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AN APPROACH FOR DECISION MAKING BASED ON LINGUISTIC INTUITIONISTIC FUZZY LATTICE DISTANCE

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Abstract. *Aiming to the linguistic information for the qualitative assessments in decision making problems under uncertain environment, the linguistic intuitionistic fuzzy lattice (LIFL) distance is utilized. As to the information losing in the linguistic process, linguistic intuitionistic lattice 2-tuple (LIL2T) model is defined to get the valid linguistic intuitionistic fuzzy lattice distance (LIFLD). Based on the LIL2T, LIFL arithmetic-mean distance and LIFL weighted-mean distance are given. Then the decision making approach is proposed based on the LIFLD. The example shows the procedure of the proposed approach and illustrates the validity for the linguistic information under uncertain environment.*

Keywords: Linguistic intuitionistic fuzzy lattice, Linguistic intuitionistic lattice 2-tuple model, LIFLD, Decision making

1. **Introduction.** In general, decision makers (DMs) prefer to use natural language instead of numerical values to give qualitative assessments for attributes in uncertain environment. Many linguistic approaches have been developed from many directions, such as type-2 based methods, hesitant fuzzy linguistic term set theory, probabilistic linguistic term set theory, 2-tuple theory and linguistic truth-valued lattice implication algebras. In fact, they are most developments and extensions of Zadeh' fuzzy set theory and computing with words (CWW) [1].

- Type-2 extended the crisp membership function to a family of type-1 function. Type-2 is more effective in modeling uncertainty while it is difficult for all individuals to agree on the same membership function in type-1 [2-4].
- Hesitant fuzzy linguistic term set (HFLTS) was introduced by Rodriguez et al. to increase the richness of linguistic elicitation based on the fuzzy linguistic approach and the use of context-free grammars by using comparative terms [5,6].
- Probabilistic linguistic term set is also a linguistic term approach. It extends the HFLTS through adding probabilities without loss of any original linguistic information provided by the DMs. The DMs can not only provide several possible linguistic values over an object (alternative or attribute), but also reflect the probabilistic information of the set of values. It brings us comprehensive and accurate preference information of the DMs [7,8].

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- To avoid the limitation of the information loss in approximate linguistic process, Herrera and Martinez proposed a 2-tuple fuzzy linguistic representation model. It develops a computational technique for computing with words (CWW) without any loss of information [9].
- Xu et al. proposed lattice implication algebra and linguistic truth-value propositional logic system. In linguistic truth-value lattice implication algebra, the hedge operator's succession was corrected reasonably, which made the order of the linguistic values is more similar to the natural language [10,11].

Inspired by linguistic truth-valued lattice implication algebra and intuitionistic fuzzy set theory [12,13], Zou et al. proposed linguistic truth-value intuitionistic fuzzy lattice implication algebra (LTV-IFLIA) [14]. The approximate reasoning, resolution method and the application in decision making problems of LTV-IFLIA have been studied [15,16]. Some approximate reasoning algorithms under uncertain environment were proposed based on LTV-IFLIA, such as layered reasoning algorithm of the linguistic truth-valued intuitionistic fuzzy lattice [17], linguistic valued lattice implication algebra TOPSIS method based on entropy weight method [18], credibility factors reasoning based on linguistic truth-valued intuitionistic fuzzy hesitancy degree [19].

The distance measure is one of the most common measure tools in decision making problems. However, the distance measure process has information loss because of the limitations of modeling and computational processes in LTV-IFLIA. As above mentioned, 2-tuple model is a computational technique without any loss of information. It has been developed especially in decision making problems with linguistic information. The concept of an intuitionistic 2-tuple linguistic information (I2LI) model was developed to provide a linguistic and computational basis. It is efficient and feasible for real-world decision making applications for the I2LI will not cause any loss of information in the process [20]. Liu and Chen proposed a new method for multi-attribute group decision making (MAGDM) with the intuitionistic 2-tuple linguistic (I2L) information based on the proposed I2L generalized aggregation (I2LGA) operator by extending the Archimedean T-norm (TN) and T-conorm (TC) to be more general [21]. Other 2-tuple models were studied by scholars such as picture 2-tuple linguistic information [22], interval-valued 2-tuple linguistic model [23], 2-dimension linguistic computational model with 2-tuples [24].

We propose a decision making approach using the LIFLD in this paper. To avoid the information loss during obtaining the distance of the linguistic truth-value intuitionistic fuzzy pairs (LTV-IFPs), we will use the thought of 2-tuple model during the linguistic computational process.

The remainder of this paper is organized as follows. In Section 2, linguistic truth-valued intuitionistic fuzzy lattice and the concept of 2-tuple model are reviewed briefly. In Section 3, the linguistic intuitionistic fuzzy lattice distance based on the 2-tuple model on linguistic truth-valued intuitionistic fuzzy lattice is discussed. In Section 4, a decision making approach is introduced. In Section 5, an example is given to illustrate the effectiveness of our method. In Section 6, conclusions are summarized.

2. Preliminaries.

In this section, we briefly review linguistic truth-valued intuitionistic fuzzy lattice in linguistic truth-valued lattice implication algebra and the 2-tuple linguistic model, seeing [14] and [9] for more details.

We discuss the linguistic truth-valued intuitionistic fuzzy lattice firstly. Intuitively, we use linguistic truth-valued intuitionistic fuzzy set instead of classical linguistic truth of propositions to express degrees of "true" and "false" of uncertain propositions in practice [14].

In the linguistic truth-valued intuitionistic fuzzy lattice $\mathcal{LI}_{2n} = (LI_{2n}, \cap, \cup)$ (Figure 1), for any $((h_i, t), (h_j, f)), ((h_k, t), (h_l, f)) \in \mathcal{LI}_{2n}$, $((h_i, t), (h_j, f)) \leq ((h_k, t), (h_l, f))$ if and only if $i \leq k$ and $j \leq l$, also

1) $((h_i, t), (h_j, f))' = ((h_{n-j+1}, t), (h_{n-i+1}, f))$;
2) $((h_i, t), (h_j, f)) \to ((h_k, t), (h_l, f)) = ((h_{\min(n,n-i+k,n-j+l}, t), (h_{\min(n,n-i+l)}, f))$;
3) $((h_i, t), (h_j, f)) \cup ((h_k, t), (h_l, f)) = ((h_{\max(i,k)}, t), (h_{\max(j,l)}, f))$;
4) $((h_i, t), (h_j, f)) \cap ((h_k, t), (h_l, f)) = ((h_{\min(i,k)}, t), (h_{\min(j,l)}, f))$.

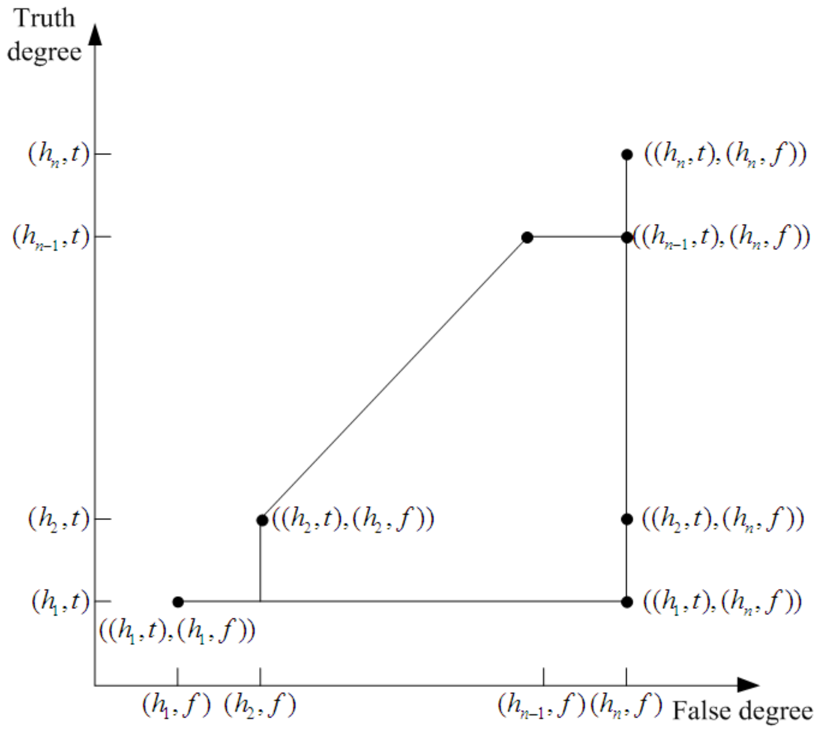

Figure 1. Hasse diagram of $\mathcal{LI}_{2n}$

In a 2-tuple linguistic information model, the linguistic information is represented by means of a 2-tuple (s_i, α), where s_i is a linguistic label from predefined linguistic term set S and α is the value of symbolic translation [9].

Definition 2.1. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $s_i \in S$ be a linguistic label. Then the function θ used to obtain the corresponding 2-tuple linguistic information of s_i is defined as*

$$\begin{aligned} &\theta : S \to S \times [-0.5, 0.5), \\ &\theta(s_i) = (s_i, 0), \ s_i \in S. \end{aligned} \tag{1}$$

Definition 2.2. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\begin{aligned} &\Delta : [0, g] \to S \times [-0.5, 0.5) \\ &\Delta(\beta) = (s_i, \alpha), \ \textit{with} \ \begin{cases} s_i, & i = round(\beta), \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases} \end{aligned} \tag{2}$$

where $round(\cdot)$ is the usual round operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.

Definition 2.3. *Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a function Δ^{-1} that can be defined, such that from a 2-tuple (s_i, α) it returns its equivalent numerical value $\beta \in [0, g]$, which is*

$$\begin{aligned} &\Delta^{-1} : S \times [-0.5, 0.5) \to [0, g], \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \end{aligned} \tag{3}$$

3. **Linguistic Intuitionistic Fuzzy Lattice Distance.** In this section, we discuss the distance measure between linguistic truth-valued intuitionistic fuzzy pairs (LT-VIFPs) for decision making under linguistic intuitionistic environment. 2-tuple linguistic terms model extended to linguistic truth-valued intuitionistic fuzzy lattice is used to deal with the information losing during aggregation procedure.

A 2-tuple model on linguistic truth-valued intuitionistic fuzzy lattice is represented as $(((h_i, t), \delta_1), ((h_j, f), \delta_2))$, where $((h_i, t), (h_j, f)) \in \mathcal{LI}_{2n}$ and $\delta_1, \delta_2 \in (0, 1)$ are linguistic information supplement, called linguistic intuitionistic lattice 2-tuple (LIL2T), denoted as $\mathcal{LIT}_{2n}$.

Definition 3.1. *Let $((h_i, t), (h_j, f)), ((h_k, t), (h_l, f)) \in \mathcal{LI}_{2n}$ be two linguistic truth-valued intuitionistic fuzzy pairs and $\alpha, \beta \in [1, n]$ be a pair of number, then the linguistic truth-valued intuitionistic fuzzy 2-tuple model that expresses the equivalent information to (α, β) is obtained with the following function:*

$$\begin{aligned} &\Delta_{\mathrm{LIF}} : [0, n] \times [0, n] \to (\mathcal{LI}_{2n} \times (0, 1)) \times (\mathcal{LI}_{2n} \times (0, 1)) \\ &\Delta_{\mathrm{LIF}}(\alpha, \beta) = \begin{cases} ((h_i, t), (h_j, f)), & \text{if } \alpha, \beta \in N, \\ (((h_i, t), \delta_1), ((h_j, f), \delta_2)), & \text{else} \end{cases} \end{aligned} \tag{4}$$

with $i = round(\alpha)$, $j = round(\beta)$, $\delta_1 = \alpha - i$, $\delta_2 = \beta - j$, where $round(\cdot)$ is the round down operation, i is the greatest integer smaller than α or equal to it, and j is the greatest integer smaller than β or equal to it.

There is always an inverse function $\Delta^{-1}_{\mathrm{LIF}}$, such that from a LIL2T it returns its equivalent numerical value pair.

$$\begin{aligned} &\Delta^{-1}_{\mathrm{LIL}} : (\mathcal{LI}_{2n} \times (0, 1)) \times (\mathcal{LI}_{2n} \times (0, 1)) \to [0, n] \times [0, n] \\ &\Delta^{-1}_{\mathrm{LIL}}(((h_i, t), \delta_1), ((h_j, f), \delta_2)) = (i + \delta_1, j + \delta_2) = (\alpha, \beta). \end{aligned} \tag{5}$$

The order relation between two LIL2T is as follows.

For any $(((h_i, t), \delta_{11}), ((h_j, f), \delta_{12})), (((h_k, t), \delta_{21}), ((h_l, f), \delta_{22})) \in \mathcal{LIT}_{2n}$, $\delta_1 = \delta_{11} + \delta_{12}$, $\delta_2 = \delta_{21} + \delta_{22}$:

1) if $((h_i, t), (h_j, f)) < ((h_k, t), (h_l, f))$, then $(((h_i, t), \delta_{11}), ((h_j, f), \delta_{12})) < (((h_k, t), \delta_{21}), ((h_l, f), \delta_{22}))$;
2) $((h_i, t), (h_j, f)) = ((h_k, t), (h_l, f))$ or $((h_i, t), (h_j, f)) \parallel ((h_k, t), (h_l, f))$, then
   a) if $\delta_1 < \delta_2$, then $(((h_i, t), \delta_{11}), ((h_j, f), \delta_{12})) < (((h_k, t), \delta_{21}), ((h_l, f), \delta_{22}))$;
   b) if $\delta_1 > \delta_2$, then $(((h_i, t), \delta_{11}), ((h_j, f), \delta_{12})) > (((h_k, t), \delta_{21}), ((h_l, f), \delta_{22}))$;
   c) if $\delta_1 = \delta_2$, then $(((h_i, t), \delta_{11}), ((h_j, f), \delta_{12})) = (((h_k, t), \delta_{21}), ((h_l, f), \delta_{22}))$.

where "$\parallel$" is a binary relation expressing "incomparable and in the same layer" on the Hass diagram of $\mathcal{LI}_{2n}$.

The distance between LT-VIFPs is defined as follows.

Definition 3.2. *Let X be a nonempty set, for any linguistic truth-valued intuitionistic fuzzy sets $\tilde{A} = \{a_1, a_2, \ldots, a_m \,|a_i \in \mathcal{LI}_{2n}\}$, $\tilde{B} = \{b_1, b_2, \ldots, b_m \,|b_i \in \mathcal{LI}_{2n}\}$, $\tilde{C} = \{c_1, c_2, \ldots, c_m \,|c_i \in \mathcal{LI}_{2n}\}$, where $a_i = \mu_{\tilde{A}}(x_i)$, $b_i = \mu_{\tilde{B}}(x_i)$, $c_i = \mu_{\tilde{C}}(x_i)$, $x_i \in X$, $i = \{1, 2, \ldots, m\}$, define the function as*

$$D_{LIF} : \mathcal{LI}_{2n} \times \mathcal{LI}_{2n} \to \mathcal{LIT}_{2n}. \tag{6}$$

$D_{LIF}(\tilde{A}, \tilde{B})$ is said to be the linguistic intuitionistic fuzzy lattice distance (LIFLD) between set $\tilde{A}$ and set $\tilde{B}$, if $D_{LIF}(\tilde{A}, \tilde{B})$ satisfies the properties:

1) $((h_1, t), (h_1, f)) \le D_{LIF}(\tilde{A}, \tilde{B}) \le ((h_n, t), (h_n, f))$;
2) $D_{LIF}(\tilde{A}, \tilde{A}) = ((h_1, t), (h_1, f))$;
3) $D_{LIF}(\tilde{A}, \tilde{B}) = D_{LIF}(\tilde{B}, \tilde{A})$;
4) $D_{LIF}(\tilde{A}, \tilde{C}) > \max(D_{LIF}(\tilde{A}, \tilde{B}), D_{LIF}(\tilde{B}, \tilde{C}))$, if $\tilde{A} \subset \tilde{B} \subset \tilde{C}$.

Aiming to applying to the decision making problems under linguistic environment, here we give some LILFD functions where the aggregation procedure is handled by the LIL2T method. Let X be a nonempty set, $\tilde{A} = \{a_1, a_2, \ldots, a_m \,|a_i \in \mathcal{LI}_{2n}\}$, $\tilde{B} = \{b_1, b_2, \ldots, b_m \,|b_i \in \mathcal{LI}_{2n}\}$, where $a_i = \mu_{\tilde{A}}(x_i)$, $b_i = \mu_{\tilde{B}}(x_i)$, $x_i \in X$, $i = \{1, 2, \ldots, m\}$, the LILFD functions are defined.

1)
$$\begin{aligned} D_{\mathit{LIF\text{-}AM}}(\tilde{A}, \tilde{B}) &= \frac{1}{m}\coprod_{k=1}^{m} d(a_k, b_k) = \frac{1}{m}\coprod_{k=1}^{m}\left((h_{i_k}, t), (h_{j_k}, f)\right) \\ &= \Delta_{\mathrm{LIF}}\left(\frac{1}{m}\sum_{k=1}^{m} i_k, \frac{1}{m}\sum_{k=1}^{m} j_k\right). \end{aligned} \tag{7}$$

where $((h_{i_k}, t), (h_{j_k}, f)) = d(a_k, b_k)$. It is called LIFL arithmetic-mean distance.

2)
$$\begin{aligned} D_{\mathit{LIF\text{-}WA}}(\tilde{A}, \tilde{B}) &= \coprod_{k=1}^{m} \omega_k \cdot d(a_k, b_k) = \coprod_{k=1}^{m} \omega_k \cdot ((h_{i_k}, t), (h_{j_k}, f)) \\ &= \Delta_{\mathrm{LIF}}\left(\sum_{k=1}^{m} \omega_k \cdot i_k, \frac{1}{m}\sum_{k=1}^{m} \omega_k \cdot j_k\right), \end{aligned} \tag{8}$$

where $W = \{\omega_1, \omega_2, \ldots, \omega_m\}$ is the weight sets with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{m} \omega_i = 1$, $((h_{i_k}, t), (h_{j_k}, f)) = d(a_k, b_k)$. It is called LIFL weighted-mean distance. $d(a_k, b_k)$ is the LIFL distance between $\mu_{\tilde{A}}(x_k)$ and $\mu_{\tilde{B}}(x_k)$. It can be obtained by the operation "$\ominus$", which is defined as follows.

Definition 3.3. *Let $a = \mu_{\tilde{A}}(x_u) = ((h_i, t), (h_j, f))$, $b = \mu_{\tilde{B}}(x_u) = ((h_k, t), (h_l, f))$ be any two linguistic truth-valued intuitionistic fuzzy pairs. The LIFL distance between $\mu_{\tilde{A}}(x_k)$ and $\mu_{\tilde{B}}(x_k)$ is obtained by the function as follows.*

$$\begin{aligned} d(a, b) &= d\left(\mu_{\tilde{A}}(x_k)\mu_{\tilde{B}}(x_k)\right) = ((h_i, t)(h_j, f)) \ominus ((h_k, t)(h_l, f)) \\ &= \begin{cases} ((h_{1+\alpha}, t), (h_{1+\beta}, f)), (k - i)(l - j) \geq 0 \\ ((h_1, t), (h_{1+\gamma}, f)), (k - i)(l - j) < 0 \end{cases} \end{aligned} \tag{9}$$

where $\alpha = \min(|i - k|, |j - l|)$, $\beta = \max(|i - k|, |j - l|)$, $\gamma = |(i - k) + (j - l)|$. Evidently, $a \parallel b$ if $\gamma = 0$.

For any $a = ((h_i, t), (h_j, f))$, $b = ((h_k, t), (h_l, f))$, $c = ((h_s, t), (h_r, f))$, linguistic truth-valued intuitionistic fuzzy difference $d(a, b)$ has the following properties:

1) if $a = b$ or $a \parallel b$, $d(a, b) = ((h_1, t), (h_1, f))$;
2) $((h_1, t), (h_1, f)) \leq d(a, b) \leq ((h_n, t), (h_n, f))$;
3) $d(a, b) = d(b, a)$;
4) if $a < b < c$, then $d(a, c) > \max(d(a, b), d(b, c))$.

To keep the paper reasonably concise, the proof of the properties of the operation "$\ominus$" and the proof of that LIFL arithmetic-mean distance and LIFL weighted-mean distance are LIFLD are not shown here, and they are easy to be proved.

4. **Decision Making Approach.** People prefer linguistic values which are uncertain, inaccurate, incomplete, both incomparable and comparable when giving assessments for attributes under uncertain environment. They often evaluate from two opposite sides, i.e., the positive side and negative side. LTV-IFPs can express the linguistic information from two sides at the same time with lattice structure. We proposed a decision making approach based on LIFLD on LTV-IFL, where the LIL2T is utilized for the aggregation procedure.

Assume the non-empty alternative set $X = \{X_1, X_2, \ldots, X_N\}$, where each alternative is defined by a nonempty set of attribute $L = \{L_1, L_2, \ldots, L_M\}$. The assessment set for attributes of the alternative is $X_i = \{x_{i1}, x_{i2}, \ldots x_{iM}\}$, $i = 1, 2, \ldots N$, $x_{ij} \in \mathcal{LI}_{2n}$.

$W = \{\omega_1, \omega_2, \ldots, \omega_M\}$, $\omega_i \in [0,1]$ and $\sum_{i=1}^{M} \omega_i = 1$ is the weight set for attributes, where ω_i is the weight for attribute L_i. The assessment set of the ideal set is denoted as $A = \{a_1, a_2, \ldots, a_M\}$.

Step 1: Calculate the LIFL distance between x_{ij} and a_j for every alternative X_i according to Equation (9).

Step 2: If the weight set is empty, that is to say, the importance degree of the attributes is not considered, calculate the LIFL arithmetic-mean distance between X_i and A; else if calculate the LIFL weighted-mean distance between X_i and A. The results are LIL2T.

Step 3: Rank the alternatives, sorting by the LIFLD in increasing order. Then on the basis of selecting the near principle, the alternative with the smallest distance with A is the best one.

5. **Illustration Example.** We give an example to show the method procedure.

Example 5.1. *We can identify the radar targets through one dimensional distance image, sound signal of radar target, (RCS, reflex cross section) of radar target and the track, speed, acceleration of the target. We need to find the most accuracy approach through comparing the identification results of the four approaches to four certain targets. Suppose there are four type aircrafts* T_1, T_2, T_3, T_4, *and we note the four approaches with* R_1, R_2, R_3, R_4 *and the ideal result with* E.

The assessment collection is on 10-elements linguistic truth-valued intuitionistic fuzzy lattice (10LTV-IFL) (Figure 2). In 10LTV-IFL, the hedge set as $H = \{h_5 = ab, h_4 = ve, h_3 = ra, h_2 = so, h_1 = li\}$, *where "ab", "ve", "ra", "so", "li" represent "absolutely", "very", "rather", "somewhat", "little" respectively.*

Table 1 shows the identification results of the four approaches to four type aircrafts. The $(i+1)^{\text{th}}$ row, $(j+1)^{\text{th}}$ column is denoted as $a_{i,j}$.

Step 1: LIFLD calculation between the approach identification attributes and the ideal attributes according to Equation (9). The calculation results list on Table 2.

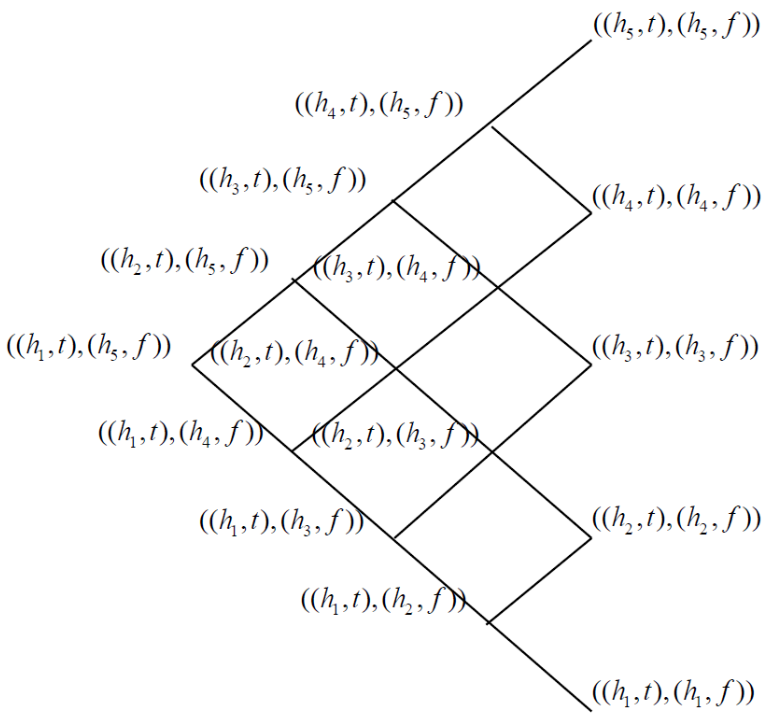

FIGURE 2. Hasse diagram of 10LTV-IFL

Table 1. The identification results of the aircrafts

	T_1	T_2	T_3	T_4
R_1	$((h_4,t)(h_4,f))$	$((h_4,t)(h_5,f))$	$((h_4,t)(h_5,f))$	$((h_5,t)(h_5,f))$
R_2	$((h_3,t)(h_5,f))$	$((h_3,t)(h_4,f))$	$((h_4,t)(h_4,f))$	$((h_4,t)(h_4,f))$
R_3	$((h_3,t)(h_3,f))$	$((h_2,t)(h_3,f))$	$((h_3,t)(h_4,f))$	$((h_4,t)(h_5,f))$
R_4	$((h_3,t)(h_5,f))$	$((h_4,t)(h_4,f))$	$((h_5,t)(h_5,f))$	$((h_4,t)(h_5,f))$
E	$((h_5,t)(h_5,f))$	$((h_5,t)(h_5,f))$	$((h_5,t)(h_5,f))$	$((h_5,t)(h_5,f))$

Table 2. Attributes LIFL distance between the alternatives and the ideal one

	T_1	T_2	T_3	T_4
R_1	$((h_2,t)(h_2,f))$	$((h_1,t)(h_2,f))$	$((h_1,t)(h_2,f))$	$((h_1,t)(h_1,f))$
R_2	$((h_1,t)(h_3,f))$	$((h_2,t)(h_3,f))$	$((h_2,t)(h_2,f))$	$((h_2,t)(h_2,f))$
R_3	$((h_3,t)(h_3,f))$	$((h_3,t)(h_4,f))$	$((h_2,t)(h_3,f))$	$((h_1,t)(h_2,f))$
R_4	$((h_1,t)(h_3,f))$	$((h_2,t)(h_2,f))$	$((h_1,t)(h_1,f))$	$((h_1,t)(h_2,f))$

The $(i+1)^{\text{th}}$ row, $(j+1)^{\text{th}}$ column represents the LIFL distance between $a_{i,j}$ and $a_{5,j}$, denoted as $d(a_{i,j}, a_{5,j})$.

Step 2: LIFL arithmetic-mean distance or LIFL weighted-mean distance calculation between R_i and E according to Equation (7) or Equation (8). Here LIFL arithmetic-mean distance is used.

$$D_{\mathit{LIF\text{-}AM}}(R_1, E) = (((h_1, t), 0.25), ((h_1, f), 0.75)),$$

$$D_{\mathit{LIF\text{-}AM}}(R_2, E) = (((h_1, t), 0.75), ((h_2, f), 0.5)),$$

$$D_{\mathit{LIF\text{-}AM}}(R_3, E) = (((h_2, t), 0.25), ((h_3, f), 0)),$$

$$D_{\mathit{LIF\text{-}AM}}(R_4, E) = (((h_1, t), 0.25), ((h_2, f), 0)).$$

Step 3: The alternatives ranking in increasing order.

$$D_{\mathit{LIF\text{-}AM}}(R_1, E) < D_{\mathit{LIF\text{-}AM}}(R_4, E) < D_{\mathit{LIF\text{-}AM}}(R_2, E) < D_{\mathit{LIF\text{-}AM}}(R_3, E).$$

On the basis of selecting the near principle, the approach R_1 is the most accurate approach for radar targets identification.

From the results of the example, LILFD is valid in decision making problems under linguistic environment. The procedure of the approach handles the linguistic values on a lattice directly without any information loss by using the LIL2T.

6. **Conclusions.** Facing the uncertain environment, people prefer linguistic assessments for the qualitative attributes, and in this paper, we proposed a decision making approach based on LILFD. The linguistic information was expressed with LT-VIFPs and the information losing in linguistic process was dealt with LIL2T. The proposed approach handles the linguistic values directly without any transformation into numbers from two opposite sides at the same time and the order of the linguistic values is more similar to the natural language. The LIL2T model avoids the limitation of information losing of LT-VIFPs. It is valid to process the decision making problems with linguistic assessments. In the future work, we will explore more approximate reasoning method and algebra method for uncertain decision making problems based on LT-VIFLIA.

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REFERENCES

[1] L. A. Zadeh, Fuzzy logic = Computing with words, *IEEE Trans. Fuzzy Systems*, vol.4, no.2, pp.103-111, 1996.

[2] L. A. Zadeh, The concept of linguistic variable and its application for approximate reasoning (1), *Information Sciences*, vol.8, pp.199-249, 1975.

[3] Y. Jiang, A general type-2 fuzzy model for computing with words, *International Journal of Intelligent Systems*, vol.33, no.4, pp.713-758, 2018.

[4] T. Wu, X. Liu and F. Liu, An interval type-2 fuzzy TOPSIS model for large scale group decision making problems with social network information, *Information Sciences*, vol.432, pp.392-410, 2018.

[5] R. M. Rodriguez, L. Martinez and F. Herrera, Hesitant fuzzy linguistic term sets for decision making, *IEEE Trans. Fuzzy Systems*, vol.20, no.1, pp.109-119, 2012.

[6] H. Wang, Z. Xu and X. Zeng, Hesitant fuzzy linguistic term sets for linguistic decision making: Current developments, issues and challenges, *Information Fusion*, vol.43, pp.1-12, 2018.

[7] Q. Pang, H. Wang and Z. Xu, Probabilistic linguistic linguistic term sets in multi-attribute group decision making, *Information Sciences*, vol.369, pp.128-143, 2016.

[8] C. Bai, R. Zhang, S. Shen, C. Huang and X. Fan, Interval-valued probabilistic linguistic term sets in multi-criteria group decision making, *International Journal of Intelligent Systems*, vol.33, no.6, pp.1301-1321, 2018.

[9] F. Herrera and L. Martinez, A 2-tuple fuzzy linguistic representation model for computing with words, *IEEE Trans. Fuzzy Systems*, vol.8, no.6, pp.746-752, 2000.

[10] Y. Xu, J. Liu, D. Ruan and X. Li, Determination of alpha-resolution in lattice-valued first-order logic $LF(X)$, *Information Sciences*, vol.181, no.10, pp.1836-1862, 2011.

[11] Y. Xu, S. Chen and J. Ma, Linguistic truth-valued lattice implication algebra and its properties, *Proc. of the IMACS Multi-Conference on Computational Engineering in Systems Applications*, Beijing, China, 2006.

[12] K. Atanassov, Intuitionistic fuzzy sets, in *VII ITKR's Session*, V. Sgurev (ed.), Sofia, 1983.

[13] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol.20, pp.87-96, 1986.

[14] L. Zou, P. Shi, Z. Pei and Y. Xu, On an algebra of linguistic truth-valued intuitionistic lattice-valued logic, *Journal of Intelligent & Fuzzy Systems*, vol.24, no.3, pp.447-456, 2013.

[15] L. Zou, X. Liu, Z. Pei and D. Huang, Implication operators on the set of V-irreducible element in the linguistic truth-valued intuitionistic fuzzy lattice, *International Journal of Machine Learning and Cybernetics*, vol.4, no.4, pp.365-372, 2013.

[16] X. Liu, Y. Wang, X. Li and L. Zou, A linguistic-valued approximate reasoning approach for financial decision making, *International Journal of Computational Intelligence Systems*, vol.10, no.1, pp.312-319, 2017.

[17] L. Zou, X. Wen and Y. Wang, Linguistic truth-valued intuitionistic fuzzy reasoning with applications in human factors engineering, *Information Sciences*, vol.327, pp.201-216, 2016.

[18] H. Wang, J. Meng and L. Zou, Linguistic-valued lattice implication algebra TOPSIS method based on entropy weight method, *The 12th International Conference on Intelligent Systems and Knowledge Engineering (ISKE)*, Nanjing, China, 2017.

[19] X. Liu, X. Tan, X. Li, Y. Zhang and L. Zou, An assessment approach with linguistic truth-valued intuitionistic fuzzy reasoning, *Proc. of the 12th International FLINS Conference on Uncertainty Modelling in Knowledge Engineering and Decision Making*, Roubaix, France, pp.31-36, 2016.

[20] I. Beg and T. Rashid, An intuitionistic 2-tuple linguistic information model and aggregation operators, *International Journal of Intelligent Systems*, vol.31, no.6, pp.569-592, 2016.

[21] P. Liu and S.-M. Chen, Multiattribute group decision making based on intuitionistic 2-tuple linguistic information, *Information Sciences*, vol.430, pp.599-619, 2018.

[22] G. Wei, F. E. Alsaadi, T. Hayat and A. Alsaedi, Picture 2-tuple linguistic aggregation operators in multiple attribute decision making, *Soft Computing*, vol.22, no.3, pp.989-1002, 2018.

[23] H. Cheng, F. Meng and K. Chen, Several generalized interval-valued 2-tuple linguistic weighted distance measures and their application, *International Journal of Fuzzy Systems*, vol.19, no.4, pp.967-981, 2017.

[24] H. Zhu, J. Zhao and Y. Xu, 2-dimension linguistic computational model with 2-tuples for multi-attribute group decision making, *Knowledge-Based Systems*, vol.103, pp.132-142, 2016.