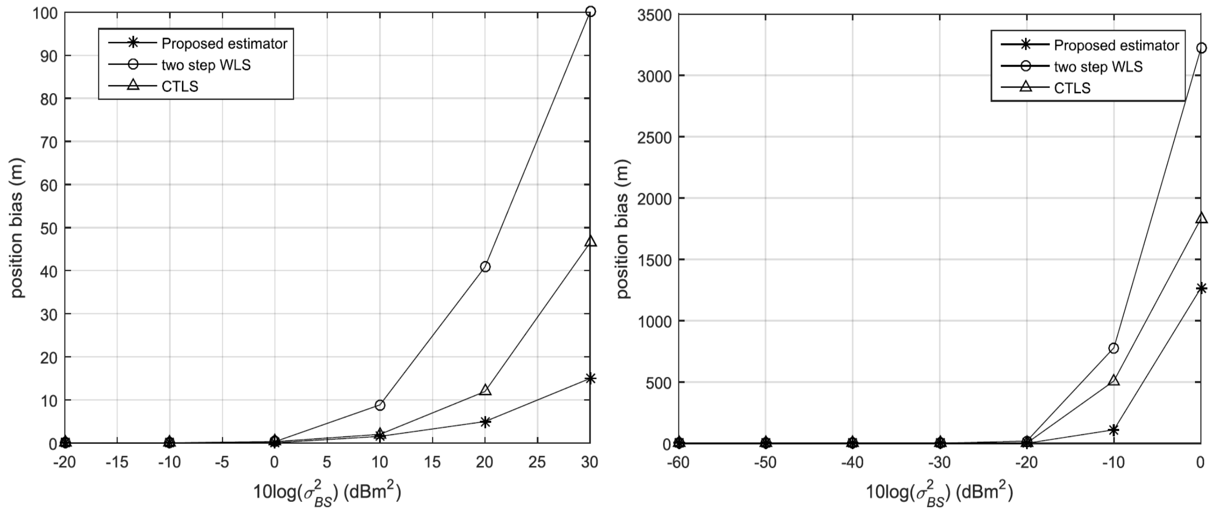


(a) (b)

FIGURE 2. Comparison of the MSE among the proposed estimator, 2WLS, CTLS and the CRLB with the presence of base station positions uncertainty versus σ_{BS}^2 for (a) near-field target (b) far-field target

Figure 3 concerns about the bias as a function of σ_{BS}^2. The bias of the proposed estimator is compared with the bias of two-step WLS and CTLS. The estimation bias is grown due to the nonlinear nature of positioning problem as we mentioned before. The figures clearly showed that the bias of the proposed algorithm performs better than 2WLS and CTLS in moderate and high base stations noise power. Figure 3(a) demonstrates the bias of the proposed method, 2WLS, and CTLS for near-field target, all algorithms almost have the same bias when $\sigma_{BS}^2 < 0$dB, while the proposed method overcomes the 2WLS and CTLS when σ_{BS}^2 exceeds 0dB. In far-field target scenario Figure 3(b), it can be seen that when $\sigma_{BS}^2 < -20$dB, the proposed method, as well as the other two algorithms, have almost equal performance, but in the case for $\sigma_{BS}^2 > -20$dB, the bias is dramatically increased for other two methods while the proposed method has a relatively small bias.

5. **Conclusion.** In this monograph, the novel weighted multidimensional scaling (MDS) was introduced with the presence of base station position uncertainties. The estimator is accurate. The proposed method achieves the CRLB at low noise power and small BSs error. In addition, the results show that the proposed method overcomes the two-step



(a) (b)

Figure 3. Comparison of the position bias for proposed estimator with 2WLS and CTLS versus σ^2_{BS} for the (a) near-field and (b) far-field target

weighted least squares and constraint total least square algorithms. The algorithm will be further extended to localize the moving target.

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Appendix (A). From Equations (25) and (26), the $\mathbf{T}_1$, $\mathbf{T}_2$, $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{F}$ and $\mathbf{F_s}$ are defined as

$$\mathbf{T}_1 = \begin{bmatrix} -\gamma_{12} & -\gamma_{13} & \cdots & -\gamma_{1M} \\ \sum_{m=1}^{M} \gamma_{2m} & -\gamma_{23} & \cdots & -\gamma_{2M} \\ -\gamma_{32} & \sum_{m=1}^{M} \gamma_{3m} & \cdots & -\gamma_{3M} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_{M2} & -\gamma_{M3} & \cdots & \sum_{m=1}^{M} \gamma_{Mm} \end{bmatrix} \tag{30}$$

where $\gamma_{mn} = a_n(d_{m1} - d_{n1})$, $m = 1, 2, \ldots, M$, $n = 2, 3, \ldots, M$ and from Equation (23), the terms $a_2, a_3, \ldots, a_M$ are defined as

$$\mathbf{a} = \mathbf{A}[1 \quad \mathbf{z}]^T = [a_1, a_2, \ldots, a_M]^T \tag{31}$$

$$\mathbf{T}_2 = \begin{bmatrix} \mathbf{0}_4 & \mathbf{0}_4 & \ldots & \mathbf{0}_4 \\ a_2 & a_3 & \ldots & a_M \end{bmatrix} \tag{32}$$

$$\mathbf{F} = [\mathbf{B}(\mathbf{I} - \mathbf{AP})]_{2:end} \cdot e_5 \tag{33}$$

e_5 represents the last row in the following equation

$$\mathbf{e} = \left(\mathbf{PP}^T\right)^{-1} [1 \quad \hat{\mathbf{z}}]^T = [e_1, e_2, e_3, e_4, e_5]^T \tag{34}$$

$$\mathbf{H}_1 = \begin{bmatrix} \sum_{m=1}^{M} \kappa_{1m} & \kappa_{12} & \cdots & \kappa_{1M} \\ \kappa_{21} & \sum_{m=1}^{M} \kappa_{2m} & \cdots & \kappa_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{M1} & \kappa_{M2} & \cdots & \sum_{m=1}^{M} \kappa_{Mm} \end{bmatrix} \tag{35}$$

where $\kappa_{mn} = a_n(\mathbf{s}_m^o - \mathbf{s}_n^o)^T$ and $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, M$.

$$\mathbf{H}_2 = \begin{bmatrix} & \mathbf{0}_{3M\times 1}^T & \\ a_1\mathbf{I}_3 & a_1\mathbf{I}_3 \ldots a_1\mathbf{I}_3 & \\ & \mathbf{0}_{3M\times 1}^T & \end{bmatrix} \tag{36}$$

$$\mathbf{F_s} = \mathbf{B}\left(\mathbf{I} - \mathbf{AP}\right) diag\left(\mathbf{e}_{2:4}^T, \mathbf{e}_{2:4}^T, \ldots, \mathbf{e}_{2:4}^T\right) \tag{37}$$