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A STUDY ON A SUPPLY CHAIN NETWORK CONTROL PROBLEM WITH AN INDEPENDENT REPLENISHMENT CYCLE POLICY UNDER INVENTORY COORDINATION

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Abstract. *We consider a supply chain network control problem with an independent replenishment cycle policy of orders. The objective of the problem is to determine the optimal replenishment quantities and cycles under minimization of the total relevant costs that consist of ordering costs, inventory holding costs, and fixed costs. For the problem statement proposed in this paper, we demonstrate that all nodes in a network except suppliers have non-negative inventory levels during network planning periods. This paper introduces some useful properties to calculate the non-negative inventories of a product. Finally, an example has been shown to describe the cost savings of the proposed policy compared with the existing policy, integer-ratio replenishment policy.*
Keywords: Supply chain network, Inventory management, Replenishment cycle

1. **Introduction.** The facts that optimizing only processes of individual companies may not be enough to sustain the companies in global competition and that the coordination between supply chain members can benefit the entire supply chain, are inspiring many researchers in research area of supply chain management. Therefore, supply chain management is steadily being carried out to provide more competitive decision making to the supply chain system in a rapidly changing market environment. Most of research demonstrated that inventory levels in supply chains can be more efficiently managed by coordinating entire supply chain network [1,2].

There are two research categories for supply chain problems with inventory management: supply chain problems under discrete planning periods (SC_D) and supply chain problems under continuous planning periods (SC_C). Most research on SC_D was conducted to save its entire costs by taking operational decisions such as lot-sizes together into traditional supply chain problems known as strategic decision-making problem [3-7]. However, there is a limit to provide a practical operating plan because the main objective of SC_D is to determine more reasonable network configuration under long-term and discrete planning periods, i.e., annual or monthly periods, rather than to offer detailed operating plans. Meanwhile, research on SC_C mainly discussed practical and detailed operating plans under continuous planning periods even though most research did not consider strategic decision-makings. Roundy [8] initially introduced a method for solving the continuous-time version of supply chain problem under *integer-ratio coordination policy*, which is a policy where a node orders to a preceding node at equally spaced points in time and each succeeding node follows an economic order quantity pattern. The objective of the research is to determine the optimal order quantity and integer-ratio replenishment cycle under single warehouse and multi retailers. Since then, many researchers have conducted researches focusing on integer-ratio coordination policy in supply chain. Tsao and Lu [9] addressed an integrated facility location and inventory allocation problem considering transportation cost discounts and proposed a heuristic to obtain the optimal order quantity and replenishment cycle. Sarkar [10] considered a supply chain problem with deterioration of volatile products and applied an algebraic approach to finding the minimum cost related to the problem. Zhao et al. [2] extended a supply chain problem with multi-stages and devised an optimal integer-ratio coordination policy for inventory replenishment across the considered supply chain.

To the best of our knowledge, although many researches investigated supply chain problems with inventory management to determine the optimal lot-sizes or the optimal order quantities with replenishment cycles, no research has been found considering independent replenishment cycles regardless of integer-ratio replenishment cycles under integrated supply chains, i.e., a supply chain network. In other words, since the traditional policy determines a replenishment cycle among integer-ratio replenishment cycles that only consist of integer multiples of one replenishment cycle taken by a preceding node, it cannot guarantee optimal for network problems. Therefore, we propose a novel supply chain network control problem that considers an independent replenishment cycle under inventory coordination.

In this paper, we demonstrate that all nodes in a network except suppliers have non-negative inventory levels for products which belong to a pathway for a single product type. Note that pathway is a path where raw materials or intermediate products pass through a series of processes in the chain to be finally the end-product. In addition, we introduce some useful properties to calculate the non-negative inventory levels.

The paper is organized as follows. Section 2 describes the problem statement of this problem. In Section 3, some useful properties for the problem are proposed. In Section 4, an example has been shown to describe the cost savings of the proposed policy. Finally, the conclusions and future studies are discussed in Section 5.

2. **Problem Statement.** The configuration of network is given in advance. The network should consist of three echelons or more, which can be represented as suppliers, plants, warehouses, distribution centers, and demand markets. Pathways of a single product type are also given in advance. Hence, all types of intermediate products and raw materials are known. In the network, a production process at one node makes a raw material or intermediate product become end-product or another intermediate product. All nodes except what represent suppliers can replenish inventories toward succeeding nodes. An edge between two nodes can take at most one replenishment cycle with a static order quantity. During planning periods of the network, one of nodes except suppliers has continuous inventory levels. The inventory levels at one node are recorded by incoming replenishments, outgoing replenishments, and production rates during planning periods. We assume that all productions commence immediately one day before replenishment and all production rates are infinite. The objective is to know which edge and how many replenishment quantities with replenishment cycles should be set.

3. **Properties.** We begin this section by introducing some useful properties which not only demonstrate that continuous inventory levels should be non-negatives values but also propose a general formulation for total inventory levels even if there are a number of periodic replenishments and schedules on production. The notations involved in following properties are given as follows:

Notations

ω	network planning periods
X	total quantity of incoming (or outgoing) replenishment during time periods
O_t^{in}	quantity of incoming replenishment at period t $\left(X = \sum_{t=1}^{\omega} O_t^{\text{in}}\right)$
O_t^{out}	quantity of outgoing replenishment at period t $\left(X = \sum_{t=1}^{\omega} O_t^{\text{out}}\right)$

I_t	inventory level at period t
C_t^{in}	cumulated quantity of incoming replenishments at period t $\left(C_t^{\text{in}} = \sum_{a=1}^{t} O_a^{\text{in}}\right)$
C_t^{out}	cumulated quantity of outgoing replenishments at period t $\left(C_t^{\text{out}} = \sum_{a=1}^{t} O_a^{\text{out}}\right)$
p	number of incoming orders
q	number of outgoing orders
T_i^{in}	replenishment cycle of incoming order i
T_j^{out}	replenishment cycle of outgoing order j
N_i^{in}	frequency of incoming order i $\left(T_i^{\text{in}} \cdot N_i^{\text{in}} = \omega\right)$
N_j^{out}	frequency of outgoing replenishment j $\left(T_j^{\text{out}} \cdot N_j^{\text{out}} = \omega\right)$
X_i^{in}	total quantity of incoming order i during time periods $\left(X = \sum_{i=1}^{p} X_i^{\text{in}}\right)$
X_j^{out}	total quantity of outgoing order j during time periods $\left(X = \sum_{j=1}^{q} X_j^{\text{out}}\right)$

In the following three properties, we discuss features of inventory of a single product belonging to a corresponding pathway by not accounting for any production as a preliminary.

Property 3.1. *The total inventory level recorded during the planning period is equal to the difference between the sum of the cumulated quantity of incoming replenishments and cumulated quantity of outgoing replenishments during the planning period.*

Proof: Let in general, inventory level at period t be $I_t = \sum_{a=1}^{t}\left(O_a^{\text{in}} - O_a^{\text{out}}\right)$. Since cumulated quantity of incoming replenishments and cumulated quantity of outgoing replenishments during the planning period are $C_t^{\text{in}} = \sum_{a=1}^{t} O_a^{\text{in}}$ and $C_t^{\text{out}} = \sum_{a=1}^{t} O_a^{\text{out}}$, respectively, we have $I_t = C_t^{\text{in}} - C_t^{\text{out}}$, i.e., $\sum_{t=1}^{\omega} I_t = \sum_{t=1}^{\omega}\left(C_t^{\text{in}} - C_t^{\text{out}}\right)$. Hence, the proposition is established. □

Property 3.2. *The inventory levels of a particular node are always non-negative if the following three conditions are met:*

a) *There are the number of incoming orders p and the number of outgoing orders q, and each incoming (or outgoing) order has its own replenishment cycle.*

b) *Initial incoming replenishment of incoming order i is completed at $t = 1$ regardless of the replenishment cycle, and the initial outgoing replenishment of outgoing order j is completed at $t = T_i^{\text{out}}$ ($T_i^{\text{out}} \neq 0$).*

c) *The total quantity of incoming replenishments is equal to the total quantity of outgoing replenishments, i.e., $\left(\sum_{i=1}^{p} X_i^{\text{in}} = \sum_{j=1}^{q} X_j^{\text{out}}\right)$.*

Proof: Let functions f, g, and h for period t and incoming order i (or outgoing order j) be as $f(t) = I_t$, $g(i,t) = \frac{X_i^{\text{in}}}{N_i^{\text{in}}}\left\lfloor 1 + \frac{t-1}{T_i^{\text{in}}}\right\rfloor$, and $h(j,t) = \frac{X_j^{\text{out}}}{N_j^{\text{out}}}\left\lfloor \frac{t}{T_j^{\text{out}}}\right\rfloor$.

Then, by refining the right side we have $f(t) = \sum_{i=1}^{p} g(i,t) - \sum_{j=1}^{q} h(j,t)$.

Furthermore, we have $f(t) = \sum_{i=1}^{p} \frac{X_i^{\text{in}}}{N_i^{\text{in}}}\left\lfloor 1 + \frac{t-1}{T_i^{\text{in}}}\right\rfloor - \sum_{j=1}^{q} \frac{X_j^{\text{out}}}{N_j^{\text{out}}}\left\lfloor \frac{t}{T_j^{\text{out}}}\right\rfloor$.

Then, by refining the right side of above equation we have

$$\sum_{i=1}^{p}\left[\frac{X_i^{\text{in}}}{N_i^{\text{in}}} - \frac{X_i^{\text{in}}\left\{(t-1) \bmod T_i^{\text{in}}\right\}}{\omega} + \frac{X_i^{\text{in}}(t-1)}{\omega}\right] - \sum_{j=1}^{q}\left[-\frac{X_j^{\text{out}}\left\{t \bmod T_j^{\text{out}}\right\}}{\omega} + \frac{X_j^{\text{out}} \cdot t}{\omega}\right]$$

$$= \left(\sum_{i=1}^{p}\frac{X_i^{\text{in}}}{N_i^{\text{in}}} - \frac{X}{\omega}\right) - \sum_{i=1}^{p}\left[\frac{X_i^{\text{in}}\left\{(t-1) \bmod T_i^{\text{in}}\right\}}{\omega}\right] + \sum_{j=1}^{q}\left[\frac{X_j^{\text{out}}\left\{t \bmod T_j^{\text{out}}\right\}}{\omega}\right].$$

Since $0 \le \sum_{i=1}^{p}\left[\frac{X_i^{\rm in}\{(t-1) \bmod T_i^{\rm in}\}}{\omega}\right] \le \sum_{i=1}^{p}\left[\frac{X_i^{\rm in}(T_i^{\rm in}-1)}{\omega}\right]$ and $0 \le \sum_{j=1}^{q}\left[\frac{X_j^{\rm out}\{t \bmod T_j^{\rm out}\}}{\omega}\right]$ $\le \sum_{j=1}^{q}\left[\frac{X_j^{\rm out}(T_j^{\rm out}-1)}{\omega}\right]$, we have $f(t) \ge \left(\sum_{i=1}^{p}\frac{X_i^{\rm in}}{N_i^{\rm in}} - \frac{X}{\omega}\right) - \sum_{i=1}^{p}\left[\frac{X_i^{\rm in}(T_i^{\rm in}-1)}{\omega}\right]$.

Then, by refining the right side of above equation we have

$$\left(\sum_{i=1}^{p}\frac{X_i^{\rm in}}{N_i^{\rm in}} - \frac{X}{\omega}\right) - \sum_{i=1}^{p}\left[\frac{X_i^{\rm in}\left(T_i^{\rm in}-1\right)}{\omega}\cdot\frac{N_i^{\rm in}}{N_i^{\rm in}}\right] = \left(\sum_{i=1}^{p}\frac{X_i^{\rm in}}{N_i^{\rm in}} - \frac{X}{\omega}\right) - \left(\sum_{i=1}^{p}\frac{X_i^{\rm in}}{N_i^{\rm in}} - \frac{X}{\omega}\right) = 0.$$

Hence, the proposition is established. □

Property 3.3. *The total inventory level of a particular node during planning period is equal to* $\sum_{i=1}^{p} X_i^{\rm in}\frac{(T_i^{\rm in}+\omega)}{2} - \sum_{j=1}^{q} X_j^{\rm out}\left(\frac{\omega-T_j^{\rm out}}{2}+1\right)$ *if the above three conditions are met.*

Proof: Let functions f, g, and h for period t and incoming order i (or outgoing order j) be as $f(t) = I_t$, $g(i,t) = \frac{X_i^{\rm in}}{N_i^{\rm in}}\left\lfloor 1 + \frac{t-1}{T_i^{\rm in}}\right\rfloor$, and $h(j,t) = \frac{X_j^{\rm out}}{N_j^{\rm out}}\left\lfloor\frac{t}{T_j^{\rm out}}\right\rfloor$.

Since the array list of the cumulated quantities of incoming replenishments is identical to an arithmetical progression where there are $T_i^{\rm in}$ of the identical terms, the sum of the cumulated quantity of incoming replenishments associated with incoming order i at period t is equal to $\sum_{t=1}^{\omega} g(i,t) = T_i^{\rm in}\cdot\frac{N_i^{\rm in}\left(\frac{X_i^{\rm in}}{N_i^{\rm in}}+X_i^{\rm in}\right)}{2} = X_i^{\rm in}\cdot\frac{(T_i^{\rm in}+\omega)}{2}$.

Similarly, the array list of the cumulated quantities of outgoing replenishments is identical to an arithmetical progression where the term 0 and $X_i^{\rm in}/N_i^{\rm in}$ are excluded and there are $T_i^{\rm out}$ of the identical terms.

However, since the number of term $X_i^{\rm in}/N_i^{\rm in}$ is unconditionally 1, the sum of the cumulated quantity of outgoing replenishments associated with outgoing order i at period t is equal to $\sum_{t=1}^{\omega} h(j,t) = T_j^{\rm out}\cdot\frac{(N_j^{\rm out}-1)\left\{\frac{X_j^{\rm out}}{N_j^{\rm out}}+\left(X_j^{\rm out}-\frac{X_j^{\rm out}}{N_j^{\rm out}}\right)\right\}}{2} + X_j^{\rm out} = X_j^{\rm out}\left(\frac{\omega-T_j^{\rm out}}{2}+1\right)$.

Since $\sum_{t=1}^{\omega} I(t) = \sum_{t=1}^{\omega}\left\{\sum_{i=1}^{p} g(i,t) - \sum_{j=1}^{q} h(j,t)\right\}$, we have $f\sum_{i=1}^{p} X_i^{\rm in}\frac{(T_i^{\rm in}+\omega)}{2} - \sum_{j=1}^{q} X_j^{\rm out}\left(\frac{\omega-T_j^{\rm out}}{2}+1\right)$.

Hence, the property is established. □

The above properties indicate that when any production rates are not considered, i.e., only incoming and outgoing replenishments for the single product are assumed, the inventory is always non-negative and simply calculated for all planning periods. We note that inventory levels are still discrete because replenishments only provide inventory information at the end of the period.

The following property discusses continuous inventory levels of intermediate products (or the end-product) when a series of production rates as well as incoming and outgoing replenishments of products is considered.

Property 3.4. *If a particular node handles a specific product under the above three conditions with the rules that all production begins one day before replenishment and the infinite production rates are allowed, the total inventory level of intermediate products (or the end-product) is defined by* $\sum_{i=1}^{p} X_i^{\rm in}\frac{(T_i^{\rm in}+\omega)}{2} - \sum_{j=1}^{q} X_j^{\rm out}\left(\frac{\omega-T_j^{\rm out}+3}{2}\right)$.

Proof: Since all production begins one day before replenishment, the sum of quantity of the product in process during a specific time interval is equal to the sum of quantity of outgoing replenishments during a specific time interval.

Hence, the total additional inventory levels across the entire planning period, which are caused by the quantities of the product in process, are equal to the total outgoing replenishments, $\sum_{j=1}^{q}\frac{X_j^{\rm out}}{2}$.

As a result, the property is established by simply adding the total outgoing replenishments to the total inventory levels during planning period. □

Finally, the relevant costs to the proposed policy, which is considering independent replenishment cycles rather than integer-ratio, can be expressed as follows:

$$\sum_{i=1}^{p}\sigma_i N_i^{\text{in}} + \sum_{j=1}^{q}\sigma_j N_j^{\text{out}} + \delta\left\{\sum_{i=1}^{p} X_i^{\text{in}}\frac{\left(T_i^{\text{in}}+\omega\right)}{2} - \sum_{j=1}^{q} X_j^{\text{out}}\left(\frac{\omega - T_j^{\text{out}}+3}{2}\right)\right\} + \theta \quad (1)$$

where σ_i and σ_j are ordering costs per each replenishment, δ is inventory handling cost per unit, and θ is fixed cost of the corresponding node.

In the next section, an example of supply chain network control problem and the resulting cost savings are described through comparison with the existing replenishment policy, which allows only integer-ratio replenishment cycles.

4. **Example.** In this section, we show a calculation experiment to compare the policy proposed in this study with the existing one allowing only integer-ratio replenishment cycles. For comparison in terms of cost, we define total network costs for a supply chain network control problem as the sum of total ordering costs, total handling costs, and total transportation costs. Note that single ordering cost and single handling cost for an edge are defined in Equation (1). Single transportation cost for an edge can be calculated by multiplying a double distance between two nodes, transportation cost parameter, and total quantity of replenishments, and then dividing them into a transportation capacity parameter. We consider an instance involving two suppliers, two retailers, and five customers with the input data shown in Table 1 and Table 2. We assume that the network planning period is set to 360 (days) and then each replenishment cycle should be set to a divisor of the network planning period such as 1, 2, 3, 4, 5, 6, 8, 9, 12, …, 180, and 360. For the same instance, we found the optimal solutions using the proposed model under

Table 1. Parameters corresponding to one supplier, one retailer, five customers, and one transportation mode

Node [a)]	S1	S2	R1	R2	C1	C2	C3	C4	C5
Daily demand (units per day)	–	–	–	–	5	3	6	7	2
Ordering cost ($ per replenishment)	35	30	80	75	–	–	–	–	–
Handling cost ($ per unit)	–	–	1	1	1	1	1	1	1
Transportation capacity (units per vehicle)	1,500								
Transportation cost ($/km/vehicle)	35								

[a)] S1 and S2 are two suppliers, R1 and R2 are two retailers and C1-C5 are five customers.

Table 2. Problem instance corresponding to one supplier, one retailer, and five customers

Node	S1	S2	R1	R2	C1	C2	C3	C4	C5
S1	–	28.48	31.13	10.41	10.62	33.43	29.43	19.84	31.65
S2	28.48	–	8.19	25.02	20.66	27.79	6.51	35.72	10.82
R1	31.13	8.19	–	24.94	25.57	20.94	14.61	33.45	2.72
R2	10.41	25.02	24.94	–	15.50	23.16	28.22	12.52	24.68
C1	10.62	20.66	25.57	15.50	–	34.60	19.98	27.71	26.92
C2	33.43	27.79	20.94	23.16	34.60	–	34.11	22.58	18.44
C3	29.43	6.51	14.61	28.22	19.98	34.11	–	39.86	17.28
C4	19.84	35.72	33.45	12.52	27.71	22.58	39.86	–	32.31
C5	31.65	10.82	2.72	24.68	26.92	18.44	17.28	32.31	–

Table 3. The optimal solutions and objectives corresponding to the proposed policy and the existing policy

Replenishment policy	Independent replenishment cycles							Integer-ratio replenishment cycles							
Node	R1	R2	C1	C2	C3	C4	C5	R1	R2	C1	C2	C3	C4	C5
Assignment		S1	R2	R2	R2	R2	R2	S2	S1	R2	R2	R1	R2	R1
Replenishment quantity	–	46	20	15	24	21	12	30	16	20	18	24	28	12
Replenishment cycle	–	2	4	5	4	3	6	2	2	4	6	4	4	6
Total ordering costs ($)	38,700							41,700						
Total handling costs ($)	28,980							32,580						
Total transportation costs ($)	11,638							9,300						
Total network costs ($)	79,318							83,510						

two replenishment cycle policies. First policy allows independent replenishment cycles and second policy allows only integer-ratio replenishment cycles. Note that the second policy refers to [8,11]. Table 3 describes the optimal solutions and the objectives. The results explain that the proposed policy in this paper offers more effective decisions in terms of minimizing the relevant costs compared with the existing policy, which allows only integer-ratio replenishment cycles.

5. **Conclusions.** In this paper, we considered a supply chain network control problem with independent replenishment cycles under inventory coordination. The objective of the problem is to determine the optimal order quantities and replenishment cycles regardless of integer-ratio replenishment cycles under minimization of the total network costs. We demonstrated that all nodes in a network except suppliers have non-negative inventories and introduced some useful properties to calculate the non-negative inventories. We showed that allowing independent replenishment cycles results in the reduction of total relevant costs as compared to considering only the integer-ratio replenishment cycles. We can expect that the proposed novel policy on independent replenishment cycles will provide more effective solution in supply chain network control problems.

In the future, we will develop a network problem considering multi-pathways of multi-products so that we can cover realistic supply chain network problems such as supply chain for chemicals or energy.

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