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ADAPTIVE MODIFIED PROJECTIVE SYNCHRONIZATION OF GYROSCOPES WITH UNKNOWN PARAMETERS AND EXTERNAL DISTURBANCES

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Abstract. *In this study, chaos synchronization problem of two gyroscopes is addressed. The outputs of state variables are affected by noise disturbances. In addition, the parameters of the system are considered unknown. Therefore, an adaptive modified projective control law and parameter estimation law are designed to control the behavior of the slave chaotic gyroscope states to track the motion trajectories of the master chaotic gyroscope system. The stability of the proposed method is verified by means of Lyapunov stability theorem. Furthermore, the convergence of the estimated parameters to their corresponding true values is evaluated. Finally the feasibility of the proposed method is shown by some numerical simulations.*
Keywords: Gyroscope chaotic system, Modified projective synchronization, Adaptive control

1. **Introduction.** This paper concentrates on the controlling and synchronization of chaotic gyroscope systems. In chaos theory, the behaviors of the nonlinear chaotic systems are studied. Chaotic systems are systems that are strongly sensitive to their initial state variables, a phenomenon which is usually known as butterfly effect [1]. The control and synchronization of chaotic systems have considerably attention during the last two decades, because of the potential applications in many scientific fields, such as electronics [2], physics [3, 4], chemistry [5], mechanics [6, 7], and secure communications [8]. In this line, many chaotic systems are derived such as Chen chaotic system [9], Lü chaotic system [10], Lorenz chaotic system [11], supply chain chaotic system [12], Chua chaotic system [13], finance chaotic system [14] and gyroscope chaotic system [15]. The gyroscope chaotic systems consist of the most famous dynamical systems in mechanical engineering. Gyroscopes have attained considerable attention in optics, navigation, aeronautics and aerospace engineering. Recently, many kinds of gyroscope systems have been introduced. In addition, many applications from synchronization of gyroscope chaotic systems arise including, secure communication [16], and controlling the spacecraft [17].

The ultimate goal of synchronization is to design an appropriate feedback controller to force the state of the slave system to track the trajectories of the master states. To this end, since the pioneering work by Pecora and Carroll in [18], a lot of synchronization methods have been developed to synchronize two identical or non-identical chaotic systems. Active method [19, 20], adaptive method [11, 21, 22], phase method [23], lag method [24], impulsive method [25], linear feedback method [26], nonlinear feedback method [27], sliding method [28, 29], and projective method [30, 31, 32] are some of the studied synchronization methods. Among them, projective method has got considerable attention due to their flexibilities and potential ability to align the synchronization error to an arbitrary scaling factor. To date, many chaos synchronizations related to the projective methods are developed to provide appropriate flexibilities. Modified projective method [33, 34], function projective method [35, 36, 37], modified function projective method [38, 39], generalized projective method [22, 40] and projective lag synchronization method [41] are some generalization of projective methods. However, almost all these projective methods can guarantee the controlling behavior of a chaotic system, especially gyroscope chaotic system, namely, the motion trajectories of the slave system can asymptotically track the motion trajectories of the master chaotic system state variables. Nevertheless, speediness in synchronization process is an important factor, which affects the effectiveness of a controlling and synchronization approach. To this end, in this paper, an adaptive modified projective control law is derived to asymptomatically synchronize the behavior of the master and slave of an identical chaotic system. For generality, the parameters of the system are considered unknown and an parameter estimation law is designed to estimate them and to achieve synchronization.

The dynamic behavior of gyro was firstly studied in [42]. Since then, the chaos control and synchronization of two gyroscope systems were studied by some researchers [43, 44, 45]. For example, a new control approach to generalized projective synchronization between symmetric gyroscopes with dead zone nonlinear inputs was proposed in [43]. And also, the projective synchronization was modified in [45], and a modified synchronization was introduced for chaotic dissipative gyroscope systems via backstepping control.

The rest of this paper is organized as follows. Some mathematical modellings are provided in Section 2, which describes a gyroscope system and its chaotic behavior. Section 3 presents the synchronization scheme. An appropriate feedback controller and parameter estimation strategy are presented to achieve synchronization. The stability of the proposed method is verified by means of Lyapunov stability theorem. Then, some numerical simulations are shown in Section 4, to validate the effectiveness of the proposed synchronization method. Finally, some concluding remarks are given in Section 5.

2. Mathematical Modelling.

In this section, some preliminaries about the gyroscope system and its chaotic behavior are given. The dynamical equation representing the behavior of a symmetric gyroscope with linear-plus-cubic damping mounted on a vibrating base can be given based on the approach in [16] as follows:

$$\ddot{\theta} + \alpha \frac{(1-\cos\theta)^2}{\sin^3\theta} - \beta \sin\theta + c_1\dot{\theta} + c_2\dot{\theta}^3 = f\sin(\omega t)\sin\theta \tag{1}$$

where θ denotes the rotation angle, the fraction $\alpha^2(1-\cos\theta)^2/\sin^2\theta$ presents the nonlinear resilience, $c_1\dot{\theta}$ indicates the linear and $c_2\dot{\theta}^2$ indicates the nonlinear damping terms, α, β, c_1 and c_2 are the parameters of the gyroscope system, which are considered unknown along this paper and also $f\sin(t)$ shows the parametric excitation that models the base excitation. Let $x_1 = \theta$ and $x_2 = \dot{\theta}$, and then the dynamical gyroscope system (1) can be rewritten in the following dynamical form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha \dfrac{(1-\cos x_1)^2}{\sin^3 x_1} + \beta\sin x_1 - c_1x_2 - c_2x_2^3 + f\sin(\omega t)\sin x_1 \end{cases} \tag{2}$$

When $\alpha = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $f = 35.5$ and $\omega = 2$, the behavior of the gyroscope system (2) is chaotic. The phase portrait of the system (2) is shown in Figure 1, with these parameters and the initial state values as: $x_1 = 6$ and $x_2 = 5.7$.

3. Synchronization.

Consider the gyroscope chaotic system (2) with uncertainty in its parameters as the master system. Then, the follower system can be given in the following

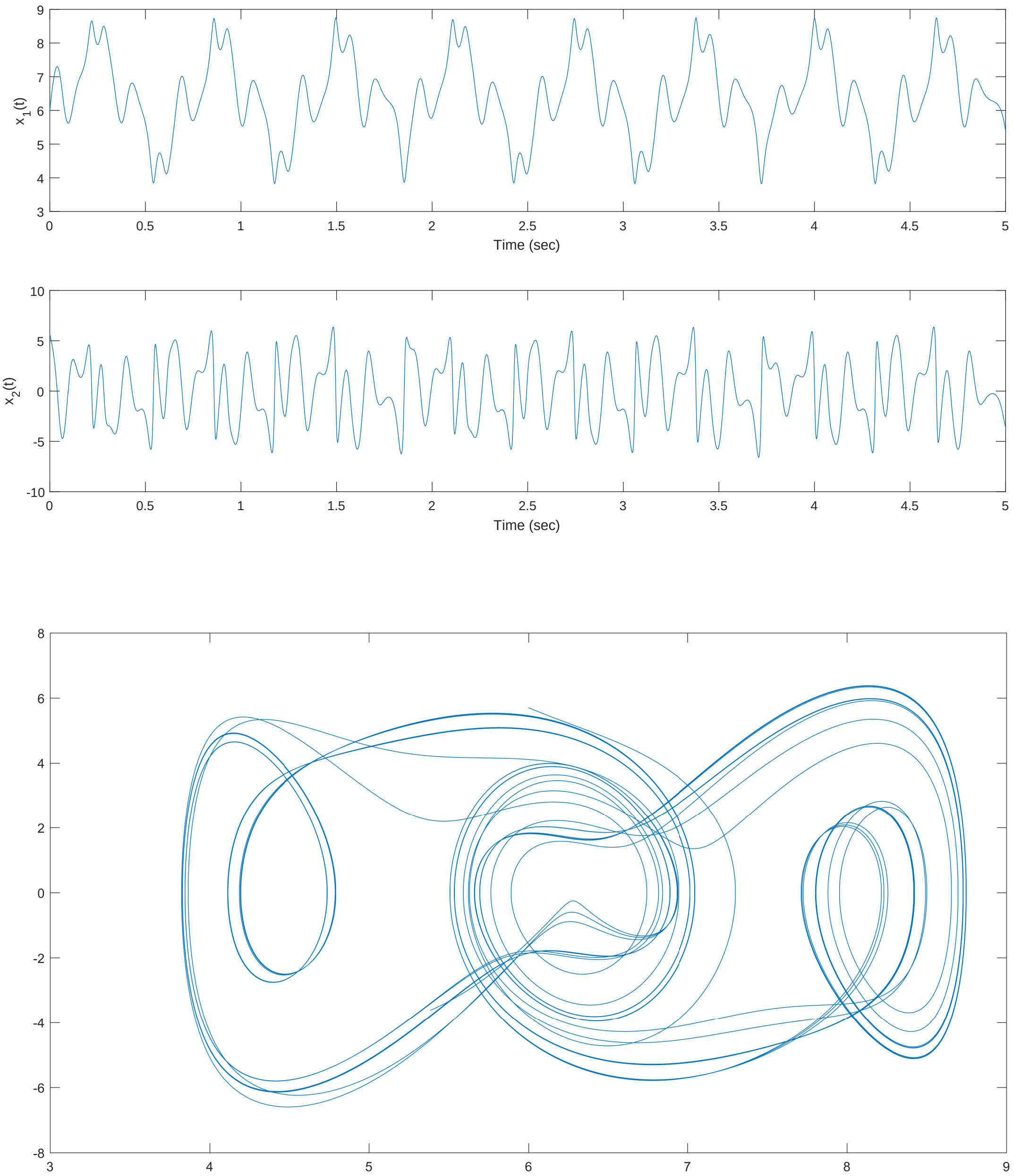

FIGURE 1. Phase portraits of the gyroscope chaotic system

form:

$$\begin{cases} \dot{y}_1 = y_2 + u_1 \\ \dot{y}_2 = -\hat{\alpha}\dfrac{(1-\cos y_1)^2}{\sin^3 y_1} + \hat{\beta}\sin y_1 - \hat{c}_1 y_2 - \hat{c}_2 y_2^3 + \hat{f}\sin(\omega t)\sin y_1 + u_2 \end{cases} \tag{3}$$

where y_1 and y_2 are the state variables of the response system and u_1 and u_2 stand for the control input of the response system (3), to be designed. $\hat{\alpha}$, $\hat{\beta}$, $\hat{c}_1$, $\hat{c}_2$ and $\hat{f}$ are the estimation of gyroscope system parameters α, β, c_1, c_2 and f, respectively.

Let the synchronization errors between the master gyroscope chaotic system (2) state variables and their corresponding slave chaotic system (3) state variables be as follows:

$$\begin{cases} e_1 = y_1 - \delta_1 x_1 \\ e_2 = y_2 - \delta_2 x_2 \end{cases} \tag{4}$$

where δ_1 and δ_2 are two different factors as modified projective scaling factors, which affect the synchronization errors. Then, the dynamical representation of the synchronization errors (4) can be obtained as follows:

$$\begin{cases} \dot{e}_1 = \dot{y}_1 - \delta_1 \dot{x}_1 \\ \dot{e}_2 = \dot{y}_2 - \delta_2 \dot{x}_2 \end{cases} \tag{5}$$

Then, the modified projective synchronization between master gyroscope chaotic system (2) and the slave system (3) can be achieved if the synchronization errors (4) converge to zero as time tends to infinity, i.e., $\lim_{t\to\infty} |e_i(t)| = 0$, $\forall i = 1, 2$.

In the following theorem, a new adaptive controller is given based on the modified projective synchronization (MPS) method to achieve such synchronization purpose.

Theorem 3.1. *The master chaotic system (2) and its identical slave system (3) would be synchronized if the control law and the parameter estimation law are set as follows:*

$$\begin{cases} u_1 = -y_2 + \delta_1 x_2 - k_1 e_1 \\ u_2 = +\,\hat{\alpha}\dfrac{(1-\cos y_1)^2}{\sin^3 y_1} - \hat{\beta}\sin y_1 + \hat{c}_1 y_2 + \hat{c}_2 y_2^3 - \hat{f}\sin(t)\sin y_1 - k_2 e_2 \\ \qquad -\,\delta_2\hat{\alpha}\dfrac{(1-\cos x_1)^2}{\sin^3 x_1} + \delta_2\hat{\beta}\sin x_1 - \delta_2\hat{c}_1 x_2 - \delta_2\hat{c}_2 x_2^3 + \delta_2\hat{f}\sin(\omega t)\sin x_1 \end{cases} \tag{6}$$

and,

$$\begin{cases} \dot{\hat{\alpha}} = +\,e_2\delta_2\dfrac{(1-\cos x_1)^2}{\sin^3 x_1} \\ \dot{\hat{\beta}} = -\,e_2\delta_2\sin x_1 \\ \dot{\hat{c}}_1 = +\,e_2\delta_2 x_2 \\ \dot{\hat{c}}_2 = +\,e_2\delta_2 x_2^3 \\ \dot{\hat{f}} = -\,e_2\delta_2\sin(\omega t)\sin x_1 \end{cases} \tag{7}$$

Proof: Let the Lyapunov stability function be as follows:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2\right) + \frac{1}{2}\left((\hat{\alpha}-\alpha)^2 + \left(\hat{\beta}-\beta\right)^2 + (\hat{c}_1 - c_1)^2 + (\hat{c}_2 - c_2)^2 + \left(\hat{f}-f\right)^2\right) \tag{8}$$

It is clear that V is positive definite, when the parameters α, β, c_1, c_2 and f and their corresponding estimations $\hat{\alpha}$, $\hat{\beta}$, $\hat{c}_1$, $\hat{c}_2$ and $\hat{f}$ are real. Then, the time derivative of the V can be described as follows:

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + (\hat{\alpha}-\alpha)\dot{\hat{\alpha}} + \left(\hat{\beta}-\beta\right)\dot{\hat{\beta}} + (\hat{c}_1 - c_1)\dot{\hat{c}}_1 + (\hat{c}_2 - c_2)\dot{\hat{c}}_2 + \left(\hat{f}-f\right)\dot{\hat{f}} \tag{9}$$

Substituting the dynamical representation of system errors (5), followed by the dynamical representations of master system (2) and slave system (3), we have:

$$\begin{aligned} \dot{V} \;=\; & e_1[y_2 - \delta_1 x_2 + u_1] + e_2\left[-\hat{\alpha}^2\frac{(1-\cos y_1)^2}{\sin^3 y_1} + \hat{\beta}\sin y_1 - \hat{c}_1 y_2 - \hat{c}_2 y_2^3 + f\sin(\omega t)\sin y_1\right. \\ & \left. + u_2 + \delta_2\alpha\frac{(1-\cos x_1)^2}{\sin^3 x_1} - \delta_2\beta\sin x_1 + \delta_2 c_1 x_2 + \delta_2 c_2 x_2^3 - \delta_2 f\sin(\omega t)\sin x_1\right] \\ & + (\hat{\alpha}-\alpha)\dot{\hat{\alpha}} + \left(\hat{\beta}-\beta\right)\dot{\hat{\beta}} + (\hat{c}_1 - c_1)\dot{\hat{c}}_1 + (\hat{c}_2 - c_2)\dot{\hat{c}}_2 + \left(\hat{f}-f\right)\dot{\hat{f}} \end{aligned} \tag{10}$$

Finally, with substituting the proposed control law and the parameter estimation law, one can get:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 \tag{11}$$

which is negative definite, when the constants k_1 and k_2 are positive. Thus the theorem is proved. Then, the motion trajectories of the slave system (3) can be asymptotically tracked by the state variables of the master gyroscope chaotic system (2), namely, the synchronization between the master gyroscope chaotic system (2) and its slave system (3) is achieved. Furthermore, the disparity amount between the parameter estimation and their corresponding true unknown values converge to zero, as time tends to infinity.

4. **Numerical Simulations.** The main goal of numerical simulation of synchronization methods is to verify the validity of the proposed controlling strategy for synchronization of the two chaotic systems. In this section, some numerical results related to the synchronization of gyroscope chaotic system are provided. The initial parameters are selected randomly, to show the effectiveness of the proposed method.

For synchronization purpose, the parameters of the master chaotic systems are chosen as: $\alpha = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $f = 35.5$ and $\omega = 2$. In addition, the initial values of system parameters are set as: $\hat{\alpha} = 56$, $\hat{\beta} = 3$, $\hat{c}_1 = 0.2$, $\hat{c}_2 = 0.1$ and $\hat{f} = 11$. The initial values of the master gyroscope chaotic system (2) are taken as: $x_1(0) = 12$ and $x_2(0) = 10$ and also the initial values of the slave system (3) are selected as: $y_1(0) = 1.5$ and $y_2(0) = 2$. The constant parameters are set as: $k_1 = 2$ and $k_2 = 2$. The proposed chaos synchronization problem of master gyroscope chaotic system (2) and its slave chaotic system (3) is carried out for modified scaling factors $(\delta_1, \delta_2) = (1.03, 1.02)$.

The behavior of the master chaotic system (2) and its follower system (3) is given in Figure 2. Furthermore, their corresponding parameter estimations are illustrated in Figure 3. As it can be seen from these figures, the anticipated synchronizations between the master gyroscope chaotic system (2) and its slave system (3) are accurately achieved. Moreover, the disparity amount between the parameters of the gyroscope chaotic system (2) and their estimations converges to zero as time goes to infinity.

5. **Conclusion.** In this paper, a chaotic controlling method is derived for synchronization of the two identical gyroscope chaotic systems. Since, the parameters of the system are considered unknown, then an adaptive control method based on the Lyapunov stability theorem, adaptive control method and modified projective synchronization method is designed to control the behavior of the gyroscope chaotic system to track the behavior of

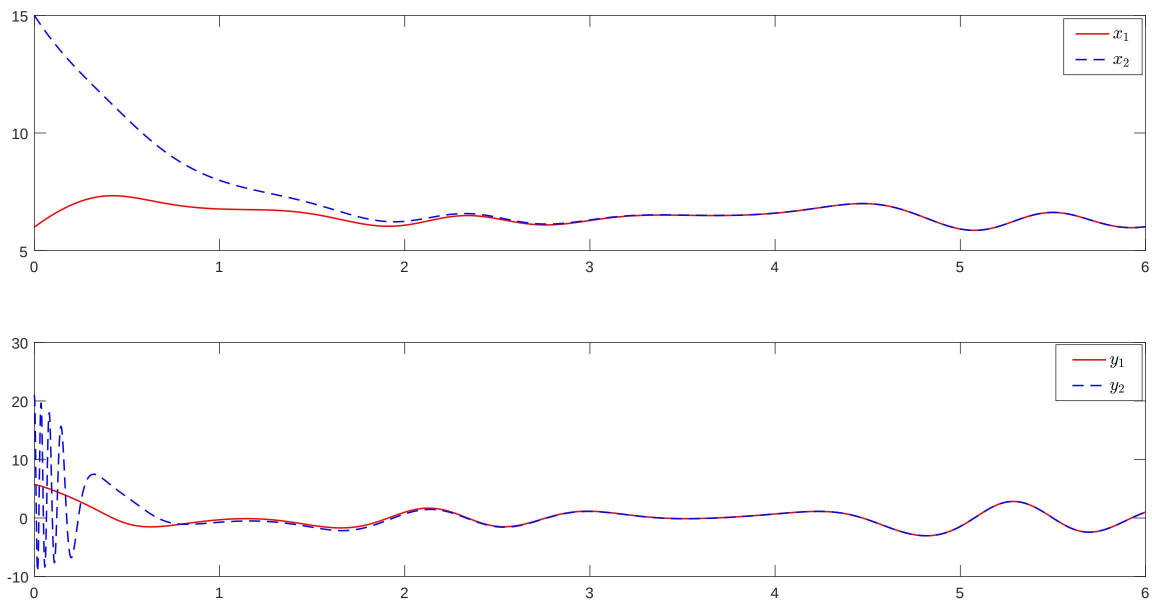

FIGURE 2. The state variables trajectories of the master gyroscope chaotic systems (2) and its slave system (3)

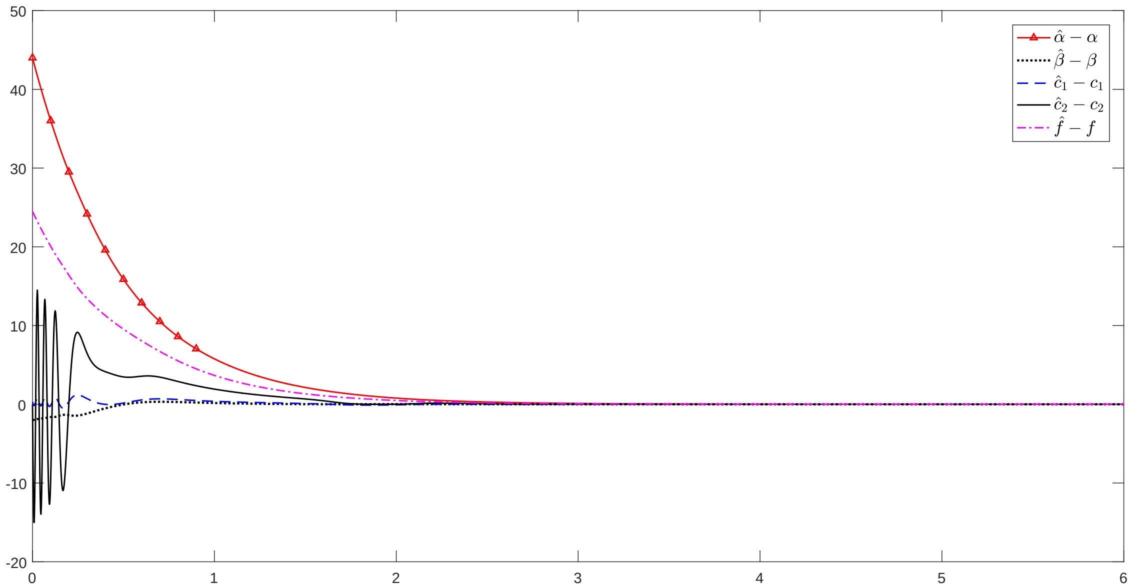

FIGURE 3. Parameter estimation errors

the master system state variables. Furthermore, the unknown parameters of the system are estimated. The validity of the proposed synchronization method is proved by means of Lyapunov stability theorem. Finally, some numerical simulations are shown to verify the effectiveness of the proposed synchronization method. The results show the effectiveness and feasibility of the synchronization scheme from both speed and accuracy points of views.

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