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LOCATION PROBLEM OF THE CUSTOMER ORDER DECOUPLING POINT WITH CONSTRAINED DELIVERY LEAD TIME

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Abstract. *To deal with the location problem of the customer order decoupling point (CODP) in a postponement production system, a manufacturer cost optimization model based on the queuing theory is proposed. Especially, the impact of delivery lead time on CODP is analyzed. The model is proved to be effective through numerical simulation. From this simulation, we obtain that higher demand arrival rate and longer production time make CODP move downstream while the product types have no significant impact on the position of CODP when delivery lead time is long.*

Keywords: Postponement strategy, Customer order decoupling point, Delivery lead time

1. Introduction.

In recent years, consumer demand for personalized products has grown steadily [1]. And the postponement strategy is often used due to the maintained scale advantages while providing customized goods [2]. Due to the customers' impatience if the original manufacturer fails to meet the negotiated delivery lead time, it is necessary and important to study the postponement strategy in order that the manufacturer can gain maximum profit considering the delivery lead time.

The location of customer order decoupling point (CODP) in the postponement strategy is a key problem. The CODP is the marketing-production interface [3]. The CODP is defined as the point in the value chain for a product, where the product is linked to a specific customer order. There are external and internal factors affecting the position of CODP. External factors are related to the supplier delivery performance [4-6] and the supply chain integration [7-9]. Internal factors are strongly linked to the company's design features, assembly and production management [10]. Researchers and practitioners propose many effective methods to study the optimal location of CODP, such as the queuing Markov chain [11,12], the Bayesian belief networks [13], and the median-joining phylogenetic networks [14]. Although postponement has recently been mentioned as a useful tool for mitigating risk, most previous studies focus on demand uncertainty. Moreover, there is a lack of theory-testing studies examining the relationship between the location of CODP and the delivery lead time. As one of the most important customer satisfaction performance indicators, the delivery lead time is a factor that cannot be neglected when we determine the position of CODP. Hence, we argue that there is an implicit and open debate within the literature that offers strong motivation for this work.

In this research, a CODP positioning model is developed, which satisfies a certain delivery lead time constraint. It aims to minimize the total cost, including the production cost, the WIP holding cost, the semi-finished items inventory holding cost and the product/process redesign cost. The effect of the delivery lead time on the CODP location is investigated. Through the numerical simulation, the impact factors such as the demand arrival rate, the production time and the product types are analyzed.

The remainder of this article is organized as follows. In Section 2, we describe the problem. Section 3 develops a mathematical model and analyzes this model. Numerical experiments are conducted to demonstrate the impact of the parameters on the optimal policy in Section 4. This is followed by some concluding discussions and suggestions for future research in Section 5.

2. **Problem Description.** We consider a manufacturer who provides a product family. The production system is divided into two stages: the generic stage (Stage 1) and the final customization stage (Stage 2). At Stage 1, semi-finished items are produced based on make-to-stock (MTS) and are stocked in warehouse. At Stage 2, once customer orders arrive, semi-finished items are customized in a make-to-order (MTO) fashion and are sent directly to the customers. The delayed production process is illustrated in Figure 1.

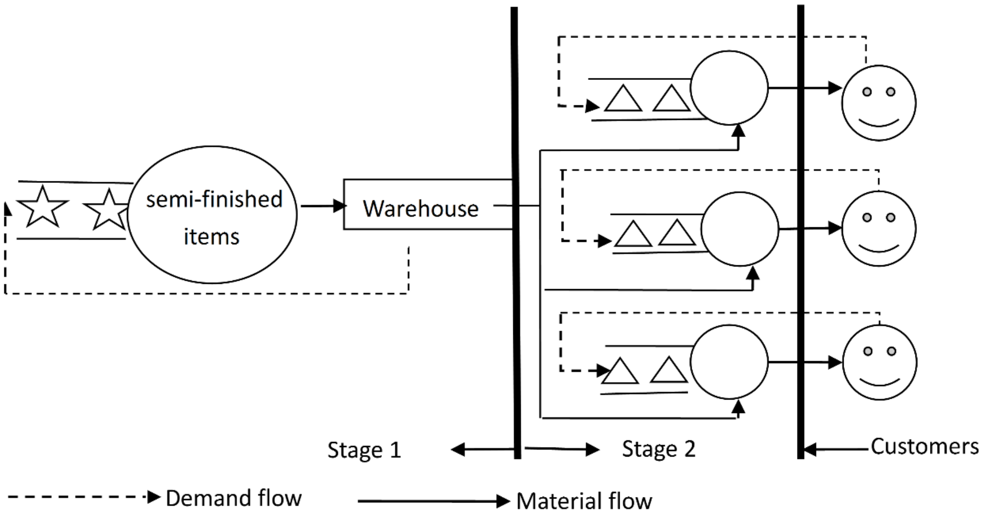


FIGURE 1. The postponement production process

For convenience, the symbols description is as follows: z is the base-stock level, α is the delivery lead time, s is the total production time, D is the demand arrival rate of all the products, λ_g is the demand arrival rate in the generic stage, $\lambda_{2,k}$ is the demand arrival rate of the kth production line in the final customization stage, $1 \leq k \leq N$, assuming N total product types.

Decision variables are as follows: r is the proportion of generalized production time in total production time, which stands for the position of CODP, $0 \leq r < 1$. For the generic stage, $c_g(r)$ is the unit production cost, $\rho_g(r)$ is the quantity of WIP, $\mu_g(r)$ is the average production rate, $w_g(r)$ is the unit cost of the WIP inventory, $h_g(r)$ is the unit semi-finished item inventory holding cost, $E[I](r)$ is the expected inventory level of the semi-finished items. For the final customization stage, $c_{2,k}(r)$ is the unit production cost, $\rho_{2,k}(r)$ is the quantity of WIP, $\mu_{2,k}(r)$ is the average production rate, and $w_{2,k}(r)$ is the unit cost of the WIP inventory.

Our model encompasses the following assumptions. (1) The production system is a continuous, uniform, value-added process. Thus, $c_g(r)$, $w_g(r)$, $h_g(r)$ and $w_{2,k}(r)$ are increasing functions of r, and $c_{2,k}(r)$ is decreasing function of r. Given $c_g(r) = c_1 r$, $w_g(r) = w_1 r$, $h_g(r) = hr$, $w_{2,k}(r) = w_0 + w_2 r$, $c_{2,k}(r) = c_2(1 - r)$, where c_1, c_2, w_0, w_1, w_2, h are constants greater than zero. (2) N types of customized products belong to one product family, and every customized product has some similarities, i.e., for two random customized products i, j, $\lambda_{2,i} = \lambda_{2,j}$, $\mu_{2,i} = \mu_{2,j}$, $c_{2,i} = c_{2,j}$, $w_{2,i} = w_{2,j}$. (3) As the CODP moves downstream, more processes are standardized and modularized. Thus, the corresponding product/process redesign cost $F(r)$ is increasing. We assume $F(r)$ is a linear increasing function of r, i.e., $F(r) = ar$, where a is a constant greater than zero. (4) The process has the characteristics of stationary, independence and universality. Both the demand arrival internal time and the production time are exponential distributions where the service rule is first come first serve. Thus, the whole production system is regarded as an M/M/1 queuing system.

3. **Mathematical Model and Analysis.** The following describes the cost optimization model used to obtain the optimal CODP. Firstly, the cost at the generic stage is calculated. Secondly, the cost at the final customization stage is derived. Thirdly, the delivery lead time is computed using the queuing theory. We begin with the cost at Stage 1.

The production time at the generic stage is rs, the average production rate $\mu_g(r)$ equals $(rs)^{-1}$, and the quantity of WIP is equal to the average utilization of Stage 1, i.e., $\rho_g(r) = \lambda_g\mu_g(r)^{-1} = Drs$. The manufacturer manages the semi-finished items using the base-stock policy, and the expected inventory quantity can be expressed as [15]:

$$E[I](r) = z - \frac{\rho_g(r)}{1-\rho_g(r)}\left[1-\rho_g(r)^z\right] \tag{1}$$

Thus, the cost at the generic stage is:

$$TC_1(r) = \lambda_g c_g(r) + \rho_g(r)w_g(r) + h_g(r)E[I](r) \tag{2}$$

The first term is the production cost, the second term is the WIP holding cost, and the third term is the inventory holding cost of the semi-finished items.

Based on assumption (1), Equation (2) can be rewritten as:

$$TC_1(r) = Dc_1r + Drsw_1r + hr\left\{z - \frac{Drs}{1-Drs}\left[1-(Drs)^z\right]\right\} \tag{3}$$

The production time at the final customization stage is $(1-r)s$, and the average production rate is $[(1-r)s]^{-1}$. By assumption (2), the demand arrival rate and the average production rate of the kth production line are respectively presented as $\lambda_{2,k} = DN^{-1}$, $\mu_{2,k}(r) = [N(1-r)s]^{-1}$. The quantity of WIP is equal to the average utilization, which means $\rho_{2,k}(r) = \lambda_{2,k}\mu_{2,k}(r)^{-1} = (1-r)Ds$. Thus, the cost at the final customization stage can be specified as:

$$TC_2(r) = \sum_{k=1}^{N}\left(\lambda_{2,k}c_{2,k}(r) + \rho_{2,k}(r)w_{2,k}(r)\right) \tag{4}$$

The first term is the production cost, and the second term is the WIP holding cost. Based on assumption (2), Equation (4) can be reformulated as:

$$TC_2(r) = Dc_2(1-r) + NDs(1-r)(w_0 + w_2r) \tag{5}$$

The total cost is the sum of three terms: the product/process redesign cost, the generic stage cost and the final customization stage cost:

$$\begin{aligned} TC(r) &= F(r) + TC_1(r) + TC_2(r) \\ &= Dc_2 + NDsw_0 + (a + Dc_1 - Dc_2 + NDsw_2 - NDsw_0)r \\ &\quad + (Dsw_1 - NDsw_2)r^2 + hr\left\{z - \frac{Drs}{1-Drs}\left[1-(Drs)^z\right]\right\} \end{aligned} \tag{6}$$

Denote $A = Dc_2 + NDsw_0$, $B = a + Dc_1 - Dc_2 + NDsw_2 - NDsw_0$, $C = Dsw_1 - NDsw_2$; thus, Equation (6) can be transferred as follows:

$$TC(r) = A + (B + hz)r + (C - hDs)r^2 - h(Ds)^2r^3 - h(Ds)^3r^4 - \cdots - h(Ds)^zr^{z+1} \tag{7}$$

Because the second stage is processed on MTO, the customers have to wait to receive products. The waiting time can be expressed by the expected sojourn time in queuing system [16]:

$$ET = \frac{1}{\sum_{k=1}^{N} \mu_{2,k} - \sum_{k=1}^{N} \lambda_{2,k}} \tag{8}$$

The focus of our research is on finding the optimal position of CODP to minimize the manufacturer's total cost with the constraint of delivery lead time. We establish the following mathematic model:

$$\begin{aligned} &\min\ TC(r) \\ &\text{s.t.} \begin{cases} ET \leq \alpha \\ 0 < Ds < 1 \\ 0 \leq r < 1 \\ w_g(r) < h_g(r) < w_{2,k}(r) \end{cases} \end{aligned} \tag{9}$$

The first constraint means the waiting time is no more than the delivery lead time. The second constraint means that the overstock is not considered. The third constraint means that the value of CODP is between 0 and 1. Since the process is value-added, the unit WIP holding cost at the generic stage is less than the unit inventory cost at the generic stage, which is less than the unit WIP holding cost at the final customization stage.

Proposition 3.1. *$TC(r)$ is strictly concave with respect to r.*

The constraint condition $ET \leq \alpha$ can be simplified as $\frac{s-\alpha+\alpha Ds}{\alpha Ds+s} \leq r$. It is easy to see that $\frac{s-\alpha+\alpha Ds}{\alpha Ds+s} < 1$, and then there exist two situations.

(1) when $\frac{s-\alpha+\alpha Ds}{\alpha Ds+s} > 0$, or equivalently, $0 < \alpha < \frac{s}{1-Ds}$, then $r \in \left[\frac{s-\alpha+\alpha Ds}{\alpha Ds+s}, 1\right)$. There exists a t that makes $TC(r = t) = TC(r \to 1)$, thus,

1) when $0 < \frac{s-\alpha+\alpha Ds}{\alpha Ds+s} < t$, $TC\left(r = \frac{s-\alpha+\alpha Ds}{\alpha Ds+s}\right) < TC(r \to 1)$, then $r^* = \frac{s-\alpha+\alpha Ds}{\alpha Ds+s}$, here, $\frac{ts-s}{Ds-1-Dst} < \alpha < \frac{s}{1-Ds}$.

2) when $t \leq \frac{s-\alpha+\alpha Ds}{\alpha Ds+s} < 1$, $TC\left(r = \frac{s-\alpha+\alpha Ds}{\alpha Ds+s}\right) \geq TC(r \to 1)$, then $r^* \to 1$, here, $0 < \alpha < \frac{ts-s}{Ds-1-Dst}$.

(2) when $\frac{s-\alpha+\alpha Ds}{\alpha Ds+s} \leq 0$, which means $\alpha \geq \frac{s}{1-Ds}$, then $r \in [0, 1)$. Because $TC(r = 0)$ is less than $TC(r \to 1)$, hence, $r^* = 0$.

From the above analysis, the optimal CODP r^* can be expressed as:

$$r^* = \begin{cases} \text{tend to 1}, & 0 < \alpha \leq \dfrac{ts-s}{Ds-1-Dst} \\ \dfrac{s-\alpha+\alpha Ds}{\alpha Ds+s}, & \dfrac{ts-s}{Ds-1-Dst} < \alpha < \dfrac{s}{1-Ds} \\ 0, & \alpha \geq \dfrac{s}{1-Ds} \end{cases} \tag{10}$$

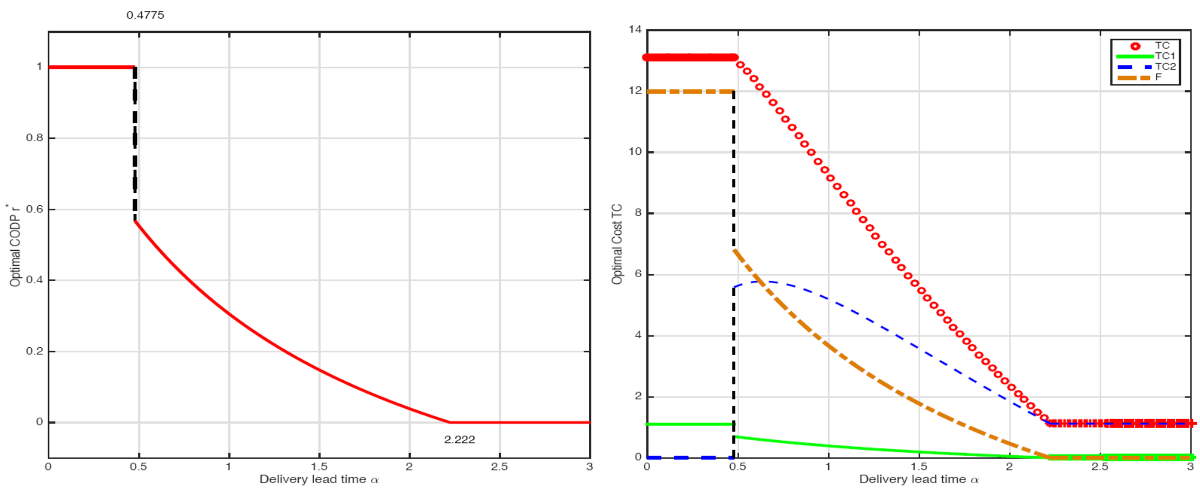
When $0 < \alpha \leq \frac{ts-s}{Ds-1-Dst}$, customers are impatient to wait. At this situation, it might be better for the manufacturer to set CODP at the end of production line. When $\alpha \geq \frac{s}{1-Ds}$, customers have enough patience to wait for product to delivery. At this situation, it could be a good choice to put CODP at the beginning of production line. When $\frac{ts-s}{Ds-1-Dst} < \alpha < \frac{s}{1-Ds}$, MTS/MTO mixed production mode is more appropriate, and the optimal CODP depends on the demand arrival rate, the total production time and the delivery lead time.

4. **Numerical Simulation.** In order to verify the proposed model, we set parameters as follows: $s = 0.8$, $D = 0.8$, $a = 12$, $c_1 = 0.9$, $w_0 = 0.01$, $w_1 = 0.15$, $w_2 = 0.65$, $h = 0.3$, $c_2 = 1$, $z = 2$, and $N = 50$. We simulate the postponement production system by Matlab software.

We can obtain the relationship between α and r^* from Equation (10):

$$r^* = \begin{cases} \text{tend to } 1, & 0 < \alpha \leq 0.4775 \\ \dfrac{0.8 - 0.36\alpha}{0.8 + 0.64\alpha}, & 0.4775 < \alpha < 2.2222 \\ 0, & \alpha \geq 2.2222 \end{cases} \tag{11}$$

Figure 2(a) is a plot of the optimal CODP r^* when the delivery lead time α increases from 0 to 3. The observation has important implications: when the delivery lead time is long, which means customers have enough patience to wait, the manufacturer should set CODP as small as possible. While the delivery lead time is quite short, the manufacturer should delay the product differentiation.



(a) Effect of delivery lead time on r^* (b) Effect of delivery lead time on TC

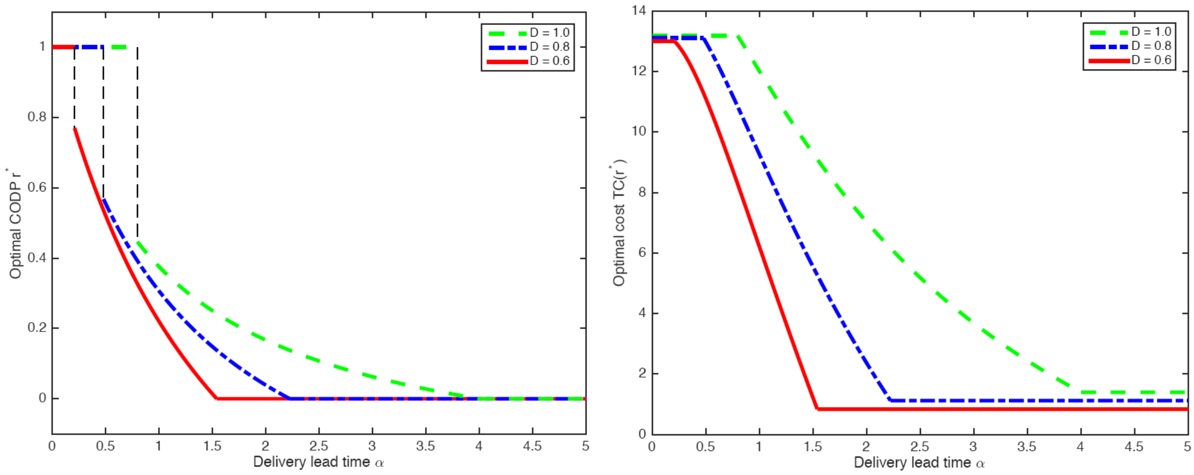
FIGURE 2. Effect of delivery lead time

The optimal cost under different values of the delivery lead time can be obtained by substituting the optimal CODP into the manufacturer's cost function (see Figure 2(b)). The relationship between the optimal total cost and the delivery lead time is similar to Figure 2(a). The implication of this result is that longer delivery lead time causes a more forward CODP position, resulting in the corresponding manufacturer's total cost to fall, and vice versa. Hence, the manufacturer hopes that customers have enough patience to wait their products to delivery. To achieve this goal, the manufacturer can surrender part of their profits to the customers, e.g., falling prices, which makes customers have lower requirement for the delivery lead time.

Then, we conduct a variety of numerical simulations to better understand how the values of D, s, N influence the manufacturer to select the optimal CODP.

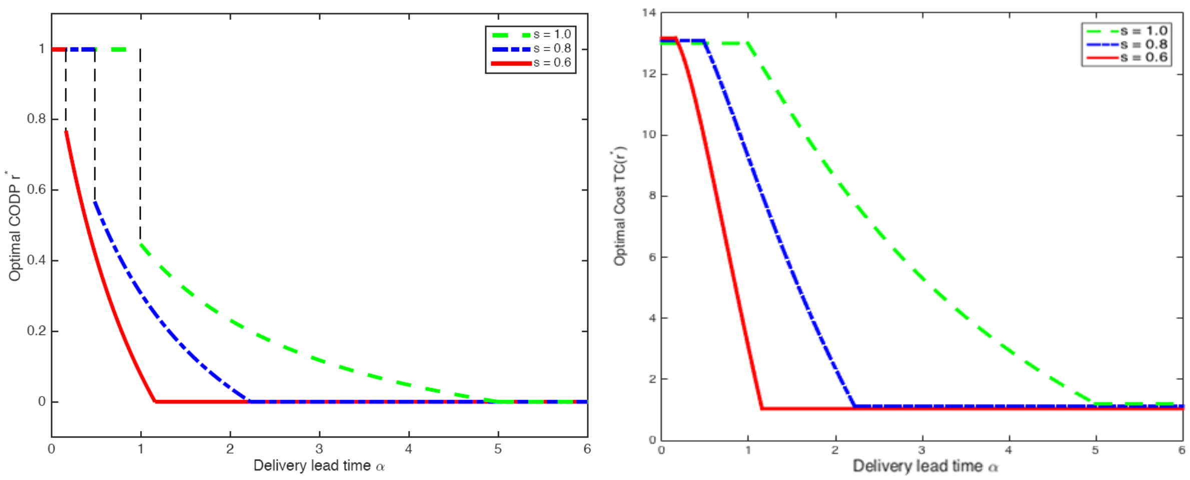
Figure 3(a) demonstrates how the demand arrival rate affects the optimal CODP over a range of the delivery lead time. It is easy to observe that as the demand arrival rate increases, the optimal policy is to delay the CODP position. The phenomenon reflects that when the delivery lead time is short, it is optimal to delay product differentiation to late in the total process as the demand arrival rate increases. On the other hand, when the delivery lead time is long, the optimal strategy is pure MTO, which is not affected by the demand arrival rate.

Substituting r^* into the total cost, as shown in Figure 3(b), we obtain that $TC(r^*)$ is increasing with the demand arrival rate. Especially, the products with medium requirement for delivery lead time are more sensitive to the demand arrival rate than low or high requirement. Therefore, the manufacturer needs to pay more attention to the demand fluctuation with the medium delivery lead time.



(a) Effect of demand arrival rate on r^* (b) Effect of demand arrival rate on TC

FIGURE 3. Effect of demand arrival rate



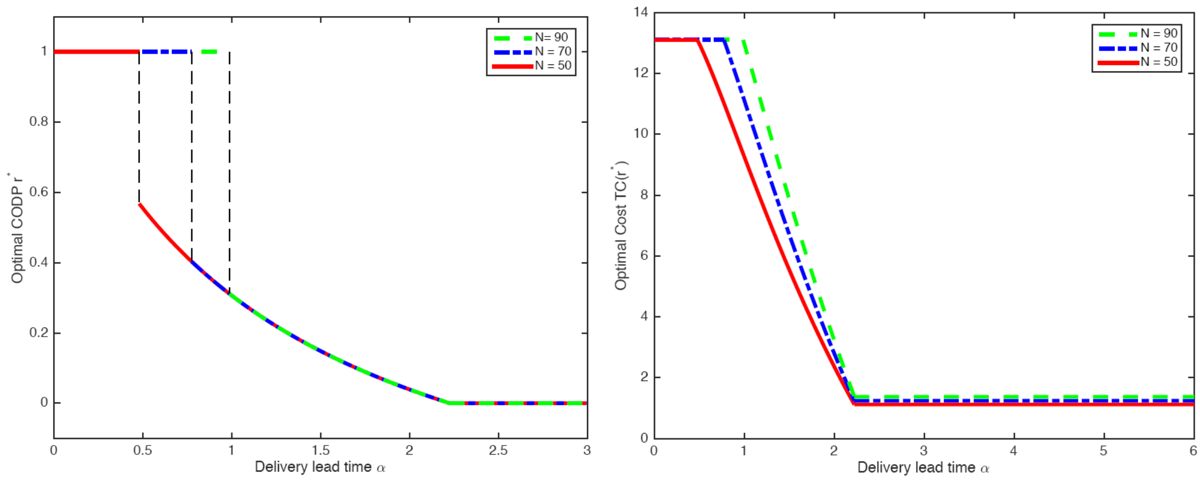
(a) Effect of production time on r^* (b) Effect of production time on TC

FIGURE 4. Effect of production time

As shown in Figure 4(a), it indicates that when delivery lead time is relatively short, the manufacturer will change his production mode from MTS/MTO mixed production mode to push CODP to the end process as the production time becomes longer. The reason is that the longer production time causes the longer queue and the lower satisfaction. Thus, the CODP should be pushed to the end of production line, which shortens the queue length.

Substitute r^* into the total cost. As shown in Figure 4(b), when the delivery lead time is short, the manufacturer's total cost will decrease slightly as the production time increases. The reason is that CODP is set at the end of production line, and the total cost is the sum of the cost of the generic stage and the cost of product/process redesign. Although the quantity of WIP at the generic stage is increasing when production time is longer, the increase rate is less than the decrease rate of inventory cost. When the delivery lead time becomes larger, $TC(r^*)$ will decrease with the production time. The analysis is just like the previous part of the demand arrival rate. The drop rate of the total cost with medium requirement for delivery lead time is obviously higher than low requirement interval. Hence, for the products with medium requirement, the manufacturer should increase investment on the production equipment and use the advanced production technology to improve productivity and reduce production time.

Figure 5(a) shows that the optimal CODP varies with delivery lead time under the different values of the product types N. It is apparent that there are different changing behaviors of r^*. When the delivery lead time is relatively short, the optimal CODP is set at the end of production line as the product types increasing. If customers are willing to wait, however, the manufacturer always chooses MTO production mode, which is also not affected by the product types.



(a) Effect of product types on r^* (b) Effect of product types on TC

Figure 5. Effect of product types

Substituting the optimal CODP into the total cost, we obtain the impact of the product types and the delivery lead time on the total cost as shown in Figure 5(b). The total cost is an increasing function of the product types, which can be found obviously from the mathematical model in Section 3. Combining the simulation result in Figure 5(b), however, the total cost is not affected by the product types when the delivery lead time is relatively short. The reason is that the final customization stage's cost equals 0 at this case; thus, the product types have no effect on total cost.

5. **Conclusions.** The CODP position is a critical question when achieving mass customization, personalization and small batch production. In this paper, we study the location problem of CODP in a postponement production system subject to a constraint on the delivery lead time. We learn that the optimal policy for the system is MTO when the customers are willing to wait. We also conduct the sensitivity analysis of three factors: the demand arrival rate, the production time and the product types. Our simulation shows that CODP moves to the end of production line when the demand arrival rate becomes larger, the production time delays or the product types increase. In addition, the manufacturer always hopes that his customers have enough patience to wait in order to minimize his total cost. In the future research, if the demand side or the supply side is disruptive, how to optimize the CODP position to strengthen self-repair ability and self-adaptive ability of supply chain may be interesting to some managers.

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