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CONSENSUS OF MULTI-AGENT SYSTEMS WITH MARKOV JUMP TOPOLOGIES AND DELAYS

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Abstract. *This paper investigates the problem of consensus for multi-agent systems with switching topologies and stochastic delays. The switching topologies and time-delays are governed by Markov chains. By using Lyapunov function approach, sufficient conditions are established such that consensus is achieved. A numerical example is provided to verify the effectiveness of the proposed approach.*
Keywords: Multi-agent systems, Consensus, Random delay, Switching topology

1. **Introduction.** With the rapid expansion of control theory [1], much attention has been paid to multi-agent systems (MASs) due to its potential applications in various areas, such as in robots, formation of unmanned vehicles [2], and attitude alignment of satellite clusters [3], and an overview of applications of MASs is also shown in [4]. Olfati-Saber and Murray gave a theoretical framework for analysis of consensus algorithms for MASs [5,6]; after that, much work has been done on MASs. In [7], group consensus of heterogeneous MASs with time delays is also studied.

Time delay is unavoidable since the physical limitations in communication channels, time-response of actuators, etc. Based on this situation, much work has been done to deal with the consensus problem subject to time delays, focusing on constant delays for all agents' interactions [8], non-uniform delays [9], and time-varying delays [10]. It is worth mentioning that once the distance between two agents exceeds a prescribed communication region, then, signal will not be received, which results in the change of topologies of the network. The changes of time delay and topology occur randomly and abruptly, which is reasonable to be described by Markov jump system (MJS).

As an important kind of hybrid system, MJS has been widely used in MASs due to its accurate description of random changes of the system structure or parameters. Some work has been done and many results have been published in literature for MASs with Markov jump parameters, with the help of MJS in [11]. In [12], robust H_∞ controller is designed for saturated uncertainties MJS. Yin et al. studied the problem of a class of extended MJS subject to time-delay and actuator saturation nonlinearity, and designed an observer-based H_∞ controller in [13]. Yin et al. focused on the design of a robust fault detection filter for a class of uncertain discrete-time MJS with non-homogeneous jump processes in [14]. In [15], results for consensus of MASs subject to Markov jump delays are obtained, and it is also extended to second-order MASs systems with Markov jump delays [16,17]. [18] focuses on the consensus of MASs where each agent is supposed to be an MJS with event-triggered protocols. [19] aims to address the stationary consensus problem for leaderless heterogenous MASs delays which are subject to Markov chain. Note that the changes of delay and communication topology are common phenomena in practical applications; however, results have been obtained only under the assumption that one of them is governed by Markov chain, see in [16-19]; motivated by this, the study in this paper takes two random factors into consideration, and it aims to address consensus problem for MASs with topologies and delays governed by Markov chain. To the authors' best knowledge, no attempt work has been done on this subject, and it can be regarded as an extension of the result in [15].

The remainder of this paper is organized as follows. In Section 2, system definitions and assumption are given. Main results for consensus are given in Section 3. In Section 4, simulation results are provided to verify the effectiveness of the proposed method. Finally, some conclusions are shown in Section 5.

Notation. 1_n and 0_n are column vectors of ones and zeros, respectively, of dimension n; $\lambda_{\max}(L)$, $\lambda_{\min}(L)$ denote the maximum and minimum eigenvalue of L. $M > 0$ ($M < 0$) means that M is a positive (negative) definition matrix; $E\{\}$ denotes the mathematical expectation. M^T means the transpose matrix of M; and * denotes the symmetric block in a matrix. Matrices, if dimensions are not indicated explicitly, are assumed to be compatible with algebraic operations. L_{a,τ_b} denotes Laplacian matrix when time-delay is τ_b and topology structure is a.

2. Problem Statement and Preliminaries.

2.1. Some graph theory.

A simple directed graph is denoted by $G = (V, E, A)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of n vertices, and E represents the set of directed edges connecting them, denote $a_{ij} = (v_i, v_j)$ where $(\cdot)$ is the directed edge function, given by $a_{ij} = (v_i, v_j) \in E$. $A = [a_{ij}]$ denotes the adjacency matrix. A spanning tree is a spanning subgraph without cycle. Obviously, in a graph with a spanning tree, there exists at least one node whose information transfers to all others, then the node is said to be globally reachable. The union of the topology set (in the switching case) has a globally reachable node. Let $L = [l_{ij}]$ with $l_{ii} = \sum_{j=1}^{n} a_{ij}$, $i = 1, 2, \ldots, n$ and $l_{ij} = -a_{ij}$, for $i \neq j$. An important property of the Laplacian matrix is $L1_n = 0_n$.

In this paper, we do study on communication network union topologies with a directed spanning tree.

2.2. Multi-agent systems.

We consider a group of n agents with first-order discrete-time dynamics,

$$x_i(k+1) = x_i(k) + u_i(k),\ i = 1, 2, \ldots, n \tag{1}$$

where $x_i(k)$ is state of the ith agent and $u_i(k)$ is the control input generated by the control law given below:

$$\begin{gathered} u_i(k) = \sum_{j \in N_i} a_{ij}\left(\tau_b^a\right)\left[x_j\left(k-\tau_b^a\right) - x_i\left(k-\tau_b^a\right)\right], \\ a \in \mathrm{T}_1 = \{1, 2, \ldots, m\}, \quad b \in \mathrm{T}_2 = \{1, 2, \ldots, q\} \end{gathered} \tag{2}$$

$N_i = \{1, 2, 3, \ldots, n\}$, $j \neq i$. τ_b^a denotes a time-delay τ_b in the ath topology structure. a and b are independent parameters and satisfy independent Markov chain. The transition probability matrix is defined as: $\pi_{rs} = P(b_k = r | b_{k-1} = s)$ is the probability of transition from time-delay τ_s at time $k-1$ to time-delay τ_r at time k, and the transition matrix is $\Pi_1 = [\pi_{rs}]$, $r, s \in \mathrm{T}_2$; $\lambda_{\alpha\beta} = P(a_k = \beta | a_{k-1} = \alpha)$ is the probability of transition from mode α at time $k-1$ to mode β at time k, and the transition matrix is $\Pi_2 = [\lambda_{\alpha\beta}]$, $\alpha, \beta \in \mathrm{T}_1$. $a_{ij}(\tau_b^a)$ denotes the case a time-delay τ_b in the ath topology structure of a_{ij}, where $0 \leq \lambda_{\alpha\beta} < 1$, $0 \leq \pi_{rs} < 1$ and $\sum_{\beta=1}^{m} \lambda_{\alpha\beta} = 1$, $\sum_{s=1}^{q} \pi_{rs} = 1$. Substituting (2) into (1), we have

$$x(k+1) = x(k) + L_{a,b}x\left(k - \tau_b^a\right) \tag{3}$$

where $x(k) = \left[x_1^T, x_2^T, \ldots, x_n^T\right]^T$ is state vector, and $L_{a,b}$ is a Laplacian matrix related to a, b.

Assumption 2.1. *τ_b^a is from a following finite integer set Γ.*

Define the error as $\delta_i(k) = x_i(k) - x_1(k)$, $i = 2, 3, \ldots, n$, and the error vector is obtained as follows: $\delta(k) = [\ \delta_2(k) \quad \delta_3(k) \quad \cdots \quad \delta_n(k)\ ]^T$. After some algebraic manipulations, we have an error dynamic system as:

$$\delta(k+1) = \delta(k) + \widehat{L}_{a,b}\delta\left(k - \tau_b^a\right) \tag{4}$$

let, $l_{ij}(\tau_b^a) = l_{ij}$, $l_{ii}(\tau_b^a) = l_{ii}$, then

$$\widehat{L}_{a,b} = -(L_{2:n,2:n} - 1_{n-1}L_{1,2:n}) = -\begin{bmatrix} l_{22} - l_{12} & l_{23} - l_{13} & \cdots & l_{2n} - l_{1n} \\ l_{32} - l_{12} & l_{33} - l_{13} & \cdots & l_{3n} - l_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ l_{n2} - l_{12} & l_{n3} - l_{13} & \cdots & l_{nn} - l_{1n} \end{bmatrix} \tag{5}$$

where $L_{2:n,2:n} = [l_{ij}] \in R^{(n-1)\times(n-1)}$, $i, j = 2, 3, \ldots, n$, $L_{1,2:n} = [\ l_{12} \quad l_{13} \quad \cdots \quad l_{1n}\ ]$. In (5), $l_{ij}(\tau_b^a) = -a_{ij}(\tau_b^a)$, $l_{ii}(\tau_b^a) = \sum_{j=1}^n a_{ij}(\tau_b^a)$, $i, j = 1, 2, \ldots, n$.

Before proceeding, we need the following definition of stability.

Definition 2.1. *For a given initial mode and state, system (3) is mean-square consensus if system (4) is stochastically stable in the mean-square sense, i.e.,* $\lim_{t\to\infty} E\left\{\delta^T(t)\delta(t)\right\} \to 0$ *holds.*

Next, we will investigate the consensus problem for system (4).

3. **The Sufficient Condition for Consensus.**

Theorem 3.1. *System (4) with random delays and switching topologies governed by Markov chains is mean square consensus if there exist matrices $P > 0$, $Q_{\rho,v} > 0$, $Z_{\rho,v} > 0$, $M_{\rho,v}$ and $\widehat{L}_{a,b}$, $v = 1, 2, \ldots, q$, $\rho = 1, 2, \ldots, m$, $\forall r = 1, 2, \ldots, q$, $\forall \alpha = 1, 2, \ldots, m$ such that it holds.*

$$\begin{bmatrix} Y_{11}(r\alpha) & Y_{12} \\ * & Y_{22} \end{bmatrix} < 0 \tag{6}$$

where

$$Y_{22} = -\mathrm{diag}\left\{Z_{1,1}, \cdots, Z_{1,q}, \cdots, Z_{m,1}, \cdots, Z_{m,q}\right\}$$

$$Y_{12} = \left[\sqrt{\tau_1^1}M_{11} \cdots \sqrt{\tau_q^1}M_{1q} \cdots \sqrt{\tau_1^m}M_{m1} \cdots \sqrt{\tau_q^m}M_{mq}\right]$$

$$Y_{11}(r\alpha) = \begin{bmatrix} \phi_0 & \psi_{1,1}(r\alpha) & \cdots & \psi_{1,q}(r\alpha) & \ldots & \psi_{m,1}(r\alpha) & \cdots & \psi_{m,q}(r\alpha) \\ * & \phi_{1,1}(r\alpha) & 0 & 0 & \ldots & 0 & \ldots & 0 \\ * & * & \ddots & \vdots & \ldots & \vdots & \ldots & \vdots \\ * & * & * & \phi_{1,q}(r\alpha) & \ldots & \vdots & \ldots & \vdots \\ * & * & * & * & \ddots & \vdots & \ldots & \vdots \\ * & * & * & * & * & \phi_{m,1}(r\alpha) & \ldots & \vdots \\ * & * & * & * & * & * & \ddots & \vdots \\ * & * & * & * & * & * & * & \phi_{m,q}(r\alpha) \end{bmatrix}$$
$$+ \left[\sum_{\rho=1}^m \sum_{v=1}^q M_{\rho,v}, -M_{1,1}, \ldots, -M_{1,q}, \ldots, -M_{m,1}, \ldots, -M_{m,q}\right]$$
$$+ \left[\sum_{\rho=1}^m \sum_{v=1}^q M_{\rho,v}, -M_{1,1}, \ldots, -M_{1,q}, \ldots, -M_{m,1}, \ldots, -M_{m,q}\right]^T$$

$$\phi_{\beta,v}(r\alpha) = \lambda_{\alpha\beta}\pi_{rv}\widehat{L}_{\beta,v}^T P\widehat{L}_{\beta,v} - Q_{\beta,v} + \lambda_{\alpha\beta}\pi_{rv}\widehat{L}_{\beta,v}^T\left(\sum_{v=1}^{q}\sum_{\rho=1}^{m}\tau_v^\rho Z_{\rho,v}\right)\widehat{L}_{\beta,v}$$

$$\phi_0 = \sum_{\rho=1}^{m}\sum_{v=1}^{q} Q_{\rho,v}, \quad \psi_{\beta,v}(r\alpha) = \lambda_{\alpha\beta}\pi_{rv}P\widehat{L}_{\beta,v}, \quad v = 1,\ldots,q, \quad \beta = 1,\ldots,m.$$

Proof: We consider the following Lyapunov function candidate:

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{7}$$

where

$$V_1(k) = \delta^T(k)P\delta(k)$$

$$V_2(k) = \sum_{\rho=1}^{m}\sum_{v=1}^{q}\sum_{i=k-\tau_{\rho,v}}^{k-1}\delta^T(i)Q_{\rho,v}\delta(i)$$

$$V_3(k) = \sum_{\rho=1}^{m}\sum_{v=1}^{q}\sum_{\xi=-\tau_{\rho,v}}^{-1}\sum_{\varphi=k+i}^{k-1}\eta^T(\varphi)Z_{\rho,v}\eta(\varphi)$$

$$\eta(\varphi) = \delta(\varphi+1) - \delta(\varphi)$$

From system (4), we have

$$V_3(k) = \sum_{\rho=1}^{m}\sum_{v=1}^{q}\sum_{\xi=-\tau_{\rho,v}}^{-1}\sum_{\varphi=k+i}^{k-1}\delta^T\left(\varphi-\tau_v^\rho\right)\widehat{L}_{\rho,v}^T Z_{\rho,v}\widehat{L}_{\rho,v}\delta\left(\varphi-\tau_v^\rho\right)$$

Let $\zeta^T(k) = \left[\delta^T(k), \delta_{1,1}^T(k), \ldots, \delta_{1,q}^T(k), \ldots, \delta_{m,1}^T(k), \ldots, \delta_{m,q}^T(k)\right]$, where $\delta_{m,q}(k)$ denotes $\delta(k-\tau_q)$ in the mth topology structure. For free matrices

$$M_{\rho,v} = \left[M_{0(\rho v)}^T, M_{11(\rho v)}^T, \ldots, M_{1q(\rho v)}^T, \ldots, M_{m1(\rho v)}^T, \ldots, M_{mq(\rho v)}^T\right]^T$$

$M_{\rho,v}$ here has the same dimensions with $\zeta(k)$, and we have the following identities:

$$\zeta^T(k)M_{\rho,v}\left[\delta(k) - \delta(k-\tau_v^\rho) - \sum_{l=k-\tau_v^\rho}^{k-1}\eta(l)\right] = 0$$

Then,

$$\begin{aligned}
E\{\Delta V(k)\} &= E\{\Delta V_1(k)\} + E\{\Delta V_2(k)\} + E\{\Delta V_3(k)\} \\
&\le 2\sum_{s=1}^{q}\pi_{rs}\sum_{\beta=1}^{m}\lambda_{\alpha\beta}\delta^T(k)\widehat{L}_{\alpha,r}\delta(k-\tau_r^\alpha) \\
&\quad + \sum_{s=1}^{q}\pi_{rs}\sum_{\beta=1}^{m}\lambda_{\alpha\beta}\delta^T(k-\tau_r^\alpha)\widehat{L}_{\alpha,r}^T P\widehat{L}_{\alpha,r}\delta(k-\tau_r^\alpha) \\
&\quad + \sum_{\rho=1}^{m}\sum_{v=1}^{q}\left[\delta^T(k)Q_{\rho,v}\delta(k) - \delta^T(k-\tau_v^\rho)Q_{\rho,v}\delta(k-\tau_v^\rho)\right] \\
&\quad + \sum_{\rho=1}^{m}\sum_{v=1}^{q}\sum_{s=1}^{q}\pi_{rs}\sum_{\beta=1}^{m}\lambda_{\alpha\beta}\delta^T(k-\tau_r^\alpha)\widehat{L}_{\alpha,r}^T\tau_{\rho,v}Z_{\rho,v}\widehat{L}_{\alpha,r}\delta(k-\tau_r^\alpha) \\
&\quad - \sum_{\rho=1}^{m}\sum_{v=1}^{q}\sum_{l=k-\tau_v^\rho}^{k-1}\delta^T(l-\tau_v^\rho)\widehat{L}_{\rho,v}^T Z_{\rho,v}\widehat{L}_{\rho,v}\delta(l-\tau_v^\rho) \\
&\quad + 2\sum_{\rho=1}^{m}\sum_{v=1}^{q}\xi^T(k)M_{\rho,v}\left[\delta(k) - \delta(k-\tau_v^\rho) - \sum_{l=k-\tau_v^\rho}^{k-1}\eta(l)\right]
\end{aligned}$$

$$+\sum_{\rho=1}^{m}\sum_{v=1}^{q}\sum_{l=k-\tau_v^\rho}^{k-1}\left[\xi^T(k)M_{\rho,v}+\eta^T(l)Z_{p,v}\right]Z_{p,v}^{-1}\left[M_{\rho,v}^T\xi(k)+Z_{\rho,v}\eta(l)\right]$$

Taking a further step, we have

$$E\{V(k+1)-V(k)\}\leq\zeta^T(k)\left[Y_{11}(r\alpha)-Y_{12}Y_{22}^{-1}Y_{12}^T\right]\zeta(k) \tag{8}$$

Then, condition (6) ensures that $E\{\Delta V(k)\}<0$.

Let $W(r\alpha)=Y_{11}(r\alpha)-Y_{12}Y_{22}^{-1}Y_{12}^T<0$. We assume that $-\lambda_{\max}(W(r\alpha))I\leq W(r\alpha)\leq -\lambda_{\min}(W(r\alpha))I$, where $\lambda_{\max}(L)>0$, $\lambda_{\min}(L)>0$.

Then

$$E\{V(k+1)-V(k)\}\leq\zeta^T(k)[W(r\alpha)]\zeta(k)\leq-\lambda_{\min}(W(r\alpha))||\zeta(k)||^2\leq-\gamma||\zeta(k)||^2 \tag{9}$$

with $0<\gamma=\min\{\lambda_{\min}(W(r\alpha)),r=1,2,\ldots,q,\alpha=1,2,\ldots,m\}$. Summing up (9) from $k=0$, it is obtained that

$$E\{V(k+1)-V(0)\}\leq-\gamma\sum_{w=0}^{\infty}E\left\{||\zeta(w)||^2\right\} \tag{10}$$

which leads to,

$$\sum_{w=0}^{k}E\left\{||\zeta(w)||^2\right\}\leq\frac{1}{\gamma}E\{V(0)-V(\infty)\}\leq\frac{1}{\gamma}E\{V(0)\} \tag{11}$$

Meanwhile, note that $||\zeta(w)||^2\geq||\delta(w)||^2$, $w=0,1,2,3,\ldots$, then, we have $\sum_{w=0}^{\infty}||\zeta(w)||^2\geq\sum_{w=0}^{\infty}||\delta(w)||^2$. Therefore, $\sum_{w=0}^{\infty}E\{||\delta(w)||\}\leq\frac{1}{\gamma}E\{V(0)\}<\infty$ from which we conclude that $\lim_{w\to\infty}E\left\{||\delta(w)||^2\right\}=0$, and from Definition 2.1, the error dynamic system (4) is mean square stable. This completes the proof.

Note that $\widehat{L}_{\beta,v}$ in (6) cannot be solved in LMI directly for the existence of $\phi_{\beta,v}(r\alpha)$. Then to deal with this problem, a separation method proposed in [8] is used here to make it solvable; for simplicity, we omit it.

4. **Numerical Example.**

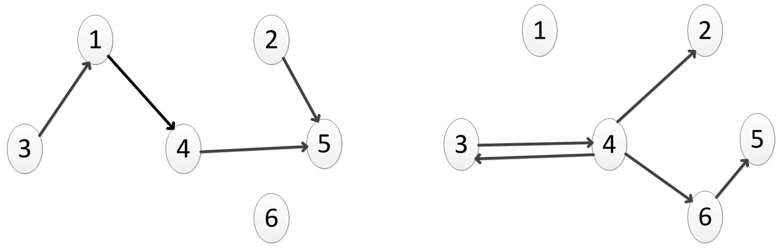
In this section, we present a numerical example to verify the design procedure and the effectiveness of the proposed method. Figure 1 shows the switching communication topology of six agents. An adjacency matrix A in 1(a) has the upper following general form and 1(b) has the lower following general form:

$$A=\begin{bmatrix}0&0&a_{13}\left(\tau_b^1\right)&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\a_{41}\left(\tau_b^1\right)&0&0&0&0&0\\0&a_{52}\left(\tau_b^1\right)&0&a_{54}\left(\tau_b^1\right)&0&0\\0&0&0&0&0&0\end{bmatrix}$$

$$A=\begin{bmatrix}0&0&0&0&0&0\\0&0&0&a_{24}\left(\tau_b^2\right)&0&0\\0&0&0&a_{34}\left(\tau_b^2\right)&0&0\\0&0&a_{43}\left(\tau_b^2\right)&0&0&0\\0&0&0&0&0&a_{56}\left(\tau_b^2\right)\\0&0&0&a_{64}\left(\tau_b^2\right)&0&0\end{bmatrix}$$

The delay set is $\Gamma=\left\{\ \tau_1^1=\tau_1^2=6s\quad\tau_2^1=\tau_2^2=16s\quad\tau_3^1=\tau_3^2=20s\ \right\}$. Switching topologies transition probability matrix Π_1 and delays transition probability matrix Π_2 are

$$\Pi_1=\begin{bmatrix}0.5&0.5\\0.3&0.7\end{bmatrix},\quad\Pi_2=\begin{bmatrix}0.2&0.5&0.3\\0.4&0.2&0.4\\0.3&0.1&0.6\end{bmatrix}$$



(a) $G(1)$ (b) $G(2)$

FIGURE 1. Multi-agent system subject to time-varying delays, composed of six agents with switching topologies $G(1)$ and $G(2)$

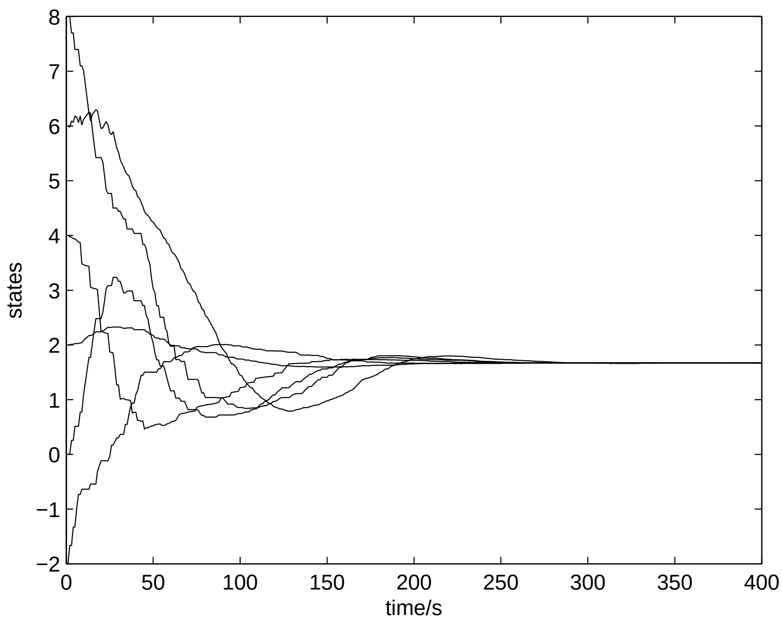


FIGURE 2. Consensus with switching adjacency matrices

The initial state of six agents is $x(0) = [\ -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8\ ]^T$. We get Laplacian matrices $L_{1,1}$, $L_{1,2}$, $L_{1,3}$, $L_{2,1}$, $L_{2,2}$ and $L_{2,3}$ by solve LMI(6).

The three matrices $L_{1,1}$, $L_{1,2}$ and $L_{1,3}$ are associated with the delays $6s$, $16s$, $20s$ respectively in case $G(1)$, and last three matrices $L_{2,1}$, $L_{2,2}$ and $L_{2,3}$ are associated with the delays $6s$, $16s$ and $20s$ in case $G(2)$. Figure 2 shows the states of multi-agent systems under control where consensus is achieved.

5. **Conclusion.** This paper aims to address consensus problem for MASs with topologies and delays governed by Markov chain in discrete system. In future works, a novel Lyapunov function is put forward in order to solve the random variation of topology and delay. The simulation results present that the consensus of MASs with jump topologies and delays can achieve when topology contains spanning tree. We hope to solve the sampling-time in un-uniform case.

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