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TIME-VARYING LYAPUNOV FUNCTIONAL FOR STABILITY OF SAMPLED-DATA SYSTEMS WITH A TIME-VARYING PERIOD

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Abstract. *In this paper, the problem of stability is investigated for a class of sampled-data linear systems with two classes of sampling period: time-varying period and constant period. This paper first models such a sampled-date input system as a continuous one, where the control input has a piecewise-continuous delay. Then, sufficient conditions in terms of linear matrix inequalities are derived by constructing a class of time-varying Lyapunov functional to achieve the stability of the closed-loop time-delay system. The feature of the constructed Lyapunov functionals is discontinuous at sampling time, but its decrease of such Lyapunov functional at sampling time is guaranteed by construction. Finally, an example is given to show the effectiveness of the proposed method.*
Keywords: Sampled-data systems, Time-delay, Delay-dependent stability, Lyapunov-Krasovskii functionals

1. **Introduction.** In the past decades, sampled-data control has played an increasingly important role in control engineering practice because modern control systems usually employ digital technology for controller implementation. Main issues on sampled-data systems have been extensively studied in the literature [1-8] and the references therein. It is known that a heavy temporary load of computation in a processor can corrupt the sampling period. The stability properties of the system will be affected by the variations of the sampling period [9-12]. In order to avoid this load, it is reasonable to schedule the sampling period in the design. Therefore, this paper will focus on developing robust stability conditions with respect to the variations of sampling period.

Sampled-data systems have been studied extensively and several methods have been proposed in the studies of the sampled-data stabilization. The first one is based on lifting technique [13], in which the problem is transformed into an equivalent finite-dimensional discrete-time problem while maintaining the inter-sampling information of the system. The second approach is based on the impulsive modeling of sampled-data systems in which a time-varying periodic Lyapunov function is used [14,15]. The third approach is based on modeling the sampled-data system as a continuous-time system with delayed control inputs [16,17], which can be applied to systems with variable sampled-data. An input delay approach using the Lyapunov-Krasovskii (LK) theorem is provided in [18]. In addition, discrete-time approaches [19-21], robust analysis techniques [16], impulsive systems formulation and the use of looped-functionals have also been developed to study the stability analysis and/or control synthesis [5, 22, 23]. [18] proposed a novel stability analysis of linear systems with sampled-data inputs by the input delay approach and the stability of impulsive systems. Nevertheless, these methods can still be improved.

This paper will propose the time-varying Lyapunov functional to study the stability of continuous linear systems with sampled-data input. Two classes of sampling period will be considered for time-varying sampling period and constant sampling period, respectively. First, we transform such continuous linear systems with sampled-data input into a continuous linear time-delay system with a piecewise-continuous delay. Then, by the developed time-varying Lyapunov functional, sufficient conditions in terms of linear matrix inequalities are proposed to achieve the decreasing not only at the sampling instants but also during the two successive sampling instants, thus to achieve the stability of the closed-loop system.

The rest of this paper is organized as follows. Section 2 presents the problem formulation. Main results are given in Section 3. Section 4 provides an example. Finally, some conclusions are drawn in Section 5.

Notations: In the paper, the notations used are fairly standard. R^n donates the n-dimensional Euclidean space. $\|\cdot\|$ denotes the Euclidean norm of vector $\cdot$. I denotes an identity matrix of appropriate dimensions. $*$ denotes the symmetric black in one symmetric matrix. $P > 0$ means that P is a real symmetric and positive definite matrix. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalue of a matrix P, respectively. The superscript T stands for matrix transposition.

2. **Problem Formulation.**

Consider the linear continuous time-invariant system:

$$\dot{x} = Ax + Bu, \tag{1}$$

where $x \in R^n$ is the system state variable, $u \in R^m$ is the input vector, A and B are constant matrices with appropriate dimensions.

The controller is actualized in discrete-time under a sampler and zero-order device:

$$u(t) = u_d(t_k) = Kx(t_k), \quad t_k \le t < t_{k+1}, \tag{2}$$

where $K \in R^{n\times m}$ is a given gain, and the sampling instants t_k, $k = 0, 1, 2, \ldots$, satisfies $0 \le t_0 < t_1 < \cdots < t_k < \cdots$. Without loss of generality, we assume that the difference between two successive sampling instants satisfies

$$\tau_1 \le t_{k+1} - t_k \le \tau_2, \quad \forall k = 0, 1, 2, \ldots. \tag{3}$$

Substituting (2) into (1), we obtain the following closed-loop system:

$$\dot{x}(t) = Ax(t) + A_d x\left(t - \tau(t)\right), \quad \tau(t) = t - t_k, \quad t_k \le t < t_{k+1}, \tag{4}$$

where $A_d = BK$, $\tau(t)$ is the time-varying delay. From (3), it follows that $\tau(t) \le \tau_2$.

The objective of this paper is to present sufficient stability conditions to guarantee stability of linear systems with sampled-data controller.

3. **Main Results.**

We will provide the asymptotically stability results for sampled-data systems with variable sampling intervals and constant sampling intervals, respectively.

3.1. **Time-varying sampling period.**

In this section, we first state a theorem to guarantee that system (4) with a time-varying sampling period satisfying (3) is asymptotically stable by employing a piecewise time-varying Lyapunov function.

Theorem 3.1. *Assume that there exist scalars $\mu \ge 1$, τ_2 and τ_1 with $\tau_2 \ge \tau_1 > 0$, $n \times n$ matrices $P_1 > 0$, $P_2 > 0$, $R > 0$ and $S > 0$ and a $2n \times n$ matrix N, such that*

$$P_2 - \mu P_1 \le 0, \tag{5}$$

$$\Pi_{lq} + \mu\tau_2\hat{\Pi} < 0, \tag{6}$$

$$\begin{bmatrix} \Pi_{lq} & \mu\tau_2 N \\ * & -\mu\tau_2 R \end{bmatrix} < 0 \tag{7}$$

hold, where

$$\vartheta_{lq} = \upsilon M_1^T P_l M_1 + M_1^T P_l M_3 + M_3^T P_l M_1 + \frac{1}{\tau_q} M_1^T (P_1 - P_2) M_1, \quad l, q = 1, 2,$$

$$\Pi_{lq} = \vartheta_{lq} - M_2^T S M_2 - N M_2 - M_2^T N^T,$$
$$\hat{\Pi} = M_2^T S M_3 + M_3^T S M_2 + M_3^T R M_3,$$

with $\upsilon = \frac{\ln \mu}{\tau_1}$ *and the matrices* $M_1 = [\, I \quad 0 \,]$, $M_2 = [\, I \quad -I \,]$, $M_3 = [\, A \quad A_d \,]$. *Then system (4) is asymptotically stable for any time-varying period less than* τ_2.

Proof: For any given sampling instant $\{t_k\}$ and the difference between two successive sampling instants satisfies $0 < \tau_1 \le t_{k+1} - t_k \le \tau_2$. When $t \in [t_k, t_{k+1})$, $k \in N$, we define two piecewise linear functions $\rho, \rho_1 \in [t_0, \infty) \to R^+$ as follows:

$$\rho(t) = \frac{t - t_k}{t_{k+1} - t_k}, \quad \tilde{\rho}(t) = 1 - \rho(t), \quad \rho_1(t) = \frac{1}{t_{k+1} - t_k}. \tag{8}$$

It is easy to see that $\rho(t) \in [0, 1)$, $\rho(t_k) = 0$, $\rho\left(t_k^-\right) = \rho\left(t_{k+1}^-\right) = 1$. Moreover, there exists a function $\rho_2(t) \in [0, 1]$ such that: $\rho_1(t) = \frac{1}{\tau_1}\rho_2(t) + \frac{1}{\tau_2}\tilde{\rho}_2(t)$, where $\tilde{\rho}_2(t) = 1 - \rho_2(t)$. For given scalar $\mu \ge 1$, define the piecewise time-varying function $\varphi(t)$ association with $\rho(t)$:

$$\varphi(t) = \mu^{\rho(t)-1}. \tag{9}$$

Then, one can verify that $\frac{1}{\mu} \le \varphi(t) < 1$, $t \ge 0$.

By using the functions $\rho(t)$ and $\varphi(t)$ defined above, when $t \in [t_k, t_{k+1})$, we construct the time-varying Lyapunov function for system (4):

$$V(t, x(t)) = V_1(t, x_t) + V_2(t, x_t), \tag{10}$$

where

$$V_1(t, x(t)) = \varphi(t) x^T(t) P(t) x(t),$$
$$V_2(t, x(t)) = (\tau_2 - \tau(t)) \xi_0^T(t) S \xi_0(t) + (\tau_2 - \tau(t)) \int_{t_k}^{t} \dot{x}^T(s) R \dot{x}(s) ds,$$

with $P(t) = \rho(t) P_1 + \tilde{\rho}(t) P_2$ and $\xi_0(t) = x(t) - x(t_k)$.

It is easy to obtain that $\frac{\lambda_2}{\mu}\|x\|^2 \le V_1(t, x) \le \lambda_1 \|x\|^2$, where $\lambda_1 = \max\{\lambda(P_l), l = 1, 2\}$ and $\lambda_2 = \min\{\lambda(P_l), l = 1, 2\}$. Since $S > 0$ and $R > 0$, then $V_2(t, x) \ge 0$.

To prove the stability of the system, we will show that V is decreasing in each sampling period and decreasing discontinuously at each sampled instant.

Next, we first show that V in (10) is decreasing discontinuously at each sampled instant.

Consider the functional V just before the sampling instant t_k, denoted as t_k^-. Then, we have that $\varphi\left(t_k^-\right) = \mu^{\rho\left(t_k^-\right)-1} = \mu^{1-1} = 1$, $\varphi\left(t_k^+\right) = \mu^{\rho\left(t_k^+\right)-1} = \mu^{0-1} = \frac{1}{\mu}$, $P\left(t_k^-\right) = \rho\left(t_k^-\right) P_1 + \tilde{\rho}\left(t_k^-\right) P_2 = P_1$ and $P\left(t_k^+\right) = \rho\left(t_k^+\right) P_1 + \tilde{\rho}\left(t_k^+\right) P_2 = P_2$. Thus one has that:

$$\begin{aligned}
& V\left(t_k^-, x\left(t_k^-\right)\right) \\
&= \varphi\left(t_k^-\right) x^T\left(t_k^-\right) P\left(t_k^-\right) x\left(t_k^-\right) + \left(\tau_2 - \tau\left(t_k^-\right)\right) \xi_0^T\left(t_k^-\right) S \xi_0\left(t_k^-\right) \\
&\quad + \left(\tau_2 - \tau\left(t_k^-\right)\right) \int_{t_{k-1}}^{t_k^-} \dot{x}^T(s) R \dot{x}(s) ds \\
&= x^T\left(t_k^-\right) P_1 x\left(t_k^-\right) + \left(\tau_2 - \tau\left(t_k^-\right)\right) \xi_0^T\left(t_k^-\right) S \xi_0\left(t_k^-\right) \\
&\quad + \left(\tau_2 - \tau\left(t_k^-\right)\right) \int_{t_{k-1}}^{t_k^-} \dot{x}^T(s) R \dot{x}(s) ds \\
&\ge x^T\left(t_k^-\right) P_1 x\left(t_k^-\right).
\end{aligned} \tag{11}$$

Just after the sampling instant, denoted as $t=t_k^+$, we have that

$$\begin{aligned}&V\left(t_k^+,x\left(t_k^+\right)\right)\\&=\varphi\left(t_k^+\right)x^T\left(t_k^+\right)P\left(t_k^+\right)x\left(t_k^+\right)+\left(\tau_2-\tau\left(t_k^+\right)\right)\xi_0^T\left(t_k^+\right)S\xi_0\left(t_k^+\right)\\&\quad+\left(\tau_2-\tau\left(t_k^+\right)\right)\int\limits_{t_k}^{t_k^+}\dot{x}^T(s)R\dot{x}(s)ds\\&=\frac{1}{\mu}x^T\left(t_k^+\right)P_2x\left(t_k^+\right).\end{aligned}\tag{12}$$

It is easy to see that the last two terms of the above functionals are zero since $\xi_0\left(t_k^+\right)=0$.

By $P_1\geq\frac{1}{\mu}P_2$ in (5), we have that

$$V\left(t_k^-,x\left(t_k^-\right)\right)\geq V\left(t_k^+,x\left(t_k^+\right)\right).\tag{13}$$

Then it means that V in (10) is decreasing discontinuously at each sampled instant.

Now, we calculate the derivative of V during each sampling period $t\in[t_k,t_{k+1})$.

For simplicity, define $\zeta(t)=\left[x^T(t),x^T\left(t_k\right)\right]^T$. Then, we have that $x(t)=M_1\zeta(t)$, $\xi_0(t)=x(t)-x(t_k)=M_2\zeta(t)$, $\dot{x}(t)=Ax(t)+A_dx(t-\tau(t))=M_3\zeta(t)$.

Since $\xi_0(t)-\int_{t_k}^t\dot{x}(s)ds=0$, then $2\zeta^T(t)NM_2\zeta(t)-2\zeta^T(t)N\int_{t_k}^t\dot{x}(s)ds=0$. Since $R>0$, then we can obtain the following inequality:

$$2\zeta^T(t)N\dot{x}(s)\leq\zeta^T(t)NR^{-1}N^T\zeta(t)+\dot{x}^T(s)R\dot{x}(s).\tag{14}$$

Integrating the previous inequality both sides in the interval $[t_k,t]$, where $\dot{x}$ is continuous, we then obtain the following inequality:

$$-\int\limits_{t_k}^{t}\dot{x}^T(s)R\dot{x}(s)ds\leq-2\zeta^T(t)NM_2\zeta(t)+\tau(t)\zeta^T(t)NR^{-1}N^T\zeta(t).\tag{15}$$

Obviously, we can have that:

$$\begin{aligned}&\dot{V}(t,x(t))\\&\leq\varphi(t)x^T(t)[\upsilon P(t)+\rho_1(t)(P_1-P_2)]x(t)+2\varphi(t)x^T(t)P(t)\left[Ax(t)+A_dx\left(t_k\right)\right]\\&\quad+2(\tau_2-\tau(t))\xi_0^T(t)S\dot{x}(t)+(\tau_2-\tau(t))\dot{x}^T(t)R\dot{x}(t)-\xi_0^T(t)S\xi_0(t)\\&\quad-\int\limits_{t_k}^{t}\dot{x}^T(s)R\dot{x}(s)ds\\&\leq\varphi(t)\zeta^T(t)\left[\upsilon M_1^TP(t)M_1+\rho_1(t)M_1^T(P_1-P_2)M_1+2M_1^TP(t)M_3\right]\zeta(t)\\&\quad+(\tau_2-\tau(t))\zeta^T(t)\left(M_2^TSM_3+M_3^TSM_2+M_3^TRM_3\right)\zeta(t)\\&\quad-\zeta^T(t)M_2^TSM_2\zeta(t)-\int\limits_{t_k}^{t}\dot{x}^T(s)R\dot{x}(s)ds\\&\leq\varphi(t)\zeta^T(t)\left\{\upsilon M_1^TP(t)M_1+\rho_1(t)M_1^T(P_1-P_2)M_1+2M_1^TP(t)M_3\right\}\zeta(t)\\&\quad+\zeta^T(t)\left\{\begin{array}{c}(\tau_2-\tau(t))\left(M_2^TSM_3+M_3^TSM_2+M_3^TRM_3\right)\\-M_2^TSM_2-NM_2-M_2^TN^T+\tau(t)NR^{-1}N^T\end{array}\right\}\zeta(t).\end{aligned}\tag{16}$$

We choose a function $\rho_2(t)\in[0,1]$ and $\rho_2(t)=1-\tilde{\rho}_2(t)$, such that $\rho_1(t)=\frac{1}{\tau_1}\rho_2(t)+\frac{1}{\tau_2}\tilde{\rho}_2(t)$. Then, we have

$$\dot{V}(t,x(t))$$

$$\leq \varphi(t)\zeta^T(t)\left\{\begin{array}{l} \upsilon M_1^T(\rho(t)P_1+\tilde{\rho}(t)P_2)M_1+M_1^T\left(\frac{1}{\tau_1}\rho_2(t)+\frac{1}{\tau_2}\tilde{\rho}_2(t)\right)(P_1-P_2)M_1 \\ +2M_1^T(\rho(t)P_1+\tilde{\rho}(t)P_2)M_3 \end{array}\right\}\zeta(t)$$
$$+\zeta^T(t)\left\{\begin{array}{l} (\tau_2-\tau(t))\left(M_2^TSM_3+M_3^TSM_2+M_3^TRM_3\right) \\ +\tau(t)NR^{-1}N^T-M_2^TSM_2-NM_2-M_2^TN^T \end{array}\right\}\zeta(t)$$
$$=\varphi(t)\zeta^T(t)\left\{\begin{array}{l} \rho(t)\left(\upsilon M_1^TP_1M_1+2M_1^TP_1M_3\right)+\tilde{\rho}(t)\left(\upsilon M_1^TP_2M_1+2M_1^TP_2M_3\right) \\ +\left(\frac{1}{\tau_1}\rho_2(t)+\frac{1}{\tau_2}\tilde{\rho}_2(t)\right)M_1^T(P_1-P_2)M_1 \end{array}\right\}\zeta(t)$$
$$+\zeta^T(t)\left\{\begin{array}{l} (\tau_2-\tau(t))\left(M_2^TSM_3+M_3^TSM_2+M_3^TRM_3\right) \\ +\tau(t)NR^{-1}N^T-M_2^TSM_2-NM_2-M_2^TN^T \end{array}\right\}\zeta(t). \tag{17}$$

It is obvious that $\varphi(t)$ satisfies $1 \leq \mu\varphi(t) < \mu$. Then, with the help of (9), we can obtain

$$\begin{aligned}
&\dot{V}(t,x(t)) \\
&\leq \zeta^T(t)[\rho(t)(\rho_2(t)\varphi(t)\vartheta_{11}+\tilde{\rho}_2(t)\varphi(t)\vartheta_{12})]\zeta(t) \\
&\quad+\zeta^T(t)[\tilde{\rho}(t)(\rho_2(t)\varphi(t)\vartheta_{21}+\tilde{\rho}_2(t)\varphi(t)\vartheta_{22})]\zeta(t) \\
&\quad+\zeta^T(t)\left[\begin{array}{l} (\tau_2-\tau(t))\left(M_2^TSM_3+M_3^TSM_2+M_3^TRM_3\right) \\ -M_2^TSM_2-NM_2-M_2^TN^T+\tau(t)NR^{-1}N^T \end{array}\right]\zeta(t) \\
&\leq \varphi(t)\zeta^T(t)\left[\rho(t)\left(\begin{array}{l} \rho_2(t)\left(\Pi_{11}+\mu\tau_2\hat{\Pi}+\mu\tau(t)\left(NR^{-1}N^T-\hat{\Pi}\right)\right) \\ +\tilde{\rho}_2(t)\left(\Pi_{12}+\mu\tau_2\hat{\Pi}+\mu\tau(t)\left(NR^{-1}N^T-\hat{\Pi}\right)\right) \end{array}\right)\right]\zeta(t) \\
&\quad+\varphi(t)\zeta^T(t)\left[\tilde{\rho}(t)\left(\begin{array}{l} \rho_2(t)\left(\Pi_{21}+\mu\tau_2\hat{\Pi}+\mu\tau(t)\left(NR^{-1}N^T-\hat{\Pi}\right)\right) \\ +\tilde{\rho}_2(t)\left(\Pi_{22}+\mu\tau_2\hat{\Pi}+\mu\tau(t)\left(NR^{-1}N^T-\hat{\Pi}\right)\right) \end{array}\right)\right]\zeta(t) \\
&< 0.
\end{aligned} \tag{18}$$

Therefore, system (4) is asymptotically stable.

Remark 3.1. *When $P_1 = P_2$ and $\mu = 1$, the Lyapunov functional V in (10) reduces to the one in [18]. If $P_1 = P_2$ and $\mu = 1$, the condition (5) in Theorem 3.1 can be removed. Thus, in this case, all conditions of Theorem 1 in [18] can be recovered by the ones of Theorem 3.1 obtained in this paper.*

3.2. **Constant sampling period.** Now, we consider the case of constant sampling period, that is, $\tau_1 = \tau_2 \equiv \tau$.

Theorem 3.2. *Assume that there exist positive scalars $\tau > 0$ and $\mu \geq 1$, and $n \times n$ matrices $P_1 > 0$, $P_2 > 0$, $R > 0$, $S > 0$ and $n \times n$ matrix U and $2n \times n$ matrix N, such that*

$$P_2 - \mu P_1 \leq 0, \tag{19}$$
$$\tilde{\Pi}_l + \mu\tau\tilde{\Pi} < 0, \tag{20}$$
$$\left[\begin{array}{cc} \tilde{\Pi}_l & \mu\tau N \\ * & -\mu\tau R \end{array}\right] < 0 \tag{21}$$

hold, where $\tilde{\Pi}_l = \Pi_l - M_4^TUM_2 - M_2^TU^TM_4$, $\tilde{\Pi} = \hat{\Pi} + M_4^TUM_3 + M_3^TU^TM_4$, with $\Pi_l = \vartheta_l - M_2^TSM_2 - NM_2 - M_2^TN^T$, $\hat{\Pi} = M_2^TSM_3 + M_3^TSM_2 + M_3^TRM_3$, $\vartheta_l = \upsilon M_1^TP_lM_1 + M_1^TP_lM_3 + M_3^TP_lM_1 + \frac{1}{\tau}M_1^T(P_1 - P_2)M_1$, $l = 1, 2$, and $\upsilon = \frac{\ln\mu}{\tau}$ and M_i for $i = 1, 2, 3$ are given in Theorem 3.1 and $M_4 = [\ 0 \quad I\ ]$. Then system (4) is asymptotically stable for the constant sampling period τ.

Proof: The proof follows the line of Theorem 3.1. Consider the Lyapunov functional:

$$\tilde{V}_1(t, x(t)) = V(t, x(t)) + \tilde{V}_2(t, x(t)), \tag{22}$$

where $V(t, x(t))$ is defined in (10) and $\tilde{V}_2(t, x(t)) = 2(\tau - \tau(t))x^T(t_k)U\xi_0(t)$. Note that $\tilde{V}_2(t, x(t))$ is not necessary positive. Due to the constant and known sampling period τ, one has:

$$\tilde{V}_2\left(t_k^-, x\left(t_k^-\right)\right) = \tilde{V}_2\left(t_k^+, x\left(t_k^+\right)\right) = 0, \quad \forall k > 0. \tag{23}$$

Nothing that $x(t_k) = (M_1 - M_2)\zeta(t) = M_4\zeta(t)$, the derivative of $\tilde{V}_1$ satisfies:

$$\begin{aligned}
&\dot{\tilde{V}}_1(t, x(t)) \\
&= \dot{V}(t, x(t)) + 2(\tau - \tau(t))\zeta^T(t)M_4^TUM_3\zeta(t) - 2\zeta^T(t)M_4^TUM_2\zeta(t) \\
&\le \zeta^T(t)[\rho(t)(\rho_2(t)\varphi(t)\vartheta_1 + \tilde{\rho}_2(t)\varphi(t)\vartheta_1)]\zeta(t) + \zeta^T(t)[\tilde{\rho}(t)(\rho_2(t)\varphi(t)\vartheta_2 + \tilde{\rho}_2(t)\varphi(t)\vartheta_2)]\zeta(t) \\
&\quad + \zeta^T(t)\begin{bmatrix} (\tau - \tau(t))\left(M_2^TSM_3 + M_3^TSM_2 + M_3^TRM_3 + M_4^TUM_3 + M_3^TU^TM_4\right) \\ - M_2^TSM_2 - NM_2 - M_2^TN^T - M_4^TU^TM_2 - M_2^TU^TM_4 \\ + \tau(t)NR^{-1}N^T \end{bmatrix}\zeta(t) \\
&\le \varphi(t)\zeta^T(t)\rho(t)\rho_2(t)\left(\tilde{\Pi}_1 + \mu\tau\tilde{\Pi} + \mu\tau(t)\left(NR^{-1}N^T - \tilde{\Pi}\right)\right)\zeta(t) \\
&\quad + \varphi(t)\zeta^T(t)\rho(t)\tilde{\rho}_2(t)\left(\tilde{\Pi}_1 + \mu\tau\tilde{\Pi} + \mu\tau(t)\left(NR^{-1}N^T - \tilde{\Pi}\right)\right)\zeta(t) \\
&\quad + \varphi(t)\zeta^T(t)\tilde{\rho}(t)\rho_2(t)\left(\tilde{\Pi}_2 + \mu\tau\tilde{\Pi} + \mu\tau(t)\left(NR^{-1}N^T - \tilde{\Pi}\right)\right)\zeta(t) \\
&\quad + \varphi(t)\zeta^T(t)\tilde{\rho}(t)\tilde{\rho}_2(t)\left(\tilde{\Pi}_2 + \mu\tau\tilde{\Pi} + \mu\tau(t)\left(NR^{-1}N^T - \tilde{\Pi}\right)\right)\zeta(t) \\
&< 0.
\end{aligned} \tag{24}$$

Then, system (4) is asymptotically stable for the constant sampling period τ satisfying (20) and (21).

Remark 3.2. *When $P_1 = P_2$ and $\mu = 1$, the Lyapunov functional $\tilde{V}_1(t, x(t))$ in (22) reduces to the one in [18]. If $P_1 = P_2$ and $\mu = 1$, the condition (19) in Theorem 3.2 can be removed. Thus, in this case, all conditions of Theorem 2 in [18] can be recovered by the ones of Theorem 3.2 obtained in this paper.*

4. **Example.**

Consider system (1) from [18] with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}$$

and

$$A_d = \begin{bmatrix} 0 & 0 \\ -0.375 & -1.15 \end{bmatrix}.$$

When $\tau = 1.71986789$, from Theorem 3.2, we obtain

$$P_1 = \begin{bmatrix} 33.6761 & 81.4631 \\ 81.4631 & 202.0685 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 33.6728 & 81.4629 \\ 81.4629 & 202.0675 \end{bmatrix},$$

$$R = \begin{bmatrix} 34.0198 & 105.5804 \\ 105.5804 & 328.6493 \end{bmatrix}, \quad S = \begin{bmatrix} 0.7145 & 0.7265 \\ 0.7265 & 0.7409 \end{bmatrix},$$

$$N = \begin{bmatrix} 0.6034 & 0.8925 \\ 24.4563 & 76.0176 \\ -35.0234 & -108.4864 \\ -106.6143 & -331.7436 \end{bmatrix}, \quad U = \begin{bmatrix} 4.3197 & 14.6320 \\ 12.7308 & 44.1289 \end{bmatrix}.$$

The maximum allowable constant sampling period obtained by Theorem 3.2 is bigger than 1.7198 obtained in [18]. Therefore, it can be seen that the results from Theorem 3.2 are less conservative than the one in [18].

5. **Conclusions.** Using the time-varying Lyapunov functional approach, the stability problem of sampled-data linear systems has been investigated in this paper. Two types of sampling period, time-varying and constant sampling period are respectively considered. First, such a sampled-date input system has been modelled as a continuous time-varying delay system, where the control input has a piecewise-continuous delay. Then, to guarantee the stability of the closed-loop delay system, sufficient conditions in terms of linear matrix inequalities are developed by using time-varying Lyapunov functional method. Based on the constructed time-varying Lyapunov functional, such a functional is forced to decrease not only at the sampling instants but also during the two successive sampling instants. In the future we will focus on researching the feature of the systems with parameter uncertainties.

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