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ROBUST OPTIMAL GUARANTEED COST CONTROL FOR A CLASS OF SWITCHED FUZZY SYSTEMS WITH TIME DELAYS

Yudong Jiang[1] and Jinming Luo[2,*]

[1]College of Aerospace Engineering
Nanjing University of Aeronautics and Astronautics
No. 29, Yudao Ave., Qinhuai Dist., Nanjing 210016, P. R. China

[2]College of Automation
Shenyang Institute of Engineering
No. 18, Puchang Rd., Shenbei New Dist., Shenyang 110136, P. R. China
[*]Corresponding author: luojinming0830@163.com

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Abstract. *This paper investigates a robust optimal guaranteed cost control problem for a class of switched fuzzy systems with time delays. The asymptotical stability of guaranteed cost control problem for subsystems is not assumed. Based on the multiple Lyapunov functions method, we design a non-fragile state feedback controller and a switching law such that the closed-loop system is asymptotically stable and the guaranteed cost function possesses an upper bound. Then, an optimization problem of the non-fragile guaranteed cost control is solved. Simulation results verify the feasibility and effectiveness of the proposed approach.*
Keywords: Switched fuzzy systems, Robust control, Guaranteed cost control, Non-fragility, Uncertainty, Multiple Lyapunov functions, Optimization, Time delays

1. **Introduction.** Switched systems belong to a special class of hybrid systems. A switched system consists of a family of continuous-time or discrete-time subsystems and a switching rule that orchestrates the switching between them. Due to their success in practical applications and importance in theory development, switched systems have been attracting considerable attention during the last decades. There are a large number of results on analysis and synthesis of switched systems [1-4].

As we know, fuzzy systems are very complex nonlinear systems. Since its first proposition in 1985 [5], Takagi-Sugeno (T-S) fuzzy model has been proven as an effective approach to represent a nonlinear system, and various works have been done [6-8]. In this type of fuzzy model, local dynamics in different state space regions are represented by linear models. The overall model of the system is achieved by fuzzy blending of these linear models through nonlinear fuzzy membership functions. Besides, time delays are inherent features of many physical process and also are big sources of instability and poor performances. The T-S fuzzy model with time delays is used as the model for the time-delay nonlinear system [9].

Especially, if each subsystem of a switched system is a fuzzy system, then the switched system is a switched fuzzy system. A switched fuzzy system revolves according to the "hard switching" between the subsystems which are fuzzy systems and the "soft switching" among the linear models within a T-S fuzzy model. These two switching strategies and their interaction lead to very complex behaviors of switched fuzzy systems, so the results for switched fuzzy systems in the literature are rather limited [10,11].

On the other hand, it is well understood that parametric uncertainties are principal factors responsible for the degraded stability and performance. We usually consider the system model uncertainty only; however, in practice, the controller has a certain degree of errors. These errors could also destabilize the closed-loop system. A controller for a given plant is thus expected to be insensitive or non-fragile to errors. Therefore, the robust stability against parametric uncertainties not only in the plant but also in the implementation of the controller is an important problem [12,13].

Furthermore, it is desirable that a system can be asymptotically stable and provided with a level of performance index. Non-fragile guaranteed cost control problems are studied in fuzzy systems [14] and in switched systems [15]. To the best of our knowledge, for uncertain switched fuzzy systems with time delays, no results on the non-fragile guaranteed cost control problem have been reported up to now.

Motivated by the above considerations, for a class of switched fuzzy systems with time delays, this paper studies the optimal guaranteed cost control problem which is robust not only for the system uncertainty but also for the controller fragility. None of subsystems are assumed to be asymptotically stable. A sufficient condition for asymptotical stability of the closed-loop system and an upper bound of the guaranteed cost function are established based on the multiple Lyapunov functions method. Meanwhile, a non-fragile state feedback controller and a switching law are designed. Compared with the existing results on switched fuzzy systems, the results of this paper have two distinct features. Firstly, the optimal guaranteed cost control problem is studied for switched fuzzy systems for the first time, especially for systems with delays. Secondly, unlike the classical guaranteed cost problem, the fuzzy controller gain uncertainty affects system matrices and enters the cost function.

2. **Problem Statement and Preliminaries.**

Consider a class of uncertain nonlinear switched systems with time delays which can be described by the following switched T-S fuzzy model.

Plant Rule:

$$\begin{aligned} R_\sigma^i &: \text{If } x_1 \text{ is } \Omega_{\sigma 1}^i,\ x_2 \text{ is } \Omega_{\sigma 2}^i,\ \ldots,\ x_n \text{ is } \Omega_{\sigma n}^i, \\ &\text{Then } \dot{x}(t) = (A_{\sigma i} + \Delta A_{\sigma i}(t))x(t) + A_{d\sigma i}x(t-\tau) + B_{\sigma i}u_\sigma(t) \end{aligned} \tag{1}$$

where $\sigma = \sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, N\}$ is a piecewise constant function called a switching signal, corresponding to it, the switching sequence

$$\sum = \{x_0; (l_0, t_0), (l_1, t_1), \ldots, (l_k, t_k), \ldots | l_k \in M\}$$

means that the l_kth subsystem is active with the initial state x_0 and initial time t_0 when $t \in [t_k, t_{k+1})$; $i = 1, 2, \ldots, r_l$, r_l is the number of inference rules; R_l^k denotes the ith fuzzy inference rule; n is the number of state variables; Ω_{ln}^i represents the fuzzy set; $x(t) \in R^n$ is the system state; $u_l(t)$ is the control input; A_{li}, B_{li} and A_{dli} are constant matrices with appropriate dimensions; τ denotes the bounded constant time delay, and τ is non-negative integer. $\Delta A_{li}(t)$ is a time varying matrix function representing the uncertainty of the system.

By the singleton fuzzification, product inference engine and center average defuzzification, the globe model of the system (1) can be denoted as

$$\dot{x}(t) = \sum_{i=1}^{r_\sigma} h_{\sigma i}(x(t))[(A_{\sigma i} + \Delta A_{\sigma i}(t))x(t) + A_{d\sigma i}x(t-\tau) + B_{\sigma i}u_\sigma(t)] \tag{2}$$

In the model (2), $h_{\sigma i}(x(t))$ can be stated as

$$h_{\sigma i}(x(t)) = \frac{\prod_{p=1}^n \mu_{\sigma p}^i(x_p)}{\sum_{i=1}^{r_\sigma} \prod_{p=1}^n \mu_{\sigma p}^i(x_p)} \tag{3}$$

where $\mu_{\sigma p}^i(x_p)$ denotes the membership function of x_p in the fuzzy set $\Omega_{\sigma p}^i$ for $p = 1, 2, \ldots, n$. For simplicity, $h_{\sigma i}(t)$ denotes $h_{\sigma i}(x(t))$, $0 \le h_{\sigma i}(t) \le 1$, $\sum_{i=1}^{r_\sigma} h_{\sigma i}(t) = 1$.

Based on the parallel distributed compensation (PDC) technology [16], the state feedback controller is considered.

Controller Rule:

$$R_\sigma^i : \text{If } x_1 \text{ is } \Omega_{\sigma 1}^i,\ x_2 \text{ is } \Omega_{\sigma 2}^i,\ \ldots,\ x_n \text{ is } \Omega_{\sigma n}^i, \\ \text{Then } u_{\sigma i}(t) = (K_{\sigma i} + \Delta K_{\sigma i}(t))x(t) \tag{4}$$

where $K_{\sigma i}$ is the controller gain to be designed, and $\Delta K_{\sigma i}(t)$ is the uncertainty of the controller gain.

Finally, the globe controller is represented by

$$u_\sigma(t) = \sum_{i=1}^{r_\sigma} h_{\sigma i}(t)(K_{\sigma i} + \Delta K_{\sigma i}(t))x(t) \tag{5}$$

Remark 2.1. *Usually, the controller gain uncertainty includes additive and multiplicative norm-bounded forms. In this paper, only an additive form is taken into consideration to solve the fragility problem.*

Definition 2.1. *The guaranteed cost function of the system (2) is given by*

$$J = \int_0^\infty \left[x^T(t)x(t) + u_\sigma^T(t)u_\sigma(t)\right]dt \tag{6}$$

Definition 2.2. *Consider the system (2). If there exist the state feedback controller $u_l(t)$ $(l \in M)$ for each subsystem, a switching law $\sigma(t)$ and a positive scalar J^* such that for all admissible uncertainties, the closed-loop system is asymptotically stable and the value of the function (6) satisfies $J \leq J^*$, then J^* is called a non-fragile guaranteed cost (NGC) and $u_l(t)$ is called a guaranteed cost control law.*

Remark 2.2. *The NGC problem is different from the classical guaranteed cost problem, because $\Delta K_{li}(t)$ not only affects system matrices but also enters the cost function J^*.*

The objective of this paper is to design a guaranteed cost control law and a switching law such that the system (2) is asymptotically stable and the guaranteed cost function (6) is no more than an NGC.

3. **Main Result.** In this section, we will present a solvability condition for an NGC problem based on the multiple Lyapunov functions method. Then, an optimal non-fragile guaranteed cost problem will be discussed.

Theorem 3.1. *For the system (2), if there exist non-positive scalar β_{lv}, positive definite matrices P_l, P_v, Q and matrices K_{li} $(l, v \in M, v \neq l)$ satisfying the following matrix inequalities*

$$\prod\nolimits_{lij} + T_{lij} + P_l S_{lij} P_l + R_{lij} + \sum_{v=1, v\neq l}^{N} \beta_{lv}(P_l - P_v) < 0 \tag{7}$$

where

$$\prod\nolimits_{lij} = P_l A_{li} + A_{li}^T P_l + P_l B_{li} K_{lj} + K_{lj}^T B_{li}^T P_l + P_l A_{lj} + A_{lj}^T P_l + P_l B_{lj} K_{li} + K_{li}^T B_{lj}^T P_l,$$

$$S_{lij} = D_{li} D_{li}^T + B_{li} D_{alj} D_{alj}^T B_{li}^T + D_{lj} D_{lj}^T + B_{lj} D_{ali} D_{ali}^T B_{lj}^T + A_{dli} Q^{-1} A_{dli}^T + A_{dlj} Q^{-1} A_{dlj}^T,$$

$$T_{lij} = 2E_{ali} E_{ali}^T + E_{li} E_{li}^T + 2E_{alj} E_{alj}^T + E_{lj} E_{lj}^T + 2I + 2Q,$$

$$R_{lij} = K_{lj}^T \left(I - D_{alj} D_{alj}^T\right)^{-1} K_{lj} + K_{li}^T \left(I - D_{ali} D_{ali}^T\right)^{-1} K_{li},$$

then the system (2) is asymptotically stable for the guaranteed cost control law (5) under the switching law

$$\sigma = \arg\min \left\{x^T(t) P_l x(t), l \in M\right\} \tag{8}$$

and the cost function (6) possesses an NGC $J^* = \min\left\{x_0^T P_l x_0, l \in M\right\}$ *for any nonzero initial state* x_0. *(I denotes identity matrix).*

Proof: Choose a Lyapunov function of the system (2)

$$V_l(x(t)) = x^T(t)P_l x(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds \tag{9}$$

and compute time derivative along the state variables of the system (2).

$$\begin{aligned}
\dot{V}_l(x(t)) &= \dot{x}^T(t)P_l x(t) + x^T(t)P_l\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) \\
&= \sum_{i=1}^{r_l}\sum_{j=1}^{r_l} h_{li}(t)h_{lj}(t)x^T(t)\{[A_{li} + \Delta A_{li}(t) + B_{li}(K_{lj} + \Delta K_{lj}(t))]P_l \\
&\quad + P_l[A_{li} + \Delta A_{li}(t) + B_{li}(K_{lj} + \Delta K_{lj}(t))]\}x(t) \\
&\quad + \sum_{i=1}^{r_l}\sum_{j=1}^{r_l} h_{li}(t)h_{lj}(t)\left[x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau)\right. \\
&\quad \left. + x^T(t-\tau)A_{dli}^T P_l x(t) + x^T(t)P_l A_{dli}x(t-\tau)\right] \\
&\le \sum_{i=1}^{r_l}\sum_{j=1}^{r_l} h_{li}(t)h_{lj}(t)x^T(t)\{[A_{li} + \Delta A_{li}(t) + B_{li}(K_{lj} + \Delta K_{lj}(t))]P_l \\
&\quad + P_l[A_{li} + \Delta A_{li}(t) + B_{li}(K_{lj} + \Delta K_{lj}(t))]\}x(t) \\
&\quad + \sum_{i=1}^{r_l}\sum_{j=1}^{r_l} h_{li}(t)h_{lj}(t)\left[x^T(t)Qx(t) + x^T(t)P_l A_{dli}Q^{-1}A_{dli}^T P_l x(t)\right] \\
&= \sum_{i=1}^{r_l} h_{li}^2(t)\left\{x^T(t)\left[I + (K_{li} + \Delta K_{li}(t))^T(K_{li} + \Delta K_{li}(t))\right.\right. \\
&\quad + P_l A_{li} + A_{li}^T P_l + P_l B_{li}K_{li} + K_{li}^T B_{li}^T P_l + P_l\Delta A_{li} + \Delta A_{li}^T P_l \\
&\quad \left. + P_l B_{li}\Delta K_{li} + \Delta K_{li}^T B_{li}^T P_l + Q + P_l A_{dli}Q^{-1}A_{dli}^T P_l\right] x(t) \\
&\quad \left. - x^T(t)x(t) - x^T(t)(K_{li} + \Delta K_{li}(t))^T(K_{li} + \Delta K_{li}(t))x(t)\right\} \\
&\quad + \sum_{i=1}^{r_l}\sum_{j=1}^{r_l} h_{li}(t)h_{lj}(t)\left\{x^T(t)\left[2I + (K_{li} + \Delta K_{li}(t))^T(K_{lj} + \Delta K_{lj}(t))\right.\right. \\
&\quad + (K_{lj} + \Delta K_{lj}(t))^T(K_{li} + \Delta K_{li}(t)) + P_l A_{li} + A_{li}^T P_l + P_l B_{li}K_{lj} \\
&\quad + K_{lj}^T B_{li}^T P_l + P_l\Delta A_{li} + \Delta A_{li}^T P_l + P_l B_{li}\Delta K_{lj} + \Delta K_{lj}^T B_{li}^T P_l + P_l A_{lj} \\
&\quad + A_{lj}^T P_l + P_l B_{lj}K_{li} + K_{li}^T B_{lj}^T P_l + P_l\Delta A_{lj} + \Delta A_{lj}^T P_l + P_l B_{lj}\Delta K_{li} \\
&\quad \left. + \Delta K_{li}^T B_{lj}^T P_l + 2Q + P_l A_{dli}Q^{-1}A_{dli}^T P_l + P_l A_{dlj}Q^{-1}A_{dlj}^T P_l\right] x(t) \\
&\quad - 2x^T(t)x(t) - x^T(t)(K_{li} + \Delta K_{li}(t))^T(K_{lj} + \Delta K_{lj}(t))x(t) \\
&\quad \left. - x^T(t)(K_{lj} + \Delta K_{lj}(t))^T(K_{li} + \Delta K_{li}(t))x(t)\right\} \\
&\le \sum_{i=1}^{r_l} h_{li}^2(t)\left\{x^T(t)\left[I + K_{li}^T\left(I - D_{ali}D_{ali}^T\right)^{-1}K_{li} + E_{ali}E_{ali}^T + P_l A_{li}\right.\right. \\
&\quad + A_{li}^T P_l + P_l B_{li}K_{li} + K_{li}^T B_{li}^T P_l + P_l D_{li}D_{li}^T P_l + P_l B_{li}D_{ali}D_{ali}^T B_{li}^T P_l \\
&\quad \left. + E_{ali}E_{ali}^T + E_{li}E_{li}^T + P_l A_{dli}Q^{-1}A_{dli}^T P_l + Q\right]x(t) \\
&\quad \left. - x^T(t)x(t) - x^T(t)(K_{li} + \Delta K_{li}(t))^T(K_{li} + \Delta K_{li}(t))x(t)\right\} \\
&\quad + \sum_{i=1}^{r_l}\sum_{j=1}^{r_l} h_{li}(t)h_{lj}(t)\left\{x^T(t)\left[2I + K_{lj}^T\left(I - D_{alj}D_{alj}^T\right)^{-1}K_{lj}\right.\right.
\end{aligned}$$

$$
\begin{aligned}
&+K_{li}^T\left(I-D_{ali}D_{ali}^T\right)^{-1}K_{li}+E_{ali}E_{ali}^T+E_{alj}E_{alj}^T+P_lA_{li}+A_{li}^TP_l\\
&+P_lB_{li}K_{li}+K_{li}^TB_{li}^TP_l+P_lD_{li}D_{li}^TP_l+P_lB_{li}D_{alj}D_{alj}^TB_{li}^TP_l+E_{alj}E_{alj}^T\\
&+E_{li}E_{li}^T+P_lA_{lj}+A_{lj}^TP_l+P_lB_{lj}K_{li}+K_{li}^TB_{lj}^TP_l+P_lD_{lj}D_{lj}^TP_l\\
&+P_lB_{lj}D_{ali}D_{ali}^TB_{lj}^TP_l+E_{ali}E_{ali}^T+E_{lj}E_{lj}^T+P_lA_{dli}Q^{-1}A_{dli}^TP_l\\
&+P_lA_{dlj}Q^{-1}A_{dlj}^TP_l+2Q\big]x(t)-2x^T(t)x(t)\\
&-x^T(t)(K_{li}+\Delta K_{li}(t))^T(K_{lj}+\Delta K_{lj}(t))x(t)\\
&-x^T(t)(K_{lj}+\Delta K_{lj}(t))^T(K_{li}+\Delta K_{li}(t))x(t)\big\}
\end{aligned}
$$

By the inequality (7), from the S-procedure, whenever $\beta_{lv}\leq 0$ we have

$$
\begin{aligned}
V_l(x(t)) <{}& \sum_{i=1}^{r_l}h_{li}^2(t)\left[-x^T(t)x(t)-x^T(t)(K_{li}+\Delta K_{li}(t))^T(K_{li}+\Delta K_{li}(t))x(t)\right]\\
&+\sum_{i<j}^{r_l}\sum_{j=1}^{r_l}h_{li}(t)h_{lj}(t)\big[-2x^T(t)x(t)-x^T(t)(K_{li}+\Delta K_{li}(t))^T(K_{lj}+\Delta K_{lj}(t))x(t)\\
&-x^T(t)(K_{lj}+\Delta K_{lj}(t))^T(K_{li}+\Delta K_{li}(t))x(t)\big]\\
={}&-\left[x^T(t)x(t)+u_l^T(t)u_l(t)\right]<0.
\end{aligned}
$$

Integrating both sides of the expression above yields

$$
\begin{aligned}
J &< -\int_0^{+\infty}\dot V_l(x(t))dt=-\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\dot V_{lk}(x(t))dt\\
&=-[V_{l_0}(x(t_1))-V_{l_0}(x(t_0))+V_{l_1}(x(t_2))-V_{l_1}(x(t_1))+\cdots]\\
&=V_{l_0}(x(t_0))=x_0^TP_{l_0}x_0\\
&=\min\left\{x_0^TP_lx_0,l\in M\right\}=J^*.
\end{aligned}
$$

Thus, the system (2) is asymptotically stable and the cost function (6) possesses an NGC under the switching law (8). This completes the proof.

Remark 3.1. *For non-switched fuzzy systems, [13] gives a stabilization result which is a special case of Theorem 3.1.*

Remark 3.2. *The different feasible solutions can result in different cost upper bounds in Theorem 3.1. It is a problem of how to optimize the matrix P_l in order to achieve the minimal NGC of the closed-loop system. This problem will be solved in the following theorem.*

Theorem 3.2. *An optimal non-fragile guaranteed cost J_{opt}^* can be achieved via solving the following optimization problem:*

$\min\gamma$ *s.t. (7) and*

$$
\begin{pmatrix}\gamma & x_0^T\\ x_0 & P_l\end{pmatrix}>0,\quad l\in M \tag{10}
$$

Proof: By Schur complement, it is easy to achieve a minimal cost upper bound $J_{opt}^*=\min\left\{x_0^TP_lx_0,l\in M\right\}$ where $\tilde P_l$ are solutions of the optimization problem.

4. **Example.** In this section, we present an example according to the theorem in previous section. Consider an uncertain switched fuzzy system composed of two subsystems.

$$
A_{11}=\begin{bmatrix}-22 & 10\\ -230 & -10\end{bmatrix},\quad A_{12}=\begin{bmatrix}-25 & 10\\ -300 & -10\end{bmatrix},\quad A_{21}=\begin{bmatrix}-22 & 10\\ 200 & -10\end{bmatrix},
$$

$$
A_{22}=\begin{bmatrix}-24 & 10\\ -300 & -10\end{bmatrix},\quad A_{d11}=A_{d12}=A_{d21}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad A_{d22}=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix},
$$

$$D_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 2 & 0 \\ 1 & -5 \end{bmatrix}, \quad D_{21} = D_{22} = \begin{bmatrix} 2 & 0 \\ 1 & -4 \end{bmatrix},$$

$$E_{11} = \begin{bmatrix} -10 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} -1 & 0.2 \\ -6 & 0 \end{bmatrix}, \quad E_{21} = E_{22} = \begin{bmatrix} 1 & -0.2 \\ -5 & 0 \end{bmatrix},$$

$$B_{11} = B_{12} = B_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D_{a11} = D_{a12} = D_{a21} = D_{a22} = \begin{bmatrix} 0.01 & -0.1 \\ 0 & 0.1 \end{bmatrix},$$

$$E_{a11} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad E_{a21} = E_{a22} = E_{a12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$M_{11}(t) = M_{12}(t) = M_{21}(t) = M_{22}(t) = M_{a11}(t) = M_{a12}(t) = M_{a21}(t) = M_{a22}(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix},$$

$$h_{11}(x_1(t)) = 1 - \frac{1}{1 + e^{-2x_1(t)}}, \quad h_{12}(x_1(t)) = \frac{1}{1 + e^{-2x_1(t)}},$$

$$h_{21}(x_1(t)) = 1 - \frac{1}{1 + e^{-2(x_1(t)-0.3)}}, \quad h_{22}(x_1(t)) = \frac{1}{1 + e^{-2(x_1(t)-0.3)}}.$$

With MATLAB LMI toolbox, the positive definite matrices P_1, P_2 and feedback gain matrices can be obtained.

$$Q = \begin{bmatrix} 1.7166 & 0.1450 \\ 0.1450 & 0.0758 \end{bmatrix}, \; P_1 = \begin{bmatrix} 5.2136 & -0.1310 \\ -0.1310 & 0.2015 \end{bmatrix}, \; P_2 = \begin{bmatrix} 4.2762 & -0.1057 \\ -0.1057 & 0.1812 \end{bmatrix},$$

$$K_{11} = \begin{bmatrix} -5.1475 & 0.1267 \\ -0.0572 & -0.1939 \end{bmatrix}, \; K_{12} = \begin{bmatrix} -5.1419 & 0.1288 \\ 0.1090 & -0.1889 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} -0.5601 & -0.5228 \\ -0.0818 & -0.1806 \end{bmatrix}, \; K_{22} = \begin{bmatrix} 1.0824 & -0.5972 \\ 0.1429 & -0.1783 \end{bmatrix}.$$

The simulation result of the system state trajectory with the initial state vector $x(0) = [\,-1 \quad 1\,]^T$ (adopted arbitrarily in the whole state space) is shown in Figure 1 for the system (2). From Figure 1, state responses converge asymptotically as demonstrated by the simulation under the switching law designed. Therefore, the system (2) is asymptotically stable.

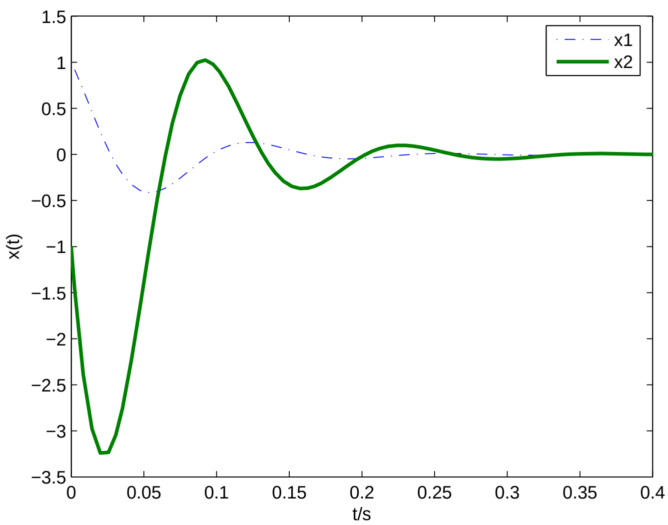

FIGURE 1. The system state trajectory of the switched fuzzy system

Additionally, a minimal cost upper bound $J_{opt}^* = 4.6686$ can be obtained from Theorem 3.2 with the initial state vector $x(0) = [\ -1 \quad 1\ ]^T$.

5. **Conclusion.** In this paper, based on the multiple Lyapunov functions method, the NGC problem for a class of uncertain switched fuzzy systems with time delays has been investigated. The method adopted can provide wider design space for switched fuzzy systems. Furthermore, an optimization problem which minimizes the NGC of the system has be solved, which can be applied to practical systems, such as the robotics system. Besides, it is assumed that all the state is measurable in this paper. When the state is not measurable or is hard to measure, the output feedback and state observer problem should be considered in future.

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