*ICIC Express Letters* ICIC International ©2018 ISSN 1881-803X
Volume **12**, Number **1**, January **2018** pp. **47–54**

# A DESIGN METHOD FOR STABILIZING MODIFIED SMITH PREDICTOR FOR NON-MINIMUM-PHASE UNSTABLE TIME-DELAY PLANTS

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Received July 2017; accepted October 2017

Abstract. *The modified Smith predictor is well known as an effective time-delay compensator for a plant with large time delays, and several papers on the modified Smith predictor have been published. The parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants is obtained by Yamada and Matsushima. However, they do not examine the parameterization of all stabilizing modified Smith predictors for non-minimum-phase time-delay plants. The purpose of this paper is to expand the result by Yamada and Matsushima and to propose the parameterization of all stabilizing modified Smith predictors for non-minimum-phase time-delay plants. Finally, control characteristics of the control system using obtained parameterization of all stabilizing modified Smith predictors are also given.*
**Keywords:** Non-minimum-phase system, Time-delay system, Smith predictor, Parameterization

1. **Introduction.** In this paper, we examine a design method for stabilizing modified Smith predictors for non-minimum-phase time-delay stable/unstable plants. Smith predictor is well known as an effective time-delay compensator for a stable plant with large time delays [1-13]. The Smith predictor in [1] cannot be used for time-delay plants having an integral mode, because a step disturbance will result in a steady state error [2-4]. To overcome this problem, Watanabe and Ito [4], Astrom et al. [9], and Matusek and Micic [10] proposed a design method for a modified Smith predictor for time-delay plants with an integrator. Watanabe and Sato expanded the result in [4] and proposed a design method for modified Smith predictors for multivariable systems with multiple delays in inputs and outputs [5].

Because the modified Smith predictor cannot be used for unstable time-delay plants [2-11], Paor [6], Paor and Egan [8] and Kwak et al. [12] proposed a design method for modified Smith predictors for unstable time-delay plants. Thus, several design methods of modified Smith predictors have been published.

On the other hand, another important control problem is the parameterization problem, the problem of finding all stabilizing controllers for a plant [14-25]. The parameterization of all stabilizing controllers for time-delay plants was considered in [22-25], but that of all stabilizing modified Smith predictors was not obtained. Yamada and Matsushima gave the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants [26]. Since the parameterization of all stabilizing modified Smith predictors was obtained, we could express previous studies of modified Smith predictors in a uniform manner and could design the modified Smith predictors systematically. Uren and Schoor illustrated the context and categories of predictive PID control strategies applied to non-minimum phase systems in [27]. In addition, Smith predictor structure for non-minimum-phase has been proposed simply in [27]. However, the parameterization of all stabilizing Smith predictor for non-minimum-phase time-delay stable/unstable plants has not been obtained in [26,27]. Many time-delay plants are of non-minimum-phase. In addition, the parameterization is a powerful tool to design controllers. The problem to obtain the parameterization of all modified Smith predictors for non-minimum-phase time-delay plants is important to solve.

The purpose of this paper is to expand the result in [26] and to propose the parameterization of all stabilizing modified Smith predictors for non-minimum-phase time-delay plants. First, the structure and necessary characteristics of modified Smith predictors described in past studies in [1-13] are defined. Next, the parameterization of all stabilizing modified Smith predictors for non-minimum-phase time-delay plants is proposed for both stable and unstable time-delay plants. Control characteristics of the control systems using this parameterization are also given. This paper is organized as follows. In Section 2, the modified Smith predictor is introduced briefly and the problem considered in this paper is explained. In Section 3 and Section 4, the parameterizations of all stabilizing modified Smith predictors for stable and unstable time-delay plants are given, respectively. The parameterization of all stabilizing modified Smith predictors in Section 3 and in Section 4 is explained based on the frequency domain. A simple numerical example is illustrated in Section 5. Finally, Section 6 concludes the paper.

Notation

| | |
|---|---|
| $R$ | The set of real numbers. |
| $C$ | The set of complex numbers. |
| $R(s)$ | The set of real rational functions with $s$. |
| $RH_\infty$ | The set of stable proper real rational functions. |

## 2. Modified Smith Predictor.

Consider the control system:

$$\begin{cases} y(s) = G(s)e^{-sT}u(s) + d(s) \\ u(s) = C(s)\left(r(s) - y(s)\right) \end{cases}, \tag{1}$$

where $G(s)e^{-sT}$ is the single-input/single-output time-delay plant with time-delay $T > 0$, $G(s) \in R(s)$, $C(s)$ is the controller, $y(s) \in R(s)$ is the output, $u(s) \in R(s)$ is the control input, $d(s) \in R(s)$ is the disturbance and $r(s) \in R(s)$ is the reference input. $G(s)$ is assumed to be coprime, that is, $G(s)$ has no polez-zeros cancellation, and of non-minimum-phase, that is, $G(s)$ has zeros in the closed right half plane.

According to [1-13], the modified Smith predictor $C(s)$ is defined by the form:

$$C(s) = \frac{C_1(s)}{1 + C_2(s)e^{-sT}}, \tag{2}$$

where $C_1(s) \in R(s)$ and $C_2(s) \in R(s)$. In addition, using the modified Smith predictor in [1-13], the transfer function from $r(s)$ to $y(s)$ of the control system in (1), written as

$$y(s) = \frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}}r(s) \tag{3}$$

has a finite number of poles. We call $C(s)$ the modified Smith predictor if $C(s)$ takes the form of (2) and the transfer function from $r(s)$ to $y(s)$ of the control system in (1) has a finite number of poles.

## 3. The Parameterization of all Stabilizing Modified Smith Predictors for Stable Time-Delay Plants.

In this section, we examine the parameterization of all stabilizing modified Smith predictors for stable time-delay plants. The parameterization is summarized in the following theorem.

**Theorem 3.1.** *$G(s)e^{-sT}$ is assumed to be stable.*

*The parameterization of all stabilizing modified Smith predictors $C(s)$ takes the form:*

$$C(s) = \frac{Q(s)}{1 - Q(s)G(s)e^{-sT}}, \tag{4}$$

*where $Q(s) \in RH_\infty$ is any function.*

**Proof:** First, the necessity is shown. From the assumption that the controller $C(s)$ in (2) makes the transfer function from $r(s)$ to $y(s)$ of the control system in (1) has a finite number of poles,

$$\frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = \frac{C_1(s)G(s)e^{-sT}}{1 + (C_2(s) + C_1(s)G(s))\, e^{-sT}} \tag{5}$$

has a finite number of poles. This implies that

$$C_2(s) = -C_1(s)G(s) \tag{6}$$

is necessary, that is,

$$C(s) = \frac{C_1(s)}{1 - C_1(s)G(s)e^{-sT}}. \tag{7}$$

From the assumption, we have

$$\frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = C_1(s)G(s)e^{-sT}, \tag{8}$$

$$\frac{C(s)}{1 + C(s)G(s)e^{-sT}} = C_1(s), \tag{9}$$

$$\frac{G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = \left(1 - C_1(s)G(s)e^{-sT}\right) G(s)e^{-sT} \tag{10}$$

and

$$\frac{1}{1 + C(s)G(s)e^{-sT}} = 1 - C_1(s)G(s)e^{-sT}. \tag{11}$$

It is obvious that the necessary condition for all the transfer functions in (8), (9), (10) and (11) to be stable is $C_1(s) \in RH_\infty$. Using $Q(s) \in RH_\infty$, letting $C_1(s)$ be

$$C_1(s) = Q(s), \tag{12}$$

we find that $C(s)$ takes the form of (4). Thus, the necessity has been shown. Next, the sufficiency is shown. That is, if $C(s)$ takes the form of (4) and $Q(s) \in RH_\infty$, from simple manipulation, we have

$$\frac{C(s)G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = Q(s)G(s)e^{-sT}, \tag{13}$$

$$\frac{C(s)}{1 + C(s)G(s)e^{-sT}} = Q(s), \tag{14}$$

$$\frac{G(s)e^{-sT}}{1 + C(s)G(s)e^{-sT}} = \left(1 - Q(s)G(s)e^{-sT}\right) G(s)e^{-sT} \tag{15}$$

and

$$\frac{1}{1 + C(s)G(s)e^{-sT}} = 1 - Q(s)G(s)e^{-sT}. \tag{16}$$

From the assumption that $G(s)e^{-sT}$ is stable and $Q(s) \in RH_\infty$, (13), (14), (15) and (16) are all stable. In addition, because the transfer function from $r(s)$ to $y(s)$ of the control system in (1) is written by (13) and $Q(s) \in RH_\infty$, the transfer function from $r(s)$ to $y(s)$ of the control system in (1) has a finite number of poles. Thus, the sufficiency has been shown. We have thus proved Theorem 3.1.

Next, we explain control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors $C(s)$ in (4). The transfer function from the reference input $r(s)$ to the output $y(s)$ of the control system in (1) takes the form

$$y(s) = Q(s)G(s)e^{-sT}r(s). \tag{17}$$

Therefore, for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady state error,

$$Q(0)G(0) = 1 \tag{18}$$

must be satisfied. In order for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady state error, $Q(s)$ is settled by

$$Q(s) = \frac{q(s)}{G_o(s)}, \tag{19}$$

where

$$q(s) = \frac{1}{(1+s\tau)^\alpha}, \tag{20}$$

$\tau > 0$, $\alpha$ is a positive integer that makes $Q(s)$ in (19) proper and $G_o(s) \in RH_\infty$ is an outer function of $G(s)$, that is, $G(s)$ is factorized as

$$G(s) = G_i(s)G_o(s), \tag{21}$$

$G_i(s) \in RH_\infty$ is an inner function satisfying $G_i(0) = 1$ and $G_o(s) \in RH_\infty$ is an outer function. Because $G_o(s)$ is of minimum phase, $Q(s)$ in (19) is $Q(s) \in RH_\infty$.

The disturbance attenuation characteristic is as follows. Using the parameterization of all stabilizing Smith predictors $C(s)$ in (4), the transfer function from the disturbance $d(s)$ to the output $y(s)$ of the control system in (1) is given by

$$y(s) = \left(1 - Q(s)G(s)e^{-sT}\right) d(s). \tag{22}$$

Therefore, in order to attenuate the disturbance $d(s)$ effectively, $Q(s)$ must satisfy

$$Q(j\omega_{di})G(j\omega_{di}) = e^{j\omega_{di}T} \quad (i = 1, \ldots, n_d), \tag{23}$$

where $\omega_{di} \in C$ $(i = 1, \ldots, n_d)$ are frequency components of the disturbance $d(s)$, that is, poles of $d(s)$ are $j\omega_{di}$ $(i = 1, \ldots, n_d)$. When $Q(s)$ is chosen as (19) and $q(s)$ in (19) is settled satisfying

$$q(j\omega_{di}) = \frac{e^{j\omega_{di}T}}{G_i(j\omega_{di})} \quad (i = 1, \ldots, n_d), \tag{24}$$

the control system in (1) can attenuate the disturbance $d(s)$ effectively.

## 4. The Parameterization of all Stabilizing Modified Smith Predictors for Unstable Time-Delay Plants.

In this section, we examine the parameterization of all stabilizing modifed Smith predictors for stable time-delay plants.

The parameterization is summarized in the following theorem.

**Theorem 4.1.** *$G(s)e^{-sT}$ is assumed to be unstable and to be of non-minimum-phase. For simplicity, the unstable poles of $G(s)e^{-sT}$ are assumed to be distinct. That is, when $s_i$*

*$(i = 1, \ldots, n)$ denote unstable poles of $G(s)$, $s_i \neq s_j$ $(i \neq j;\ i = 1, \ldots, n;\ j = 1, \ldots, n)$. Under these assumptions, there exists $\bar{G}_u(s) \in RH_\infty$ satisfying*

$$\bar{G}_u(s_i) = \frac{1}{G_s(s_i)\, e^{-s_i T}} \quad (i = 1, \ldots, n), \tag{25}$$

*where $G_s(s)$ is a stable non-minimum-phase function of $G(s)$, that is, when $G(s)$ is factorized as*

$$G(s) = G_u(s) G_s(s), \tag{26}$$

*$G_u(s)$ is an unstable biproper minimum-phase function and $G_s(s)$ is a stable non-minimum-phase function. Using these functions, the parameterization of all stabilizing modified Smith predictors $C(s)$ is written as*

$$C(s) = \frac{C_f(s)}{1 - C_f(s) G(s) e^{-sT}}, \tag{27}$$

*where $C_f(s)$ is given by*

$$C_f(s) = \frac{1}{G_u(s)} \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) \tag{28}$$

*and $Q(s) \in RH_\infty$ is any function. (Proof is omitted on account of space limitation).*

Next, we explain control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors $C(s)$ in (27). The transfer function from the reference input $r(s)$ to the output $y(s)$ of the control system in (1) is written as

$$y(s) = \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) G_s(s) e^{-sT} r(s). \tag{29}$$

Therefore, when $G(s)$ has a pole at the origin, for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady state error,

$$\bar{G}_u(0) G_s(0) = 1 \tag{30}$$

must be satisfied. Because $\bar{G}_u(s) \in RH_\infty$ satisfies (25), (30) holds true. This implies that when $G(s)$ has a pole at the origin, the output $y(s)$ follows the step reference input $r(s) = 1/s$ without steady state error, independent of $Q(s) \in RH_\infty$ in (27). On the other hand, when $G(s)$ has no pole at the origin, for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ without steady state error,

$$\left( \bar{G}_u(0) + \frac{Q(0)}{G_u(0)} \right) G_s(0) = 1 \tag{31}$$

must hold. From simple manipulation, if $Q(s)$ is chosen satisfying

$$Q(0) = G_u(0) \left( \frac{1}{G_s(0)} - \bar{G}_u(0) \right), \tag{32}$$

then the output $y(s)$ follows the step reference input $r(s) = 1/s$ without steady state error.

The disturbance attenuation characteristic is as follows: Using the parameterization of all stabilizing Smith predictor $C(s)$ in (27), the transfer function from the disturbance $d(s)$ to the output $y(s)$ of the control system in (1) is given by

$$y(s) = \left\{ 1 - \left( \bar{G}_u(s) + \frac{Q(s)}{G_u(s)} \right) G_s(s) e^{-sT} \right\} d(s). \tag{33}$$

Therefore, in order to attenuate the disturbance $d(s)$ effectively, $Q(s)$ is settled satisfying

$$\left( \bar{G}_u(j\omega_{di}) + \frac{Q(j\omega_{di})}{G_u(j\omega_{di})} \right) G_s(j\omega_{di}) e^{-j\omega_{di} T} = 1 \quad (i = 1, \ldots, n_d), \tag{34}$$

where $\omega_{di}$ $(i = 1, \dots, n_d)$ are frequency components of the disturbance $d(s)$, that is, poles of $d(s)$ are $j\omega_{di}$ $(i = 1, \dots, n_d)$. When $Q(s)$ is chosen satisfying (32), the step disturbance $d(s)$ is attenuated effectively.

5. **Numerical Example.** In this section, a numerical example for unstable non-minimum-phase time-delay plant is shown to illustrate the effectiveness of the proposed parameterization of all stabilizing modified Smith predictors.

Consider the problem finding the parameterization of all stabilizing modified Smith predictors for the unstable non-minimum-phase time-delay plant $G(s)e^{-sT}$ written as

$$G(s)e^{-sT} = \frac{s-20}{(s+2)(2s-1)}e^{-0.5s}, \tag{35}$$

where

$$G(s) = \frac{s-20}{(s+2)(2s-1)} \tag{36}$$

and $T = 0.5$ [s]. $G(s)$ is factorized by (26) as

$$G_u(s) = \frac{2s+1}{2s-1} \tag{37}$$

and

$$G_s(s) = \frac{s-20}{(s+2)(2s+1)}. \tag{38}$$

One of $\bar{G}_u(s)$ satisfying (25) is given by

$$\bar{G}_u(s) = -\frac{10.27(s+0.125)}{s+19}. \tag{39}$$

From Theorem 4.1, the parameterization of all stabilizing modified Smith predictors for unstable non-minimum-phase time-delay plant $G(s)e^{-sT}$ in (35) is given by

$$C(s) = \frac{C_f(s)}{1 - C_f(s)\dfrac{s-20}{(s+2)(2s-1)}e^{-0.5s}}, \tag{40}$$

where

$$C_f(s) = -\frac{2s-1}{2s+1}\left(\frac{10.27(s+0.125)}{s+19} - \frac{2s-1}{2s+1}Q(s)\right) \tag{41}$$

and $Q(s)$ is any function.

In order for the output $y(s)$ to follow the step reference input $r(s) = 1/s$ and in order to attenuate the step disturbance $d(s) = 1/s$ effectively, $Q(s) \in RH_\infty$ is settled by

$$Q(s) = \frac{0.2(s+0.3242)}{s+2}. \tag{42}$$

Substituting (42) for (41), we have

$$C_f(s) = -\frac{10.07(s+1.385)(s+0.7324)(s-0.5)(s+0.186)}{(s+19)(s+2)(s+0.5)^2}. \tag{43}$$

Using the obtained modified Smith predictor $C(s)$ in (40) with $C_f(s)$ in (43), the response of the output $y(t)$ for the step reference input $r(t) = 1$ is shown in Figure 1. Figure 1 shows that the control system in (1) is stable and the output $y(t)$ follows the step reference input $r(t) = 1$ without steady state error.

When the disturbance $d(t) = 1$ exists, the response of the output $y(t)$ is shown in Figure 2. Figure 2 shows that the control system in Figure 1 can attenuate the step disturbance effectively.

In this way, we can design stabilizing modified Smith predictors easily using obtained parameterization.

6. **Conclusions.** In this paper, we proposed the parameterization of all stabilizing modified Smith predictor for non-minimum-phase time-delay plants. First, the parameterization of all stabilizing modified Smith predictors for stable time-delay plants was proposed. Next, we expended the result of the parameterization for stable time-delay plants and proposed the parameterization of all stabilizing modified Smith predictors for unstable time-delay plants. Control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors were also given. In the future, we are going to consider multiple input/output instead of single input/output of all stabilizing

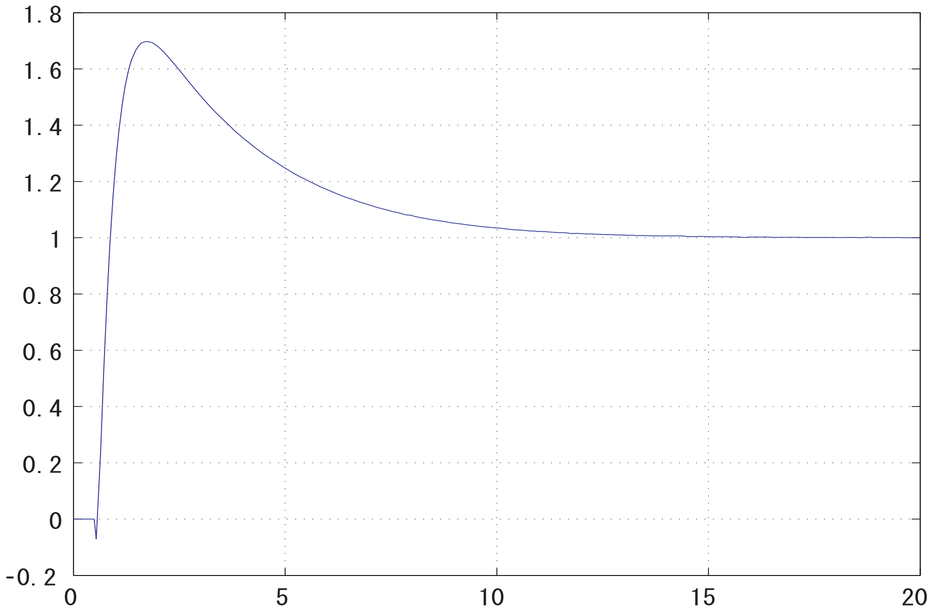


FIGURE 1. Response for step reference input $r(t) = 1$

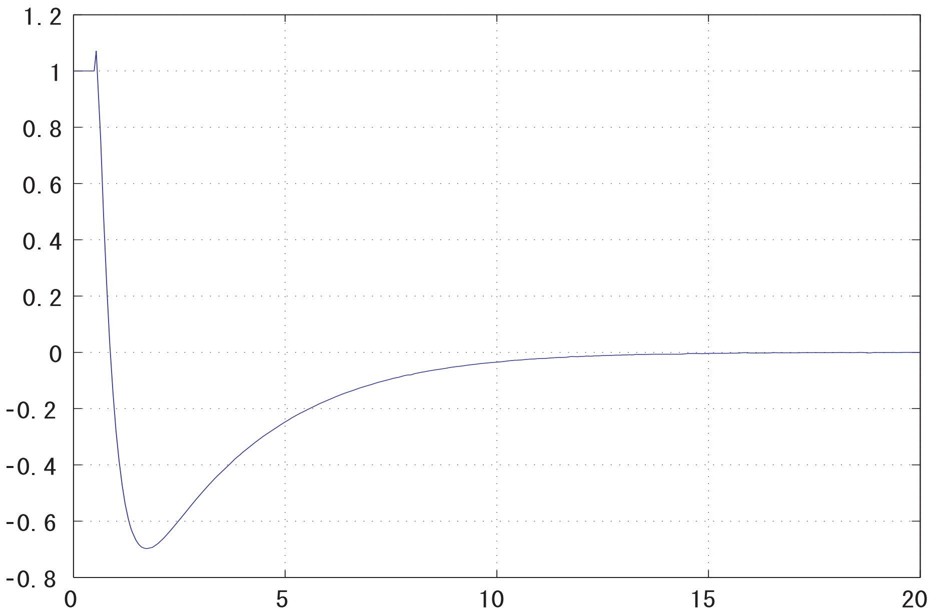


FIGURE 2. Response for disturbance $d(t) = 1$

modified Smith predictors for non-minimum-phase unstable time-delay plants and the parameterization will be given.

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