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CONTROL OF A MAGNETIC LEVITATION SYSTEM BY ASPR BASED ADAPTIVE OUTPUT FEEDBACK CONTROL WITH AN ADAPTIVE PARALLEL FEEDFORWARD COMPENSATOR

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Abstract. *In this paper, experimental validation of a control scheme of adaptive output feedback based output tracking control with an adaptive parallel feedforward compensator (PFC) for minimum-phase systems is shown. It is well recognized that one can design a simple and robust adaptive output feedback control system for uncertain system by introducing a parallel feedforward compensator (PFC) so as to render the resulting augmented system with the PFC almost strictly positive real (ASPR). We will confirm the effectiveness of adaptive output feedback control system with adaptive PFC, which remains the almost strictly positive realness (ASPR-ness) of the augmented system adaptively, through experiments for control of a magnetic levitation system.*

Keywords: Adaptive control, Parallel feedforward compensator, Magnetic levitation system

1. **Introduction.** It is well recognized that one can design a simple and robust adaptive output feedback control system for uncertain system by introducing a parallel feedforward compensator (PFC) so as to render the resulting augmented system with the PFC 'almost strictly positive real (ASPR)' [1, 2, 3, 4, 5]. The system is said to be ASPR if there exists a static output feedback such that the resulting closed-loop system is strictly positive real (SPR) [6], and the conditions for the system to be ASPR are (1) the system is minimum-phase, (2) the system has a relative degree of 1, (3) the high-frequency gain of the system is positive.

For ASPR system, one can easily design stable adaptive output feedback control system unlike the conventional adaptive controls including model reference adaptive control (MRAC). So called 'Simple Adaptive Control' [2] is the typical adaptive control strategy utilizing ASPR-ness of the system. Unfortunately, however, since most practical systems do not satisfy the ASPR condition, it was severe restriction to practical application of the ASPR based adaptive control strategy. With this in mind, a strategy introducing a PFC [1, 4, 7, 8] has been proposed and the effectiveness of the adaptive output feedback control with the PFCs has been confirmed through several kinds of numerical simulations and experiments [2, 3, 4, 5, 9, 10]. However, how to design adequate PFC has been still open problems.

As a solution to this issue, adaptive-type PFC design schemes including data-driven PFC design scheme [11, 12, 13] and direct adaptive PFC design scheme [13] have been proposed. However, the methods in [11, 12] did not guarantee the stability of the control system exactly and the method in [13] was only for regulation problem. Recently, the method in [13] has been expanded for output tracking problem with adequate feedforward input for the output tracking to the original practical system, not for the augmented system with a PFC, and the effectiveness of the method has been confirmed through numerical simulations [14].

In this paper, we show the experimental control results of magnetic levitation system with the method in [14] and validate the effectiveness of the adaptive output feedback control with adaptive PFC. The main objective of this paper is to show availability of adaptive control in practical situation through magnetic levitation system.

The organization of the paper is as follows. In Section 2, the magnetic levitation system dealt with in this work is shown. The adaptive control system design procedure is presented in Section 3. In Section 3, adaptive PFC design and overall adaptive control system will be given. Experimental validation results are shown in Section 4 and the effectiveness of the proposed adaptive control method will be validated. Finally, conclusions are given in Section 5.

2. **Magnetic Levitation System.** Consider a magnetic levitation system as shown in Figure 1.

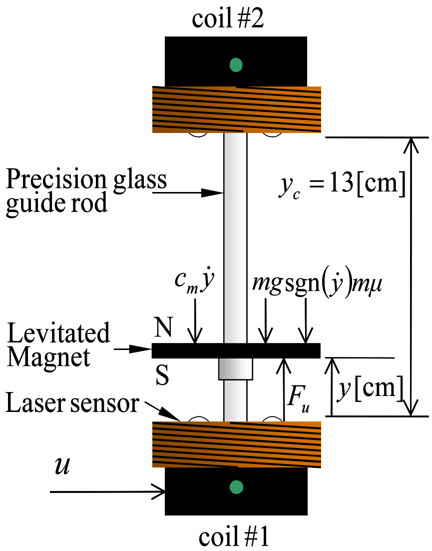


FIGURE 1. Magnetic levitation system

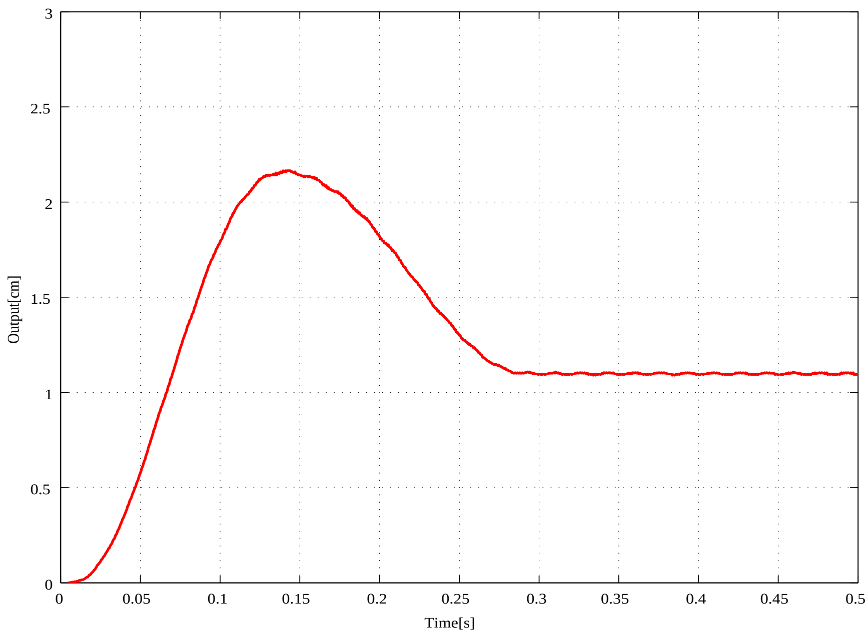
The equation of motion for this magnetic levitation system can be obtained as

$$\begin{aligned} m\ddot{x}_1(t) &= F_u(t) - c_m\dot{x}_1(t) - mg - \mathrm{sgn}(\dot{x}_1)m\mu \\ F_u(t) &= \frac{u(t)}{a(x_1(t)+b)^N}, \quad y(t) = 10^2 x_1(t) \end{aligned} \tag{1}$$

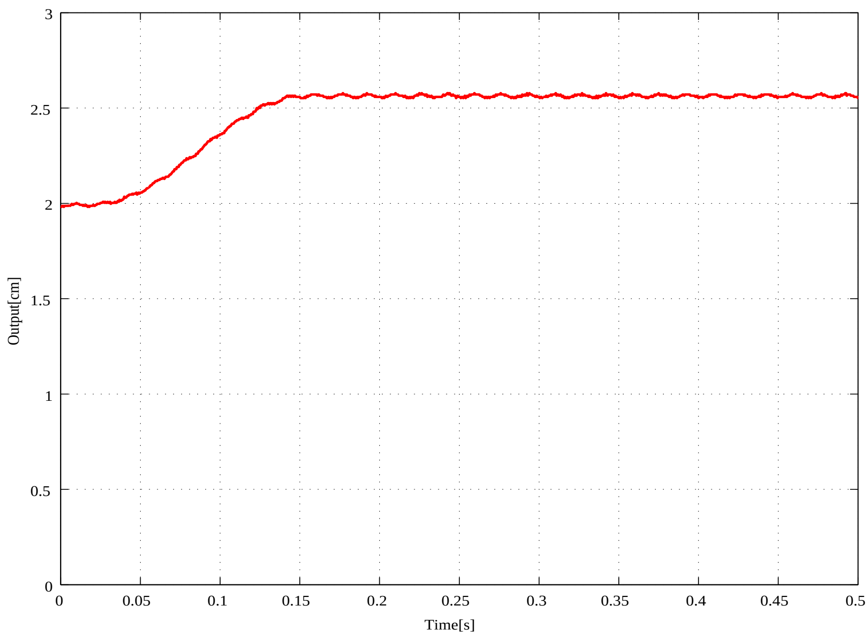
where $y(t)$ is the displacement of the magnet, m (0.12[kg]) is the mass of the magnet, g is the attraction of gravity, c_m [Ns/m] is the damping coefficient, μ is the coefficient of friction between the glass rod and the magnetic, and $F_u(t)$ [N] is the magnetic force generated from the coil. $u(t)$ [V] is the control input voltage, a and b are appropriate constants and N is an appropriate integer. We suppose that all the system parameters m, c_m, μ and a, b, N are unknown.

The magnetic levitation system has strong nonlinearity as given by $F_u(t)$ in (1). Figure 2 shows experimental results of step response of the considered magnetic levitation system. According to the difference of the set point magnitude, the step response shows quite different response for the nonlinearity of the magnetic levitation system.

The objective considered here is to design an adaptive control system of this magnetic levitation system via adaptive output feedback control based on ASPR properties of the augmented system with adaptively adjusted parallel feedforward compensator [14].



(a) Initial magnet position of 0.0 cm



(b) Initial magnet position of 2.0 cm

FIGURE 2. Step responses of the magnetic levitation system

3. **Adaptive Control System Design.** For the control of magnetic levitation system, in this paper, we consider designing a two-degree of freedom ASPR based adaptive output feedback control system as shown in Figure 3, where $G(s)$ is a linear approximated model of the magnetic levitation system around any operating point and $H(s, \boldsymbol{\rho})$ is a PFC rendering the resulting augmented control system ASPR. Since the magnetic levitation system is a nonlinear system, we consider adaptively adjusting PFC parameter vector $\boldsymbol{\rho}$ so as to maintain the ASPR-ness of the augmented system with the PFC.

3.1. Design of PFC output signal $\bar{y}_f(t)$.

3.1.1. *Ideal PFC representation.* Suppose that the considered magnetic levitation system can be modelled by a linear system $G(s)$ at any operating point. For this approximated

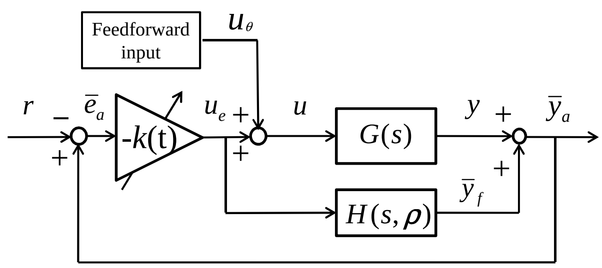


FIGURE 3. Block diagram of considered adaptive control system

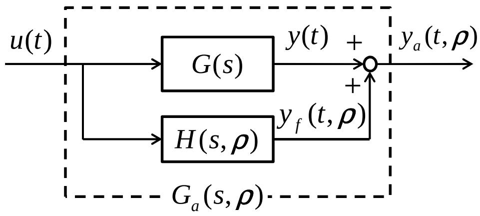


FIGURE 4. Augmented system with a PFC

model, consider introducing a PFC: $H(s, \boldsymbol{\rho})$ which is parameterized by $\boldsymbol{\rho}$, in parallel with the system $G(s)$ as shown in Figure 4.

The resulting augmented system denoted by $G_a(s, \boldsymbol{\rho})$, with $y_a(t, \boldsymbol{\rho})$ as the output and $u(t)$ as the input, can be given by

$$G_a(s, \boldsymbol{\rho}) = G(s) + H(s, \boldsymbol{\rho}) \tag{2}$$

Now, define the ideal output for a given ideal ASPR model $G_a^*(s)$ with the input $u(t)$ by

$$y_a^*(t) := G_a^*(s)[u(t)] \tag{3}$$

where the notation of $W(s)[u(t)]$ denotes the output of the system $W(s)$ with the input $u(t)$.

Note that $G_a^*(s)$ is a desired ASPR model which is given by control system designer. Since $G_a^*(s)$ is known, the signal $y_a^*(t)$ is available, so that ideal PFC output defined by

$$y_f^*(t) := y_a^*(t) - y(t) \tag{4}$$

using the practical output $y(t)$ of the controlled system is also available.

If the ideal PFC model, which is unknown, is given by the following form:

$$H^*(s) = \frac{N_H^*(s)}{D_H^*(s)} = \frac{b_1^* s^{n_h-1} + b_2^* s^{n_h-2} + \cdots + b_{n_h}^*}{s^{n_h} + a_1^* s^{n_h-1} + \cdots + a_{n_h}^*} \tag{5}$$

as an n_hth order compensator, the ideal PFC output is also given by

$$y_f^*(t) = H^*(s)[u(t)] \tag{6}$$

Introducing the following stable filter of order of n_h to both sides in (6):

$$\frac{1}{F(s)} = \frac{1}{s^{n_h} + f_1 s^{n_h-1} + \cdots + f_{n_h}} \tag{7}$$

we have

$$\frac{D_H^*(s)}{F(s)}\left[y_f^*(t)\right] = \frac{N_H^*(s)}{F(s)}[u(t)] \tag{8}$$

and thus it follows that the ideal PFC output can be represented by

$$y_f^*(t) = \frac{z_1^* s^{n_h-1} + \cdots + z_{n_h}^*}{F(s)} \left[y_f^*(t)\right] + \frac{N_H^*(s)}{F(s)}[u(t)] = \boldsymbol{\rho}^{*T} \boldsymbol{z}\left(y_f^*, u\right) \tag{9}$$

where

$$\boldsymbol{\rho}^* = \left[z_1^* \ z_2^* \ \cdots \ z_{n_h}^* \ b_1^* \ b_2^* \ \cdots \ b_{n_h}^*\right]^T, \quad (z_i^* = f_i - a_i^*)$$

and

$$\boldsymbol{z}(y_f^*, u) = \left[\frac{s^{n_h-1}}{F(s)}\left[y_f^*\right], \ \cdots, \ \frac{1}{F(s)}\left[y_f^*\right], \frac{s^{n_h-1}}{F(s)}[u], \ \cdots, \ \frac{1}{F(s)}[u]\right]^T \tag{10}$$

This is a parametric representation of the ideal PFC.

3.1.2. *Design of PFC signal.* The PFC signal $\bar{y}_f(t)$ in the control system given in Figure 3 is designed as follows.

Define a signal $y_f(t, \boldsymbol{\rho})$ with a parameter $\boldsymbol{\rho}$ by

$$y_f(t, \boldsymbol{\rho}) = \boldsymbol{\rho}^T \boldsymbol{z}(t), \quad \boldsymbol{z}(t) = \boldsymbol{z}\left(y_f^*, u\right) \tag{11}$$

It follows from (9) that $y_f(t, \rho^*) = y_f^*(t)$ with $\boldsymbol{\rho} \equiv \boldsymbol{\rho}^*$.

Further define $y_{f\theta}(t, \boldsymbol{\rho})$ as a PFC output with the parameter $\boldsymbol{\rho}$ and input $u_\theta(t)$ by

$$y_{f\theta}(t, \boldsymbol{\rho}) = \boldsymbol{\rho}^T \boldsymbol{z}_\theta(t) \tag{12}$$

where

$$\boldsymbol{z}_\theta(t) = \boldsymbol{z}_\theta\left(y_{f\theta}, u_\theta\right) = \left[\frac{s^{n-1}}{F(s)}[y_{f\theta}], \ \cdots, \ \frac{1}{F(s)}[y_{f\theta}], \ \frac{s^{n-1}}{F(s)}[u_\theta], \ \cdots, \ \frac{1}{F(s)}[u_\theta]\right]^T \tag{13}$$

$\boldsymbol{\rho} = \boldsymbol{\rho}^*$ leads that $y_f^*(t) = y_{f\theta}(t, \boldsymbol{\rho}^*) = \boldsymbol{\rho}^{*T} \boldsymbol{z}_\theta^*(t)$ with $\boldsymbol{z}_\theta^*(t) = \boldsymbol{z}_\theta\left(y_{f\theta}^*, u_\theta\right)$.

With these definitions, the ideal PFC output $\bar{y}_f^*(t) = \bar{y}_f(t, \boldsymbol{\rho}^*)$ with the input $u_e(t) = u(t) - u_\theta(t)$ is obtained by

$$\begin{aligned} \bar{y}_f^*(t) &= \bar{y}_f(t, \boldsymbol{\rho}^*) = H(s, \boldsymbol{\rho}^*)[u_e(t)] = H(s, \boldsymbol{\rho}^*)[u(t) - u_\theta(t)] \\ &= y_f(t, \boldsymbol{\rho}^*) - y_{f\theta}(t, \boldsymbol{\rho}^*) = y_f^*(t) - y_{f\theta}^*(t) \end{aligned} \tag{14}$$

where $H(s, \boldsymbol{\rho}^*) = H^*(s)$.

Since this ideal PFC output is not available, we design a PFC output signal as follows by using the adaptively estimated parameter $\boldsymbol{\rho}(t)$ of $\boldsymbol{\rho}^*$.

$$\bar{y}_f(t) = y_f(t) - y_{f\theta}(t) \tag{15}$$

with

$$y_f(t) = G_a^*(s)\left[\boldsymbol{\rho}(t)^T \bar{\boldsymbol{z}}(t)\right], \quad \bar{\boldsymbol{z}}(t) = G_a^{*-1}(s)[\boldsymbol{z}(t)] \tag{16}$$

and

$$y_{f\theta}(t) = G_a^*(s)\left[\boldsymbol{\rho}(t)^T \bar{\boldsymbol{z}}_\theta(t)\right], \quad \bar{\boldsymbol{z}}_\theta(t) = G_a^{*-1}(s)[\boldsymbol{z}_\theta(t)] \tag{17}$$

3.2. **Adaptive controller design.** Using the designed PFC signal $\bar{y}_f(t)$ in (15), the adaptive controller is designed as follows:

$$\begin{aligned} u(t) &= u_e(t) + u_\theta(t) \\ u_e(t) &= -k(t)\bar{e}_a(t), \quad u_\theta(t) = u_a^*(t) + \boldsymbol{\rho}^T(t)\bar{\boldsymbol{z}}_\theta(t) \\ \bar{e}_a(t) &= \bar{y}_a(t) - r(t), \quad \bar{y}_a(t) = y(t) + \bar{y}_f(t) \end{aligned} \tag{18}$$

where $u_a^*(t)$ is ideal control input satisfying $G_a^*(s)[u_a^*(t)] = r(t)$, i.e., $u_a^*(t) = G_a^{*-1}(s)[r(t)]$, $r(t)$ is a reference signal for which the systems output is required to follow, and feedback gain $k(t)$ and PFC parameter $\boldsymbol{\rho}(t)$ are adaptively adjusted by the following parameter adjusting laws:

$$\dot{k}(t) = \gamma \bar{e}_a^2(t) - \sigma k(t) \tag{19}$$

$$\dot{\boldsymbol{\rho}}(t) = -\Gamma_H \bar{\boldsymbol{z}}(t)\bar{e}_a(t) - \sigma_H \boldsymbol{\rho}(t) \tag{20}$$

Remark 3.1. *In the case where* $\boldsymbol{\rho}(t) \equiv \boldsymbol{\rho}^*$*, the feedforward input is obtained by*

$$\begin{aligned} u_\theta(t) &= u_a^*(t) + \boldsymbol{\rho}^{*T} G_a^{*-1}(s)[\boldsymbol{z}_\theta(t)] \\ &= u_a^*(t) + G_a^{*-1}(s)\left[y_{f\theta}(t, \boldsymbol{\rho}^*)\right] \\ &= u_a^*(t) + G_a^{*-1}(s)[H(s, \boldsymbol{\rho}^*)[u_\theta(t)]] \end{aligned} \tag{21}$$

and thus

$$G_a^{*-1}(s)\left(G_a^*(s) - H(s, \boldsymbol{\rho}^*)\right)[u_\theta(t)] = u_a^*(t) \tag{22}$$

It follows from the fact that $G_a^*(s) - H(s, \boldsymbol{\rho}^*) = G(s)$

$$u_\theta(t) = G^{-1}(s) G_a^*(s)[u_a^*(t)] = G^{-1}(s)[r(t)] \tag{23}$$

This means that $u_\theta(t)$ *with* $\boldsymbol{\rho}(t) \equiv \boldsymbol{\rho}^*$ *is the ideal feedforward input which attains* $r(t) = G(s)[u_\theta(t)]$*, and in the method,* $G(s)^{-1}(s)$ *should be stable for obtaining bounded ideal feedforward input.*

Concerning the boundedness of all signals in the obtained adaptive control system, we have the following theorem.

Theorem 3.1. *For a minimum-phase controlled system, design adaptive control system as shown in Figure 3 with adaptive controller (18) and adaptive adjusting law (20). Then all the signals in the control system are bounded.*

Proof: See [14].

4. Experimental Results of Magnetic Levitation System Control.

Figure 5 shows the experimental device of magnetic levitation system. In the experiment, we suppose that all the parameters of the system are unknown, but we know that the system can roughly be approximated by second order system.

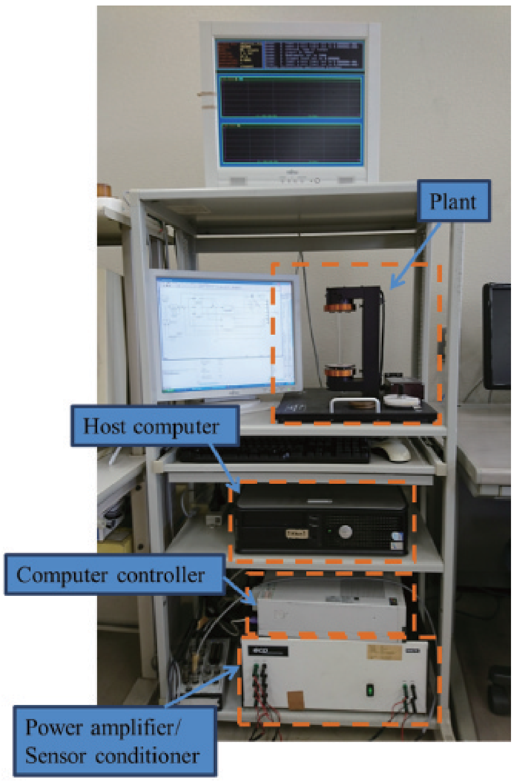


FIGURE 5. Experimental device of magnetic levitation system

4.1. **Control system setup.** For the magnetic levitation system, we set

$$G_a^*(s) = \frac{200s + 5}{s^2 + 60s + 5} \tag{24}$$

as a given desired ASPR model, and second order PFC was considered as the one which should be estimated.

Furthermore, the stable filter for estimating PFC parameter and designing control system was designed as follows in this experiment by supposing that the ideal PFC is given as second order.

$$\frac{1}{F(s)} = \frac{1}{s^2 + 50s + 1000} \tag{25}$$

Remark 4.1. *In order to design adequate PFC adaptively, the ideal ASPR augmented system's response should be close to the original system's response so as to alleviate the effects from the obtained PFC output to actual output. So, for the approximated second order system, we set second order system as an ideal ASPR model.*

4.2. **Experimental results.** In order to confirm the effectiveness of the proposed method, we implemented the following experiment.

We consider the following combined reference signal with sinusoidal wave and step-type signal was considered.

$$r(t) = R(s)[u_r(t)]\ , \quad R(s) = \frac{1}{0.2s + 1}$$

$$u_r(t) = \begin{cases} 1.5 \ (0 \le t < 2.25) \\ \sin 7t + 1.5 \ (2.25 \le t < 4) \\ 1.0 \ (4 \le t < 5.5) \\ 2.0 \ (5.5 \le t < 7) \\ 1.3 \ (7 \le t < 8.5) \\ 1.7 \ (8.5 \le t < 12.25) \\ \sin 7t + 1.5 \ (12.25 \le t < 14) \\ 0.75 \ (14 \le t < 15.5) \\ 2.25 \ (15.5 \le t < 17) \\ 1.2 \ (17 \le t < 18.5) \\ 1.8 \ (18.5 \le t) \end{cases}$$

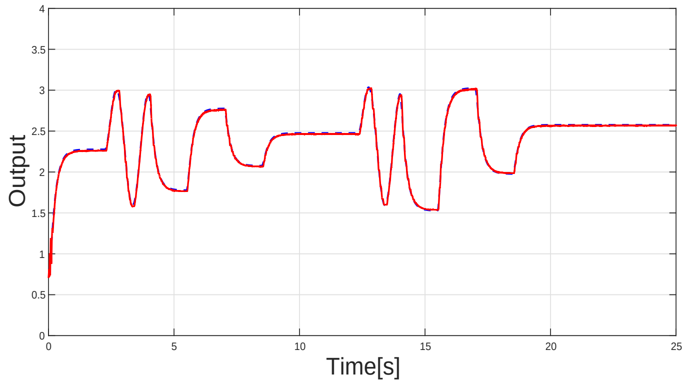
The design parameters for adaptive controller were set by

$$\begin{aligned} \gamma &= 3.0 \times 10^4, \quad \sigma = 0.02, \quad \sigma_H = 0.05 \\ \Gamma_H &= \mathrm{diag}\left[5.0 \times 10^4,\ 2.5 \times 10^4,\ 2.5 \times 10^5,\ 3.5 \times 10^4\right] \end{aligned}$$

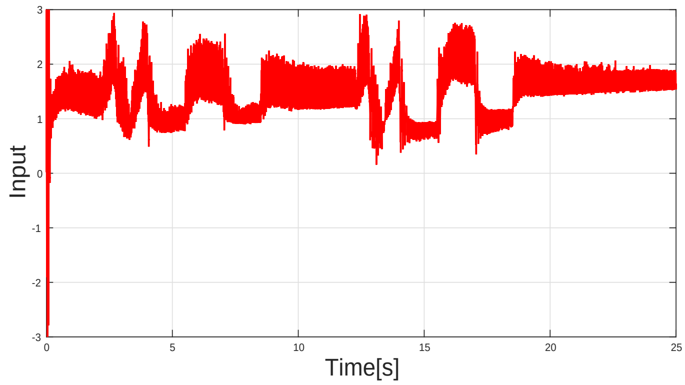
For this case, we validated the control performance of the considered adaptive control by comparing with conventional PID controller. The PID controller was set as follows:

$$\begin{aligned} u(t) &= \left(K_P + K_D s + K_I \frac{1}{s}\right)[r(t) - y(t)] \\ K_P &= 0.08, \quad K_D = 8, \quad K_I = 25 \end{aligned}$$

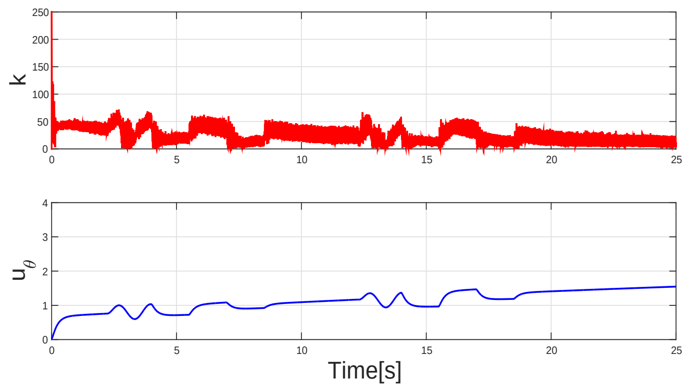
Figure 6 shows the experimental results with the proposed adaptive control method and Figure 7 shows the experimental results with PID controller. Although PID control shows not so bad results by tuning the PID controller adequately, a tangible error appears in the transient state and large variance of the control input appears. On the other hand, the proposed adaptive controller shows better results in the whole operation with small input variance. This shows the practical availability of the proposed adaptive control scheme.



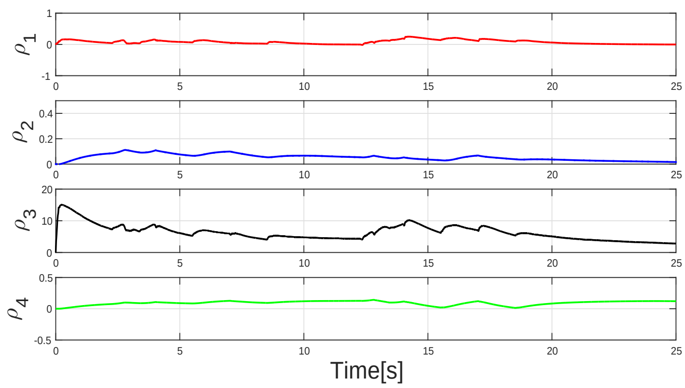
(a) Output $y(t)$ (solid line) and reference signal $r(t)$ (dashed line)



(b) Control input $u(t)$

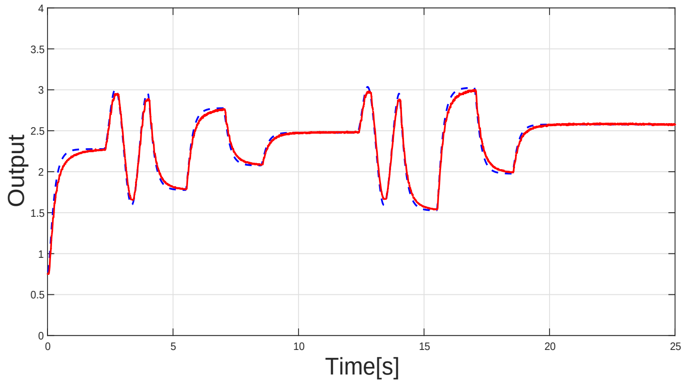


(c) Feedback gain $k(t)$ and feedforward input $u_\theta(t)$

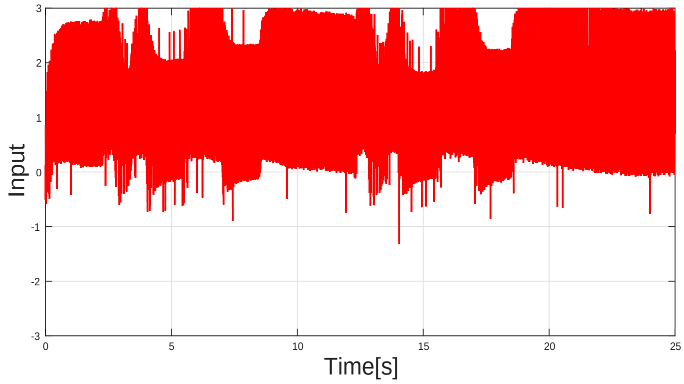


(d) PFC parameter vector $\boldsymbol{\rho}(t)$

FIGURE 6. Output of the magnetic levitation system with proposed adaptive control



(a) Output $y(t)$ (solid line) and reference signal $r(t)$ (dashed line)



(b) Control input $u(t)$

FIGURE 7. Output of the magnetic levitation system with PID

5. **Conclusions.** In this paper, a two degree of freedom adaptive output feedback control based on augmented system's ASPR-ness with adaptive PFC is considered in order to control a magnetic levitation system. The effectiveness of the considered adaptive control was validated through experiments and it was confirmed that by adaptively adjusting PFC parameters and feedback gain, one can attain a better control performance for any operating point to nonlinear magnetic levitation system. Through this experimental validation, we showed availability of adaptive control in practical situation through magnetic levitation system. The presented method is a useful and powerful method; however, it is available only for minimum-phase systems and basically for linear systems. It will be future work to expand the method for non-minimum phase and/or nonlinear systems.

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REFERENCES

[1] I. Barkana, Parallel feedforward and simplified adaptive control, *International Journal of Adaptive Control and Signal Processing*, vol.1, no.2, pp.95-109, 1987.
[2] H. Kaufman, I. Barkana and K. Sobel, *Direct Adaptive Control Algorithms*, 2nd Edition, Springer, 1997.
[3] I. Barkana, I. Rusnak and H. Weiss, Almost passivity and simple adaptive control in discrete-time systems, *Asian Journal of Control*, vol.16, no.4, pp.1-12, 2014.
[4] I. Mizumoto and Z. Iwai, Simplified adaptive model output following control for plants with unmodelled dynamics, *Int. J. of Control*, vol.64, no.1, pp.61-80, 1996.
[5] A. L. Fradkov and D. J. Hill, Exponentially feedback passivity and stabilizability of nonlinear systems, *Automatica*, vol.34, no.6, pp.697-703, 1998.

[6] I. Barkana, Positive realness in multivariable continuous-time systems, *Journal of the Franklin Institute*, vol.328, no.4, pp.403-418, 1991.
[7] Z. Iwai and I. Mizumoto, Realization of simple adaptive control by using parallel feedforward compensator, *Int. J. of Control*, vol.59, no.6, pp.1543-1565, 1994.
[8] A. L. Fradkov, Shunt output feedback adaptive controller for nonlinear plants, *Proc. of the 13th IFAC World Congress*, San-Francisco, vol.K, pp.367-372, 1996.
[9] I. Mizumoto, D. Ikeda, T. Hirahata and Z. Iwai, Design of discrete time adaptive PID control systems with parallel feedforward compensator, *Control Engineering Practice*, vol.18, no.2, pp.168-176, 2010.
[10] H. Kim, S. Kim, J. Back, H. Shim and J. Seo, Design of stable parallel feedforward compensator and its application to synchronization problem, *Automatica*, vol.64, pp.208-216, 2016.
[11] T. Takagi, S. Fukui, K. Yamanaka, I. Mizumoto and S. L. Shah, Performance-driven adaptive output feedback control for discrete-time systems with a PFC designed via FRIT approach, *ICIC Express Letters*, vol.6, no.5, pp.1147-1154, 2012.
[12] T. Takagi, I. Mizumoto and J. Tsunematsu, Performance-driven adaptive output feedback control with direct design of PFC, *Journal of Robotics and Mechatronics*, vol.27, no.5, pp.461-468, 2015.
[13] T. Takagi and I. Mizumoto, Adaptive output feedback control with adaptive PFC and its stability analysis, *ICIC Express Letters, Part B: Applications*, vol.6, no.12, pp.3189-3195, 2015.
[14] I. Mizumoto, N. Kawabe and M. Sano, ASPR based adaptive output tracking control system design with an adaptive PFC for minimum-phase systems, *Proc. of the 2016 International Conference on Advanced Mechatronic Systems*, Melbourne, Australia, pp.224-229, 2016.