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COMPOSITE ANTI-DISTURBANCE CONTROL FOR SINGULAR STOCHASTIC MARKOVIAN JUMP SYSTEM

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Abstract. *In this paper, the issue of composite anti-disturbance control is investigated for singular stochastic Markovian jump system with multiple disturbances. Two disturbances are considered: one is time-varying disturbance and described by an external system; the other is multiplicative stochastic white noise. The method of composite anti-disturbance control is constructed to ensure that the closed-loop system is asymptotically bounded in mean square or asymptotically stable in mean square. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.*
Keywords: Singular Markovian jump system, Multiple disturbances, Stochastic disturbance, Composite anti-disturbance control

1. **Introduction.** In many actual systems, because disturbances exist due to many internal and external factors, the performances of control systems are often degraded, or even result in instability of system. To deal with this problem, lots of meaningful control methods have been presented, such as disturbance-observer-based robust control method [1], robust adaptive neural network control strategy [2] and robust H_∞ control method [3]. Many good control performances could be guaranteed for the systems which suffer from single disturbance by using these mentioned methods. However, with the raising requirements for control precision, multiple disturbances should be considered when the controller design of some actual systems is discussed. In [4,5], the composite anti-disturbance control schemes were studied. Nevertheless, the Markovian jump systems have not been presented in the results above.

Markovian jump systems have raised concern for many well-known scholars, and many meaningful results have been reported [6-8]. Recently, many papers are devoted to the multiple disturbances problem's study for Markovian jump systems, for example, in [9-12]. In [11,12], the authors of this paper have investigated the composite anti-disturbance resilient control problem for Markovian jump systems. In actual system, singular Markovian jump systems are able to better describe the physical system than regular ones, and are noted by many experts, such as [13,14]. The problem of multiple disturbances has been considered for singular Markovian jump nonlinear systems in [15]. However, the issues of multiple disturbances have been rarely studied for singular stochastic Markovian jump systems.

In this paper, the problem of the multiple disturbances is discussed for singular stochastic Markovian jump systems by using the composite anti-disturbance control method. The paper organization is as follows. The problem formulation and some preliminaries of singular stochastic Markovian jump systems are presented in Section 2. The main results obtained by composite anti-disturbance control are shown in Section 3. Section 4 gives a numerical example to illustrate the effectiveness of the proposed approach. Finally, the conclusions of the paper are provided in Section 5.

2. **Problem Statement and Preliminaries.** Consider the following singular stochastic Markovian system in probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$E\dot{x}(t) = A(r_t)x(t) + B(r_t)(u(t) + d(t)) + F(r_t)x(t)\xi_1(t), \tag{1}$$

where $x(t) \in \mathcal{R}^n$ and $u(t) \in \mathcal{R}^m$ are system state and control input, respectively. $d(t) \in \mathcal{R}^m$ is a disturbance and assumed to be given by an exogenous system in Assumption 2.1, and $\xi_1(t)$ is a one dimensional stochastic white noise as another disturbance. $E \in \mathcal{R}^{n\times n}$ may be singular, and $A(r_t)$, $B(r_t)$ and $F(r_t)$ are known matrices with appropriate dimension. And $\{r_t\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $S = \{1, 2, \dots, N\}$. The transition probability matrix is given by

$$Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } j \neq i, \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } j = i, \end{cases}$$

where $\Delta > 0$, $o(\Delta)$ satisfies $\lim\limits_{\Delta\to 0} \left(\frac{o(\Delta)}{\Delta}\right) = 0$, and γ_{ij} is the transition rate from mode i at time t to mode j at time $t + \Delta$, $\gamma_{ij} \geq 0$ when $i \neq j$ and $\gamma_{ii} = -\sum\limits_{j=1, j\neq i}^{N} \gamma_{ij}$.

Assumption 2.1. *The disturbance $d(t)$ is given as the following exogenous system:*

$$\begin{aligned} d(t) &= V(r_t)w(t) + H(r_t)\xi_2(t), \\ \dot{w}(t) &= W(r_t)w(t) + G(r_t)\delta(t), \end{aligned} \tag{2}$$

where $V(r_t)$, $W(r_t)$, $G(r_t)$ and $H(r_t)$ are known matrices, $w(t)$ is the internal state of the system, $\xi_2(t)$ is a one dimensional white noise, which is independent of the $\xi_1(t)$, and $\delta(t)$ belongs to the space of $\mathcal{L}_2[0, \infty)$.

For simplicity, for $r_t = i$, the matrix $A(r_t)$ will be denoted by A_i, $i \in S$; and the same setting for the other matrices, for example, $B(r_t)$ is expressed as B_i.

Assumption 2.2. *(E, A_i, B_i) is impulse controllable and (W_i, B_iV_i) is observable.*

Definition 2.1. [16] *Let $p > 0$. System (1) is said to be asymptotically bounded in the pth moment if there is a positive constant H such that*

$$\lim_{t\to\infty} \sup \mathcal{E}|x(t; t_0, x_0, r_0)|^p \leq H, \tag{3}$$

for all $(t_0, x_0, r_0) \in \mathcal{R}_+ \times \mathcal{R}^n \times S$, when $p = 2$, we say that the stochastic system (1) is asymptotically bounded in mean square.

Definition 2.2. [16] *For $p > 0$, the trivial solution of system (1) is said to be asymptotically stable in the pth moment if*

$$\lim_{t\to\infty} \mathcal{E}\left(|x(t; t_0, x_0, r_0)|^p\right) = 0,$$

for all $(t_0, x_0, r_0) \in \mathcal{R}_+ \times \mathcal{R}^n \times S$, when $p = 2$, it is said to be asymptotically stable in mean square.

Lemma 2.1. [1] *Assume that X and Y are vectors or matrices with appropriate dimension. The following inequality*

$$X^TY + Y^TX \leq \alpha X^TX + \alpha^{-1}Y^TY,$$

holds for any constant $\alpha > 0$.

In this section, the composite anti-disturbance control method is presented for the system (1). The disturbance observer is constructed as

$$\begin{aligned}&\hat{d}(t) = V_i\hat{w}(t),\ \hat{w}(t) = v(t) - L_iEx(t),\\ &\dot{v}(t) = (W_i + L_iB_iV_i)\hat{w}(t) + L_i(A_ix(t) + B_iu(t)) + \sum_{j=1}^{N}\gamma_{ij}L_jEx(t),\end{aligned} \tag{4}$$

where L_i is observer gain. Then, the following composite anti-disturbance controller is built based on the disturbance observer (4):

$$u(t) = K_ix(t) - \hat{d}(t), \tag{5}$$

where K_i is the gain of anti-disturbance controller.

The estimation error is denoted as $e_w(t) = w(t) - \hat{w}(t)$. Then, the error system is given by

$$\dot{e}_w(t) = \dot{w}(t) - \mathcal{A}\hat{w}(t) = (W_i + L_iB_iV_i)\, e_w(t) + L_iF_ix(t)\xi_1(t) + L_iB_iH_i\xi_2(t) + G_i\delta(t), \tag{6}$$

where $\mathcal{A}$ denotes the weak infinitesimal generator [18] of the random process $\{x(t), r(t)\}$.

Then, the composite closed-loop system is given as

$$\overline{E}\dot{\eta}(t) = \overline{A}_i\eta(t) + \overline{F}_i\eta(t)\xi_1(t) + \overline{H}_i\xi_2(t) + \overline{G}_i\delta(t), \tag{7}$$

where $\eta(t) = \left(x^T(t)\ \ e_w^T(t)\right)^T$, and

$$\overline{E} = \begin{pmatrix} E & 0\\ 0 & I\end{pmatrix},\ \overline{A}_i = \begin{pmatrix} A_i + B_iK_i & B_iV_i\\ 0 & W_i + L_iB_iV_i\end{pmatrix},\ \overline{F}_i = \begin{pmatrix} F_i & 0\\ L_iF_i & 0\end{pmatrix},$$

$$\overline{H}_i = \begin{pmatrix} B_iH_i\\ L_iB_iH_i\end{pmatrix},\ \overline{G}_i = \begin{pmatrix} 0\\ G_i\end{pmatrix}.$$

According to [17], replace $\xi_1(t)$ and $\xi_2(t)$ with $\frac{dW_1(t)}{dt}$ and $\frac{dW_2(t)}{dt}$, respectively, and Equation (7) is written as

$$\overline{E}d\eta(t) = \overline{A}_i\eta(t)dt + \overline{F}_i\eta(t)d\overline{W}_1(t) + \overline{H}_id\overline{W}_2(t) + \overline{G}_i\delta(t)dt, \tag{8}$$

where $\overline{W}_1(t)$ and $\overline{W}_2(t)$ are independent standard Wiener processes, because we assume that the white noises $\xi_1(t)$ and $\xi_2(t)$ are independent for each other in Assumption 2.1.

3. **Main Results.** Now, the asymptotical boundedness in mean square and asymptotical stability in mean square of the composite closed-loop system (8) are discussed by using the composite anti-disturbance control method and linear matrix inequalities (LMIs) technology. The following theorem is given for the control design.

Theorem 3.1. *Given parameter $\lambda > 0$, the system (8) is asymptotically bounded in mean square, if there exist parameters $\varepsilon_i > 0$ and matrices P_i, $i \in S$, such that*

$$\overline{E}^TP_i = P_i^T\overline{E} \geq 0, \tag{9}$$

$$\overline{A}_i^TP_i + P_i^T\overline{A}_i + \overline{F}_i^T\overline{E}^TP_i\overline{F}_i + \sum_{j=1}^{N}\gamma_{ij}\overline{E}^TP_j + \varepsilon_i^{-1}P_i^T\overline{G}_i\overline{G}_i^TP_i + \lambda\overline{E}^TP_i < 0. \tag{10}$$

Proof: Based on (9) and (10), we have

$$\overline{A}_i^TP_i + P_i^T\overline{A}_i + \sum_{j=0}^{N}\gamma_{ij}\overline{E}^TP_j + \lambda\overline{E}^TP_i < 0. \tag{11}$$

It is easy to choose two nonsingular matrices $\hat{M}$ and $\hat{N}$ such that $\hat{M}\overline{E}\hat{N} = \begin{pmatrix} I & 0\\ 0 & 0\end{pmatrix}$. Then, the same proof process as [14], we obtain the system (8) is regular and impulse free.

Choose some nonsingular matrices M_i, $i \in S$ and N, such that

$$M_i\overline{F}_iN = \begin{pmatrix} F_{1i} & F_{2i} \\ 0 & F_{3i} \end{pmatrix},\ M_i\overline{A}_iN = \begin{pmatrix} A_{1i} & 0 \\ 0 & I \end{pmatrix},\ M_i\overline{E}N = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix},$$

$$M_i\overline{H}_i = \begin{pmatrix} H_{1i} \\ H_{2i} \end{pmatrix},\ M_i\overline{G}_i = \begin{pmatrix} G_{1i} \\ G_{2i} \end{pmatrix}.$$

Thus, the system (8) is equivalent with the following system:

$$d\zeta_1(t) = A_{1i}\zeta_1(t)dt + F_{1i}\zeta_1(t)dW_1(t) + F_{2i}\zeta_2(t)dW_1(t) + H_{1i}dW_2(t) + G_{1i}\delta(t)dt, \quad (12)$$

$$0 = \zeta_2(t)dt + F_{3i}\zeta_2(t)dW_1(t) + H_{2i}dW_2(t) + G_{2i}\delta(t)dt, \quad (13)$$

where $\zeta(t) = \begin{pmatrix} \zeta_1(t) \\ \zeta_2(t) \end{pmatrix} = N^{-1}\eta(t)$. From (13), it yields

$$\mathcal{E}\left\{\zeta_2^T(t)\zeta_2(t)\right\} \le \kappa\delta^2(t) \le H_1, \quad (14)$$

where $\kappa > 0$. Choose Lyapunov function candidate as

$$V(\eta(t), r(t), t) = \eta^T(t)\overline{E}^T P_i\eta(t). \quad (15)$$

According to (8) and the generalized Itô formula, it yields

$$\begin{aligned}\mathcal{A}V(\eta(t), i, t) = {} & \eta^T(t)\left(\overline{A}_i^T P_i + P_i^T\overline{A}_i + \overline{F}_i^T\overline{E}^T P_i\overline{F}_i + \sum_{j\in S}\gamma_{ij}\overline{E}P_j\right)\eta(t) \\ & + \eta^T(t)P_i^T\overline{G}_i\delta(t) + \delta^T(t)\overline{G}_i^T P_i\eta(t) + Tr\left(\overline{H}_i^T\overline{E}^T P_i\overline{H}_i\right).\end{aligned} \quad (16)$$

Using Lemma 2.1 for (16), it yields

$$\begin{aligned}\mathcal{A}V(\eta(t), i, t) \le {} & \eta^T(t)\left(\overline{A}_i^T P_i + P_i^T\overline{A}_i + \overline{F}_i^T\overline{E}^T P_i\overline{F}_i + \sum_{j\in S}\gamma_{ij}\overline{E}P_j + \varepsilon_i^{-1}P_i^T\overline{G}_i\overline{G}_i^T P_i\right)\eta(t) \\ & + \varepsilon_i\delta^T(t)\delta(t) + Tr\left(\overline{H}_i^T\overline{E}^T P_i\overline{H}_i\right).\end{aligned} \quad (17)$$

Since that $\delta(t)$ is a bounded disturbance, there exists a constant β, and from (10), it yields

$$\mathcal{A}V(\eta(t), i, t) \le -\lambda V(\eta(t), i, t) + \beta. \quad (18)$$

Based on (9) and (15), for a positive integer k, according to the generalized Itô formula, for any $0 \le t_0 \le t$, we have

$$\begin{aligned}\mathcal{E}\left\{e^{-\lambda t}k(\|E\eta(t)\|^2)\right\} & \le \mathcal{E}\left\{e^{-\lambda t}V(\eta(t), r_t, t)\right\} \\ & \le V(\eta(t_0), r_0, t_0) + \frac{\beta}{\lambda}\int_{t_0}^{t} e^{-\lambda s}ds \\ & \le V(\eta(t_0), r_0, t_0) + \frac{\beta}{-\lambda}\left[e^{-\lambda t} - 1\right].\end{aligned}$$

Then, with $t \to \infty$, there exists a positive number β', such that

$$\mathcal{E}\left\{\|E\eta(t)\|^2\right\} \le \beta'. \quad (19)$$

From $\eta(t) = N\zeta(t)$, for some appropriate parameters c_i, $i \in S$, we have

$$\|M_iEN\zeta(t)\|^2 = \zeta^T(t)N^TE^TM_i^TM_iEN\zeta(t) = \zeta_1^T(t)\zeta_1(t) = \|\zeta_1(t)\|^2 \le c_i\beta' = H_2.$$

From (14), we have $\|\zeta(t)\|^2 \le H$ with $H \ge \max\{H_1, H_2\}$. Then, according to $\eta(t) = N\zeta(t)$ and Definition 2.1, we obtain (8) is asymptotically bounded in mean square.

Theorem 3.2. *Given parameter $\lambda > 0$, under the disturbance observer and the controller in form of (4) and (5). The system (8) is asymptotically bounded in mean square, if there exist parameters $\varepsilon_i > 0$, $\alpha_{1i} > 0$, $\alpha_{2i} > 0$ and $\beta_i > 0$, matrices Q_i, $P_{2i} > 0$, $i \in S$, such that*

$$Q_i^T E^T = E Q_i \geq 0, \tag{20}$$

$$Q_i^T E - Q_i^T - Q_i < -\alpha_{1i} I, \quad P_{2i} \leq \alpha_{2i} I, \tag{21}$$

$$\begin{pmatrix} \Pi_{11i} & B_i V_i & Q_i^T F_i & 0 & \Pi_{15i} & 0 \\ * & \Pi_{22i} & 0 & 0 & 0 & P_{2i} G_i \\ * & * & -\alpha_{1i} I & 0 & 0 & 0 \\ * & * & * & \Pi_{44i} & 0 & 0 \\ * & * & * & * & \Pi_{55i} & 0 \\ * & * & * & * & * & -\varepsilon_i I \end{pmatrix} < 0, \tag{22}$$

where

$$\Pi_{11i} = Q_i^T A_i^T + A_i Q_i + B_i X_i + X_i^T B_i^T + \beta_i I + (\lambda + \gamma_{ii}) Q_i^T E^T,$$

$$\Pi_{22i} = W_i^T P_{2i} + P_{2i} W_i + V_i^T B_i^T Y_i^T + Y_i B_i V_i + \lambda P_{2i} + \sum_{j=1}^{N} \gamma_{ij} P_{2j},$$

$$\Pi_{15i} = \begin{pmatrix} \sqrt{\gamma_{i1}} Q_i^T & \cdots & \sqrt{\gamma_{iN}} Q_i^T \end{pmatrix}, \ \Pi_{55i} = -diag \begin{pmatrix} \alpha_{1i} I & \ldots & \alpha_{1j} I & \ldots & \alpha_{1N} I \end{pmatrix}_{j \neq i},$$

$$\Pi_{44i} = \begin{pmatrix} -\alpha_{2i} I & Y_i F_i & 0 \\ * & -2I & Q_i^T \\ * & * & -\beta_i I \end{pmatrix}.$$

Moreover, the gains of controller and observer are given by $K_i = X_i Q_i^{-1}$ and $L_i = P_{2i}^{-1} Y_i$.

Proof: Noting (21), and defining $Y_i = P_{2i} L_i$, we have

$$\alpha_{1i}^{-1} Q_i^T F_i^T F_i Q_i \geq Q_i^T F_i^T E^T Q_j^{-1} F_i Q_i \text{ and } \alpha_{2i}^{-1} Q_i^T F_i^T Y_i^T Y_i F_i Q_i \geq Q_i^T F_i^T Y_i^T Y_i F_i Q_i. \tag{23}$$

By using Lemma 2.1 and Schur complement for (22), define $X_i = K_i Q_i$ and $Q_i = P_{1i}$. Premultiplying $\text{diag}\begin{pmatrix} P_{1i}^T & I \end{pmatrix}$ and post-multiplying simultaneously by $\text{diag}(P_{1i} \ I)$ with (22), and combining with (23), it yields

$$\begin{pmatrix} \Pi_{11i'} & B_i V_i \\ * & \Pi_{22i'} \end{pmatrix} < 0, \tag{24}$$

$$\begin{aligned} \Pi_{11i'} = {} & A_i^T P_{1i} + P_{1i}^T A_i + P_{1i}^T B_i K_i + K_i^T B_i^T P_{1i} + \lambda E^T P_{1i} + F_i^T E^T P_{1j} F_i \\ & + \sum_{j=1}^{N} \gamma_{ij} E^T P_{1j} + F_i^T L_i^T P_{2i} L_i F_i, \end{aligned}$$

$$\Pi_{22i'} = W_i^T P_{2i} + P_{2i} W_i + V_i^T B_i^T Y_i^T + Y_i B_i V_i + \lambda P_{2i} + \sum_{j=1}^{N} \gamma_{ij} P_{2j} + \varepsilon_i^{-1} P_{2i} G_i G_i^T P_{2i}.$$

Hence, form (9), letting $P_i = \begin{pmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{pmatrix}$, we deduce (10). On the other hand, the condition (9) holds under (21) and $P_{2i} > 0$. According to Theorem 2.1, the system (8) is asymptotically bounded in mean square.

In the following theorem, some sufficient conditions are presented to ensure the asymptotic stability in mean square of system (8) under the composite anti-disturbance control method.

Theorem 3.3. *For $H_i = 0$, $G_i = 0$ given parameter $\lambda > 0$, under the disturbance observer and the controller in form of (4) and (5). The system (8) is asymptotically stable in mean* *square, if there exist parameters* $\varepsilon_i > 0$, $\alpha_{1i} > 0$, $\alpha_{2i} > 0$ *and* $\beta_i > 0$, *matrices* Q_i, $P_{2i} > 0$, $i \in S$, *such that*

$$Q_i^T E^T = EQ_i \geq 0, \tag{25}$$

$$Q_i^T E - Q_i^T - Q_i < -\alpha_{1i} I, \quad P_{2i} \leq \alpha_{2i} I, \tag{26}$$

$$\begin{pmatrix} \Pi_{11i} & B_i V_i & Q_i^T F_i & \Pi_{14i} & \Pi_{15i} \\ * & \Pi_{22i} & 0 & 0 & 0 \\ * & * & -\alpha_{1i} I & 0 & 0 \\ * & * & * & \Pi_{44i} & 0 \\ * & * & * & * & \Pi_{55i} \end{pmatrix} < 0. \tag{27}$$

Moreover, the gains of controller and observer are given by $K_i = X_i Q_i^{-1}$ *and* $L_i = P_{2i}^{-1} Y_i$.

Proof: According to Theorem 3.1 and Theorem 3.2, for $H_i = 0$, $G_i = 0$, we can obtain $\mathcal{A}V(\eta(t), i, t) \leq -\lambda V(\eta(t), i, t)$. Hence, based on Definition 2.2, system (8) is asymptotically stable in mean square.

4. **Simulation Example.** In this section, a numerical example is presented to demonstrate the effectiveness of the proposed scheme in two aspects, i.e., asymptotic boundedness in mean square and asymptotic stability in mean square. Consider singular stochastic Markovian jump systems under multiple disturbances (1) with $E = \begin{pmatrix} 1 & -2 \\ 0.5 & -1 \end{pmatrix}$, and the other parameters are presented as follows:

Mode 1:

$$A_1 = \begin{pmatrix} 1 & -0.3 \\ 0.6 & 1 \end{pmatrix}, \; B_1 = \begin{pmatrix} -0.9 \\ 1 \end{pmatrix}, \; G_1 = \begin{pmatrix} 1.3 \\ -0.6 \end{pmatrix}, \; F_1 = \begin{pmatrix} 1 & -1 \\ 0.5 & 0.7 \end{pmatrix},$$

$$W_1 = \begin{pmatrix} 0 & 0.8 \\ -0.9 & 0 \end{pmatrix}, \; H_1 = 1, \; V_1 = \begin{pmatrix} 1 & -1.9 \end{pmatrix}.$$

Mode 2:

$$A_2 = \begin{pmatrix} 1 & -1 \\ 0.3 & -1 \end{pmatrix}, \; B_2 = \begin{pmatrix} -1.2 \\ 1 \end{pmatrix}, \; G_2 = \begin{pmatrix} 0.5 \\ 1.2 \end{pmatrix}, \; F_2 = \begin{pmatrix} -0.3 & 0.8 \\ 0.8 & -0.4 \end{pmatrix},$$

$$W_2 = \begin{pmatrix} 0 & -1.4 \\ 0.9 & 0 \end{pmatrix}, \; H_2 = 0.9, \; V_2 = \begin{pmatrix} 1 & -2 \end{pmatrix}.$$

Mode 3:

$$A_3 = \begin{pmatrix} -1 & 0.8 \\ 0.5 & 1 \end{pmatrix}, \; B_3 = \begin{pmatrix} 1.2 \\ 2.2 \end{pmatrix}, \; G_3 = \begin{pmatrix} 0.7 \\ -0.6 \end{pmatrix}, \; F_3 = \begin{pmatrix} -0.3 & 0.6 \\ 0 & 1 \end{pmatrix},$$

$$W_3 = \begin{pmatrix} 0 & 1.6 \\ -1.1 & 0 \end{pmatrix}, \; H_3 = 1.3, \; V_3 = \begin{pmatrix} 0.8 & 1 \end{pmatrix}.$$

Mode 4:

$$A_4 = \begin{pmatrix} -1.2 & 0.5 \\ 0 & 1.1 \end{pmatrix}, \; B_4 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \; G_4 = \begin{pmatrix} 0.4 \\ -0.9 \end{pmatrix}, \; F_4 = \begin{pmatrix} 1 & 0.9 \\ 0.4 & 0.5 \end{pmatrix},$$

$$W_4 = \begin{pmatrix} 0 & -2.5 \\ 2.1 & 0 \end{pmatrix}, \; H_4 = -0.9, \; V_4 = \begin{pmatrix} 1 & -1 \end{pmatrix}.$$

The uncertain matrix $S_i(t)$ is given as $S_i(t) = \cos(t)$, $i \in \{1, 2, 3, 4\}$, the additional disturbance is given as $\delta(t) = \delta_0(t) = \delta_1(t) = e^{-t}\sin(t)$, and the nonlinear function is chosen as $f(x(t), t) = x_1(t)\sin(t)$. Here, we choose the transition rate matrix as

$$\Gamma = \begin{pmatrix} -0.7 & 0.2 & 0.2 & 0.3 \\ 0.4 & -0.9 & 0.2 & 0.3 \\ 0.2 & 0.4 & -0.8 & 0.2 \\ 0.3 & 0.3 & 0.3 & -0.9 \end{pmatrix}.$$

4.1. **Asymptotical boundedness in mean square.** According to Theorem 3.2, we obtain the gains of controller and disturbance observer to ensure the asymptotical boundedness in mean square of system (8), which are

$$K_1 = \begin{pmatrix} 4.0495 & -5.7554 \end{pmatrix},\ K_2 = \begin{pmatrix} 2.5301 & -2.8137 \end{pmatrix},\ K_3 = \begin{pmatrix} 0.7618 & 2.6841 \end{pmatrix},$$

$$K_4 = \begin{pmatrix} 2.5836 & 5.7241 \end{pmatrix},\ L_1 = \begin{pmatrix} -0.8547 & -4.4204 \\ 0.3981 & 2.0603 \end{pmatrix},\ L_2 = \begin{pmatrix} 0.0745 & 0.8751 \\ 0.2816 & 3.3232 \end{pmatrix},$$

$$L_3 = \begin{pmatrix} 0.0971 & -0.2655 \\ 0.4989 & -1.3651 \end{pmatrix},\ L_4 = \begin{pmatrix} 0.3804 & -1.0550 \\ -0.5161 & 1.4315 \end{pmatrix}.$$

The simulation results are given in Figures 1-3. In Figure 1, we can see that the system still has a high control precision by the designed controller under the multiply disturbances. Figure 2 shows that the disturbance $d(t)$ can be efficiently estimated by the disturbance observer. The control input is presented in Figure 3.

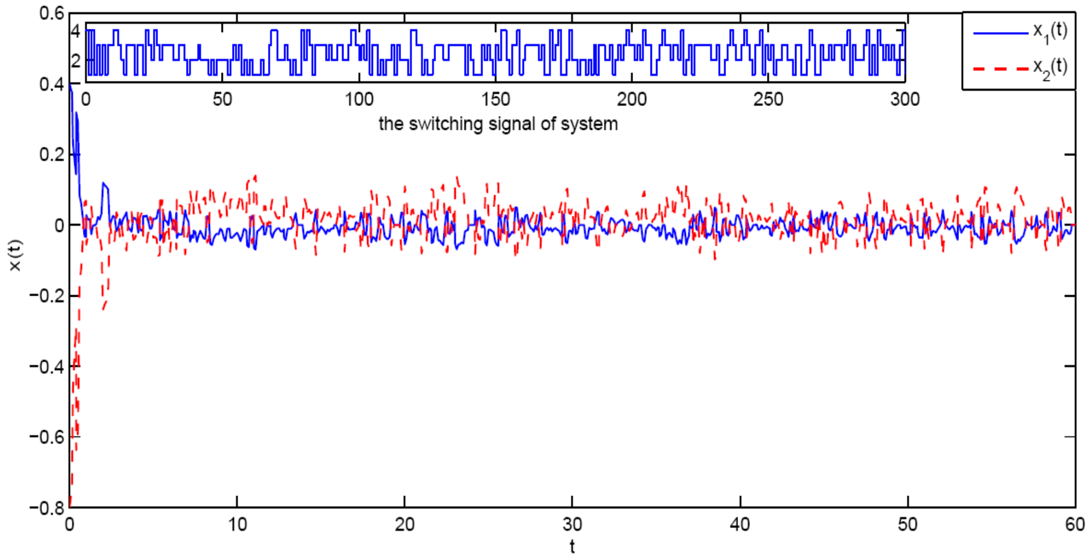

Figure 1. Response curves of system state

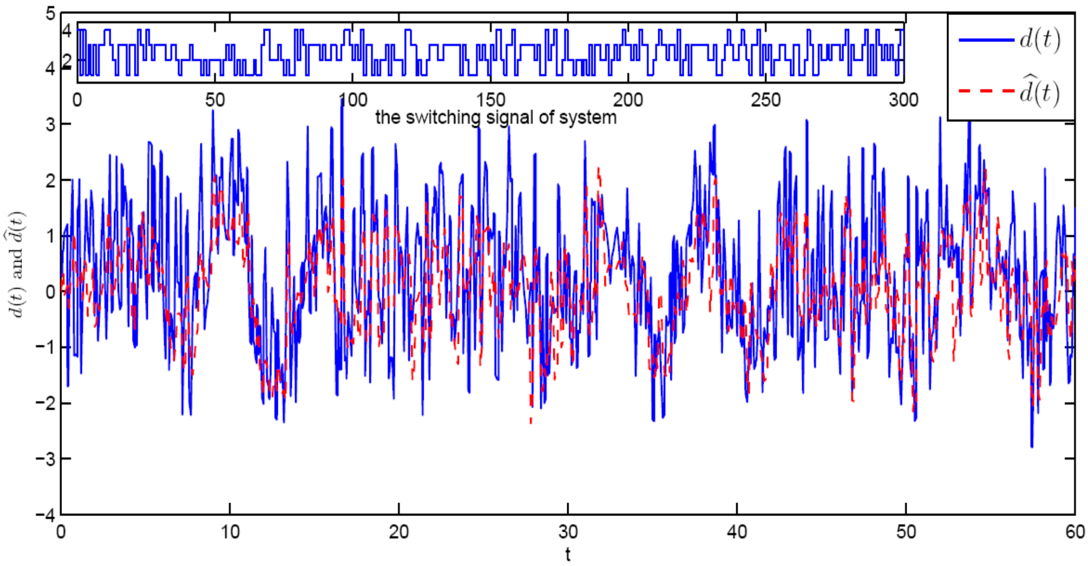

Figure 2. Curves of the disturbance and its estimation

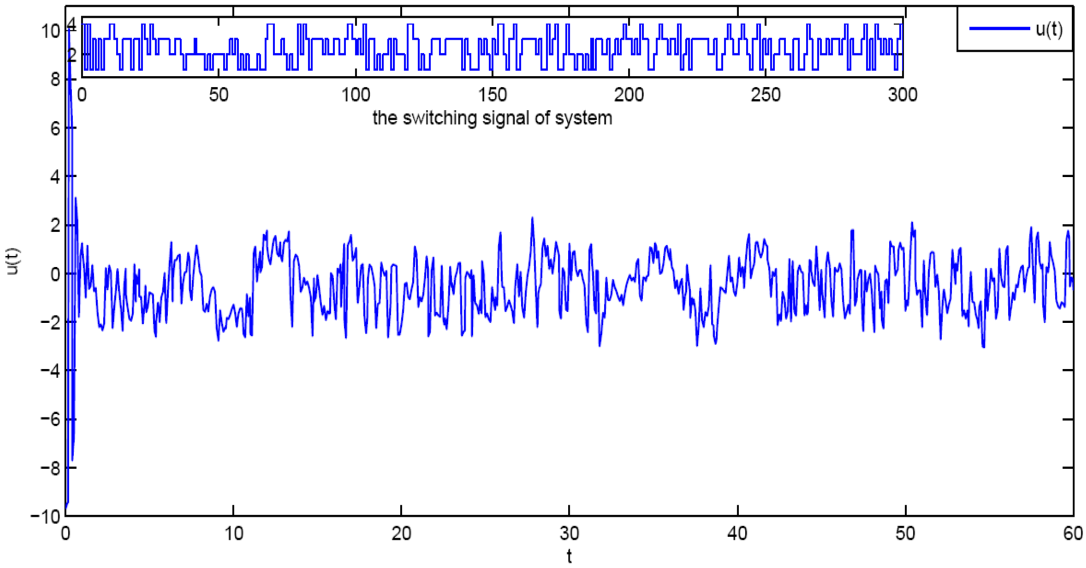

FIGURE 3. Curve of control input of system

4.2. **Asymptotic stability in mean square.** By using Theorem 3.3, the gains of controller and disturbance observer are given to guarantee the asymptotical stability in mean square of system (8), which are

$$K_1' = \begin{pmatrix} 4.0647 & -5.9955 \end{pmatrix}, \ K_2' = \begin{pmatrix} 2.5378 & -3.0447 \end{pmatrix}, \ K_3' = \begin{pmatrix} 0.9494 & 3.1216 \end{pmatrix},$$

$$K_4' = \begin{pmatrix} 2.3840 & 4.9735 \end{pmatrix}, L_1' = \begin{pmatrix} -0.2475 & -1.3350 \\ 0.2718 & 1.4659 \end{pmatrix}, \ L_2' = \begin{pmatrix} -0.0475 & -0.7548 \\ 0.1062 & 1.6874 \end{pmatrix},$$

$$L_3' = \begin{pmatrix} 0.2818 & -0.7566 \\ 0.3555 & -0.9526 \end{pmatrix}, \ L_4' = \begin{pmatrix} 0.2978 & -0.8187 \\ -0.4103 & 1.1278 \end{pmatrix}.$$

The simulation results are presented in Figures 4-6. From Figure 4, we can see, the system is asymptotically stable in mean square under composite anti-disturbance control method. In Figure 5, the disturbance observer can efficiently estimate disturbance. The control input is presented in Figure 6.

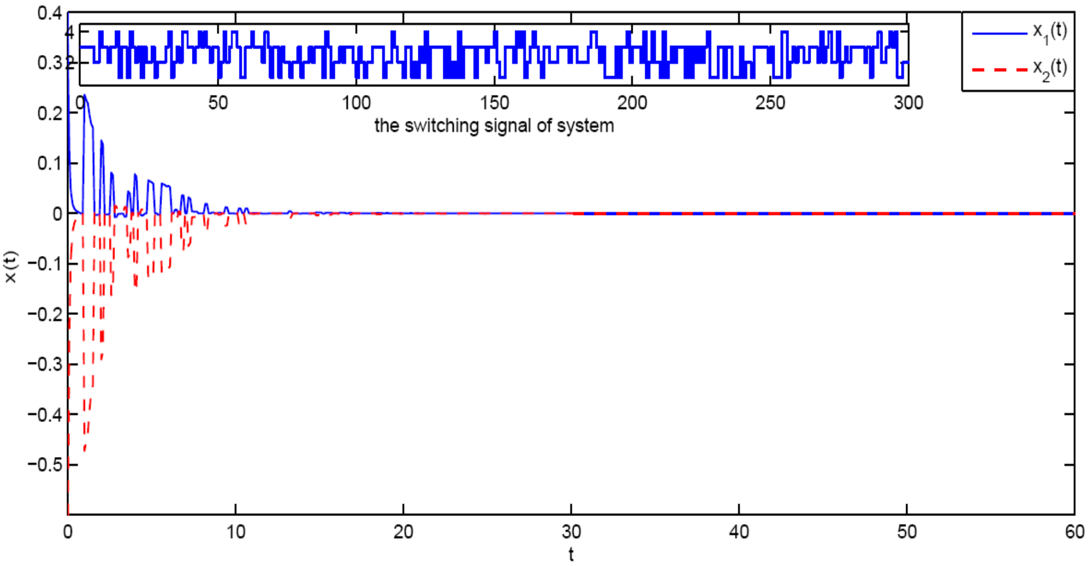

FIGURE 4. Response curves of system state

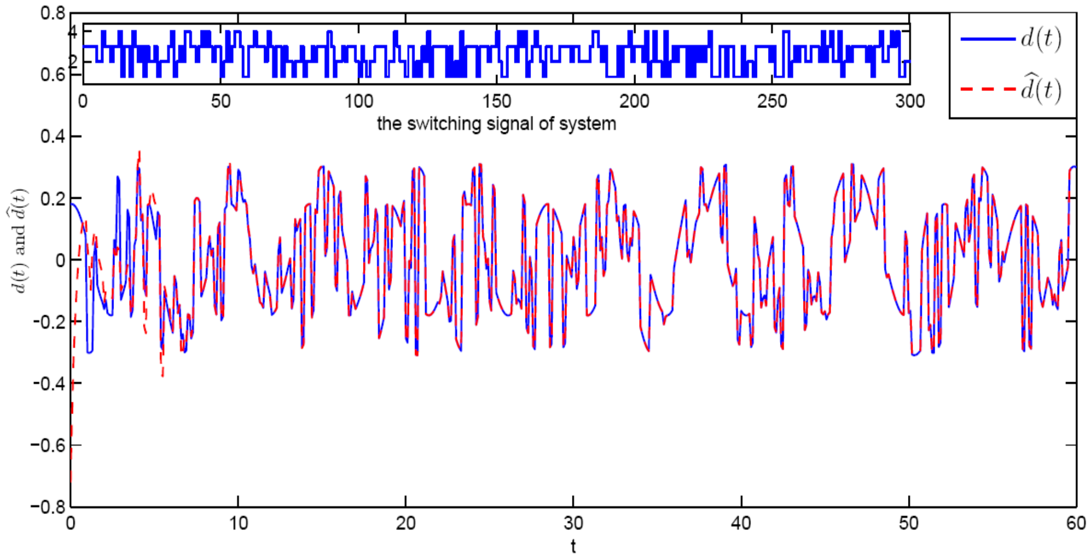

FIGURE 5. Curves of the disturbance and its estimation

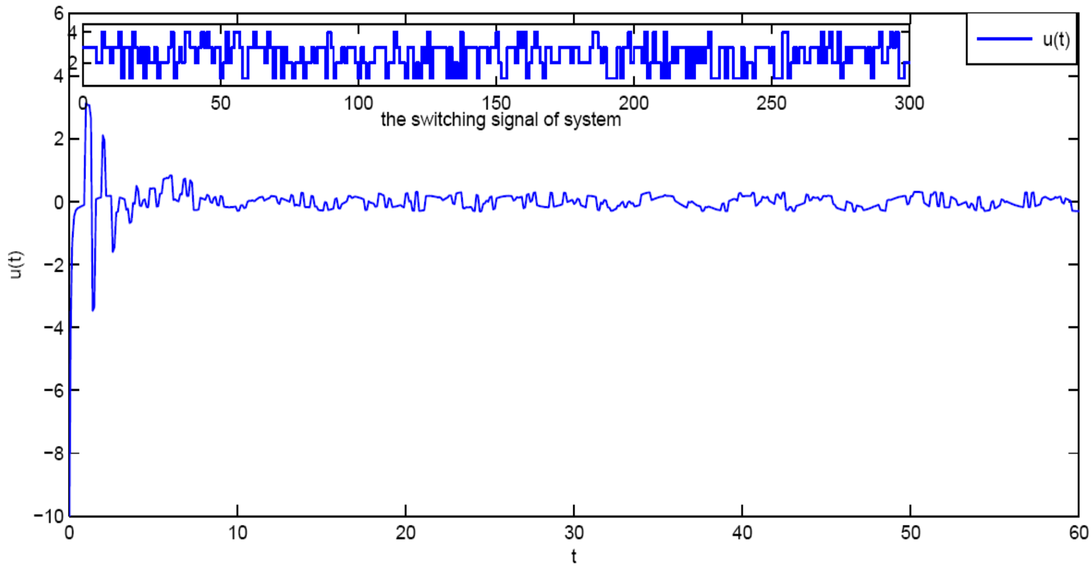

FIGURE 6. Curve of control input of system

5. **Conclusions.** In this paper, the method of composite anti-disturbance control has been developed for the problem of singular stochastic Markovian jump systems with multiple disturbances. The methods of Lyapunov stability and linear matrix inequalities have been used to guarantee the asymptotically bounded in mean square or asymptotically stable in mean square of the closed-loop system. Finally, a numerical example is given to illustrate the effectiveness of proposed approach.

In the future, we will consider to combine the dynamical output feedback control strategy with the disturbance-observer-based-control method to handle the problem of singular stochastic Markovian jump systems with multiple disturbances and unmeasurable states.

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REFERENCES

[1] M. Chen and W. H. Chen, Disturbance-observer-based robust control for time delay uncertain systems, *International Journal of Control, Automation and Systems*, vol.8, no.2, pp.445-453, 2010.

[2] M. Chen, S. S. Ge and B. V. How, Robust adaptive neural network control for a class of uncertain MIMO nonlinear systems with input nonlinearities, *IEEE Trans. Neural Networks*, vol.21, no.5, pp.796-812, 2010.

[3] G. D. Zong, S. Y. Xu and Y. Q. Wu, Robust H_∞ stabilization for uncertain switched impulsive control systems with state delay: An LMI approach, *Nonlinear Analysis: Hybrid Systems*, vol.2, no.3, pp.1287-1300, 2008.

[4] L. Guo and S. Y. Cao, *Anti-Disturbance Control for Systems with Multiple Disturbances*, CRC Press, Boca Raton, 2013.

[5] X. J. Wei, Z. J. Wu and H. R. Karimi, Disturbance observer-based disturbance attenuation control for a class of stochastic systems, *Automatica*, vol.63, pp.21-25, 2016.

[6] H. Y. Li, P. Shi, D. Y. Yao and L. G. Wu, Observer-based adaptive sliding mode control for nonlinear Markovian jump systems, *Automatica*, vol.64, pp.133-142, 2016.

[7] L. G. Wu, P. Shi and H. J. Gao, State estimation and sliding-mode control of Markovian jump singular systems, *IEEE Trans. Automatic Control*, vol.55, no.5, pp.1213-1219, 2010.

[8] P. Shi, Y. Yin, F. Liu and J. Zhang, Robust control on saturated Markov jump systems with missing information, *Information Sciences*, vol.265, pp.123-138, 2014.

[9] X. M. Yao and L. Guo, Composite anti-disturbance control for Markovian jump nonlinear systems via disturbance observer, *Automatica*, vol.49, no.8, pp.2538-2545, 2013.

[10] X. M. Yao and L. Guo, Composite disturbance-observer-based output feedback control and passive control for Markovian jump systems with multiple disturbances, *IET Control Theory and Applications*, vol.8, no.10, pp.873-881, 2014.

[11] Y. Li, H. B. Sun, G. D. Zong and L. L. Hou, Disturbance-observer-based-control and L_2-L_∞ resilient control for Markovian jump non-linear systems with multiple disturbances and its application to single robot arm system, *IET Control Theory and Applications*, vol.10, no.2, pp.226-233, 2016.

[12] Y. Li, H. B. Sun, G. D. Zong and L. L. Hou, Composite adaptive anti-disturbance resilient control for Markovian jump systems with partly known transition rate and multiple disturbances, *International Journal of Adaptive Control and Signal Processing*, 2016.

[13] Y. Zhao and W. H. Zhang, New results on stability of singular stochastic Markov jump systems with state-dependent noise, *International Journal of Robust and Nonlinear Control*, vol.26, pp.2169-2186, 2015.

[14] E. K. Boukas, S. Y. Xu and J. Lam, On stability and stabilizability of singular stochastic systems with delays, *Journal of Optimization Theory and Applications*, vol.127, no.2, pp.249-262, 2005.

[15] X. M. Yao, L. Q. Zhu and L. Guo, Disturbance-observer-based control and H_∞ control for non-linear Markovian jump singular systems with multiple disturbances, *IET Control Theory and Applications*, vol.8, pp.1689-1697, 2014.

[16] X. Mao and C. Yuan, *Stochastic Differential Equations with Markovian Switching*, Imperial College Press, London, 2006.

[17] B. Øksendal, *Stochastic Differential Equations – An Introduction with Applications*, Springer-Verlag, New York, 2003.

[18] E. D. Dynkin, *Markov Process*, Springer, Berlin, 1965.