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PREMISE REDUCTION OF MISO FUZZY SYSTEMS BASED ON ENTROPY WEIGHTS

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Abstract. *There exists deviation caused by some subjective experience for rule premise reduction in reasoning process of MISO fuzzy systems, and how to objectively determine the weights of rule antecedent components in MISO fuzzy systems is worth studying. In this paper, information entropy is used to determine the weights of antecedent components in rule premise reduction, which not only shows relative importance of antecedent components in fuzzy reasoning, but avoids the subjective deviation by dynamically adjusting weights. In addition, it is proved that the reduction does not change the interpolation approximation of the improved MISO fuzzy systems. Therefore, this premise reduction method is practicable.*

Keywords: Fuzzy reasoning, MISO fuzzy system, Entropy weight, Multifactorial function, Interpolation approximation

1. **Introduction.** Reasonable and effective design methods of fuzzy inference engine are very important for fuzzy control systems. In 1973, Zadeh proposed CRI (Compositional Rule of Inference) [1] such that the research of fuzzy inference and fuzzy systems develops rapidly. Furthermore, Mamdani et al. discussed the mechanism of fuzzy control systems and found some significant conclusions [2-13]. Particularly, Li proved that a fuzzy control system is an interpolation function in mathematical essence [14], and proposed variable universe adaptive fuzzy control theory [15,16].

Obviously, the application of fuzzy systems has achieved remarkable success in practice for the past few decades. However, the logical foundation and some details in inference engine of fuzzy systems, especially MISO (Multiple-Input Single-Output) fuzzy systems, are worthy of further study [17-20]. From the various expressions of the output functions of MISO fuzzy systems, although the inference methods are different, the output functions of existing MISO fuzzy systems can be represented approximately as weighted average functions of the center of the peak points of rule consequents [19], in which the determination of the weights is very critical, and the weights are closely related to the reduction of antecedent state values in reasoning process.

In traditional premise reduction methods, the weights of rule antecedents are usually given by specialists in various fields, which causes subjective deviation in premise reduction. This paper focuses on the premise reduction of MISO fuzzy systems and tries to use information entropy to solve the deviation issue.

Shannon proposed information entropy firstly in 1948 [21]. Information entropy is a measurement of systemic confusion that has many important applications in cybernetics, probability theory, number theory, astrophysics, life science, information science and so on [22-26]. In multiple objective decision making, entropy can reduce the subjective deviation in weight determination [24]. Entropy is simple and feasible that calculates weights objectively according to the information given by index data. We can reduce the state values of antecedent components of fuzzy rules by using entropy weights so that the weights of antecedent components can be dynamically adjusted according to various input values. In this paper, a new premise reduction algorithm for MISO fuzzy systems based on entropy weights is given.

The paper is organized as follows. In Section 2, we give some preliminary knowledge. In Section 3, according to the distribution of membership degrees of input values for antecedent fuzzy sets, we use the entropy to determine the weights of all the antecedent components in reasoning process, and give a new premise reduction algorithm for MISO fuzzy systems. In Section 4, we discuss the interpolation approximation property of the improved MISO fuzzy systems based on entropy weights, followed by conclusion in Section 5.

2. **Preliminaries.**

2.1. **General form of output functions of MISO fuzzy systems.**

By analyzing mathematical expressions of output functions of existing fuzzy systems, we give the following general form of MISO fuzzy systems. Firstly, it is necessary to know several signs. Let $X_1, X_2, \ldots, X_m$ be m input universes, and Y be an output universe. $\mathcal{A}_1 = \{A_{i_1 1} \mid 1 \leq i_1 \leq n_1\}$, $\mathcal{A}_2 = \{A_{i_2 2} \mid 1 \leq i_2 \leq n_2\}$, …, $\mathcal{A}_m = \{A_{i_m m} \mid 1 \leq i_m \leq n_m\}$, $\mathcal{B} = \{B_{i_1 i_2 \ldots i_m} \mid 1 \leq i_j \leq n_j, j = 1, 2, \ldots, m\}$ are the fuzzy partitions to $X_1, X_2, \ldots, X_m, Y$, respectively. For convenience, we suppose $X_1, X_2, \ldots, X_m, Y$ are all $[0, 1]$, and $x_{i_j j}$, $y_{i_1 i_2 \ldots i_m}$ are the peak points of $A_{i_j j}$, $B_{i_1 i_2 \ldots i_m}$ $(1 \leq i_j \leq n_j,\ j = 1, 2, \ldots, m)$ respectively, where

$$0 \leq x_{11} < \cdots < x_{n_1 1} \leq 1, \ldots, 0 \leq x_{1m} < \cdots < x_{n_m m} \leq 1, 0 \leq y_1 < \cdots < y_{n_1 n_2 \ldots n_m} \leq 1.$$

$\mathcal{A}_1, \ldots, \mathcal{A}_m, \mathcal{B}$ can be regarded as linguistic variables. Then $n_1 n_2 \ldots n_m$ fuzzy rules are generated as follows,

If x_1 is $A_{i_1 1}$ and x_2 is $A_{i_2 2}$ and $\cdots$ and x_m is $A_{i_m m}$ then y is $B_{i_1 i_2 \ldots i_m}$.

Most of the output functions of existing fuzzy systems can be represented approximately as weighted average functions of the center of the peak points of rule consequents, where the center of the peak point y of a rule consequent satisfies $B_{i_1 i_2 \ldots i_m}(y) = 1$. The $i_1 i_2 \ldots i_m$th weight is equal to the activation degree of the $i_1 i_2 \ldots i_m$th rule, denoted by $\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)$, $i_j = 1, 2, \ldots, n_j$; $j = 1, 2, \ldots, m$. The output function of a MISO fuzzy system can be approximated as follows,

$$f(x_1, x_2, \ldots, x_m) = \frac{\sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_m=1}^{n_m} \mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m) y_{i_1 i_2 \ldots i_m}}{\sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_m=1}^{n_m} \mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)}. \tag{1}$$

In various fuzzy inference engines, output functions of some familiar fuzzy reasoning algorithms can be approximately reduced to Equation (1). These algorithms include CRI (Compositional Rules of Inference) algorithm, $(+, \cdot)$-centroid algorithm, simple reasoning algorithm, function reasoning algorithm, characteristic expansion reasoning algorithm, full implication triple I algorithm, and so on [14,27]. $\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)$ is different from one another in these algorithms, but the output functions have one thing in common, that is, $\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)$ is related to the membership degrees of the input values for the antecedents of every rule. Input-output data pairs can be considered as the central values of the peak points of antecedents and consequents in reasoning rules. Therefore, the key to design of MISO fuzzy systems lies in how to calculate $\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)$. Multifactorial functions are valid tools for decreasing dimensions in factor spaces [17], and $\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)$ can be constructed by multifactorial functions [19].

Remark 2.1. *In this paper, all of antecedent fuzzy sets satisfy consistency, i.e., if there exists fuzzy set* A_k *such that* $A_k(x_0) = 1$*, then* $A_j(x_0) = 0$ *for* $\forall j \neq k$.

2.2. **Entropy weights.** Entropy is used as a measurement of uncertainty in information theory [26]. The larger the entropy value, the greater the uncertainty.

Definition 2.1. *Suppose a system is in* m *states* $S_1, S_2, \ldots, S_m$*,* p_i *is the probability that the system is in the state* S_i*, where* $i = 1, 2, \ldots, m$*;* $0 \leq p_i \leq 1$*, and* $\sum_{i=1}^{m} p_i = 1$*. The entropy* $H(p_1, p_2, \ldots, p_n)$ *can be calculated by the following formula,*

$$H(p_1, p_2, \ldots, p_n) = -k \sum_{i=1}^{n} p_i \ln p_i. \tag{2}$$

Particularly, the entropy H *has the unique form*

$$H(p_1, p_2, \ldots, p_n) = -\sum_{i=1}^{n} p_i \ln p_i, \tag{3}$$

if H *satisfies the following three conditions,*

(1) $H(p_1, p_2, \ldots, p_n) \leq H\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ *(extremum property);*

(2) $H(p_1, p_2, \ldots, p_n) = H(p_1, p_2, \ldots, p_n, 0)$*;*

(3) $H(A, B) = H(A) + H(B/A)$.

Remark 2.2. *In Equation (2) and Equation (3), we set* $0 \ln 0 = 0$ *when* $\exists p_i = 0$ $(i = 1, 2, \ldots, n)$.

3. **MISO Fuzzy Systems Based on Entropy Weights.** In [19], multifactorial functions are used to calculate $\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)$. The following lists several common forms of $\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m)$ based on multifactorial functions.

Example 3.1.

$$\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m) = \sum_{j=1}^{m} \omega_j A_{i_j j}(x_j),$$

where ω_j *is the weight of the* j*th antecedent component in fuzzy reasoning for an input variable* $(x_1, x_2, \ldots, x_m)$*,* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{m} \omega_j = 1$.

Example 3.2.

$$\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m) = \left[\sum_{j=1}^{m} \omega_j \left(A_{i_j j}(x_j)\right)^p\right]^{\frac{1}{p}},$$

where $p > 0$*,* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{m} \omega_j = 1$.

Example 3.3.

$$\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m) = \bigvee_{j=1}^{m} \omega_j A_{i_j j}(x_j),$$

where $\omega_j \in [0, 1]$ *and* $\bigvee_{j=1}^{m} \omega_j = 1$.

Example 3.4.

$$\mu_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m) = \bigvee_{j=1}^{m} \left(\omega_j \wedge A_{i_j j}(x_j)\right),$$

where $\omega_j \in [0, 1]$ *and* $\bigvee_{j=1}^{m} \omega_j = 1$.

Entropy can avoid the subjective deviation in traditional weighting methods, so it can objectively reflect the values of ω_j $(j = 1, 2, \ldots, m)$ in reasoning process.

Therefore, we can adjust the values of ω_j $(j = 1, 2, \ldots, m)$ dynamically according to input values $x_1, x_2, \ldots, x_m$ with entropy. If the membership degrees of the input component value x_j for fuzzy sets $A_{i_j j}$ $(i_j = 1, 2, \ldots, n_j)$ have great difference, then the jth antecedent component has a great effect on reasoning. On the contrary, according to the extremum property of entropy weights, if the membership degrees of the input component value x_j for fuzzy sets $A_{i_j j}$ $(i_j = 1, 2, \ldots, n_j)$ are almost equal, then the jth antecedent component has little effect on reasoning.

According to the analysis above, let

$$H_j = -\sum_{i_j=1}^{n_j} A_{i_j j}(x_j) \ln A_{i_j j}(x_j), \quad (j = 1, 2, \ldots, m). \tag{4}$$

The entropy value is the maximum when the values of $A_{i_j j}(x_j)$ are equal for all $i_j = 1, 2, \ldots, n_j$, i.e., $\max H_j = \ln n_j$. The normalized entropy value e_j of the jth antecedent component is given as follows,

$$e_j = \frac{1}{\ln n_j} \cdot H_j, \quad (j = 1, 2, \ldots, m). \tag{5}$$

The maximum value of e_j is 1 when the values of $A_{i_j j}(x_j)$ are equal for all $i_j = 1, 2, \ldots, n_j$, and $0 \le e_j \le 1$. Thus, we can define the weight of the jth antecedent component in reasoning for the input value x_j, that is,

$$w_j = \frac{1}{m - \sum\limits_{j=1}^{m} e_j}(1 - e_j), \tag{6}$$

where $0 \le w_j \le 1$ and $\sum_{j=1}^{m} w_j = 1$.

4. **Interpolation Approximation of the MISO Fuzzy Systems Based on Entropy Weights.** We discuss whether the MISO fuzzy systems based on entropy weights have interpolation property or not from the point of view of function approximation.

For a given $x^* \triangleq (x_1^*, x_2^*, \ldots, x_m^*) \in X_1 \times X_2 \times \cdots \times X_m$, we use singleton fuzzification, i.e.,

$$A^*(x) = \begin{cases} 1, & x = x^*, \\ 0, & x \neq x^*. \end{cases}$$

Theorem 4.1. *On the basis of above-mentioned assumption, there exists a group of base elements* $\Phi = \{\phi_{i_1 i_2 \ldots i_m} \mid 1 \le i_1 \le n_1, \ldots, 1 \le i_m \le n_m\}$ *such that the MISO fuzzy systems based on entropy weights can be expressed as some piecewise interpolation functions that take* $\phi_{i_1 i_2 \ldots i_m}$ *as their base functions, that is,*

$$f(x_1, x_2, \ldots, x_m) = \sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_m=1}^{n_m} \phi_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m) y_{i_1 i_2 \ldots i_m}. \tag{7}$$

Proof: For convenience, we take Example 3.1 to prove. Examples 3.2, 3.3 and 3.4 have similar proofs. According to the interpolation mechanism of fuzzy systems in [14], for the given $(x_1^*, x_2^* \ldots, x_m^*) \in X_1 \times X_2 \times \cdots \times X_m$, the output of the fuzzy systems should be

$$y^* = f(x_1^*, x_2^*, \ldots, x_m^*) = \frac{\sum\limits_{i_1=1}^{n_1} \sum\limits_{i_2=1}^{n_2} \cdots \sum\limits_{i_m=1}^{n_m} \mu_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*) y_{i_1 i_2 \ldots i_m}}{\sum\limits_{i_1=1}^{n_1} \sum\limits_{i_2=1}^{n_2} \cdots \sum\limits_{i_m=1}^{n_m} \mu_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*)}. \tag{8}$$

Let

$$\alpha_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*) = \mu_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*),$$

$$\beta_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*) = \frac{1}{\sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_m=1}^{n_m} \alpha_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*)},$$

$$\phi_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*) = \beta_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*) \cdot \alpha_{i_1 i_2 \ldots i_m}(x_1^*, x_2^*, \ldots, x_m^*).$$

When $x_1^* = x_{i_1 1}, x_2^* = x_{i_2 2}, \ldots, x_m^* = x_{i_m m}$, then $A_{i_1 1}(x_{i_1 1}) = A_{i_2 2}(x_{i_2 2}) = \cdots = A_{i_m m}(x_{i_m m}) = 1$, $A_{k_1 1}(x_{i_1 1}) = A_{k_2 2}(x_{i_2 2}) = \cdots = A_{k_m m}(x_{i_m m}) = 0$ $(k \neq i)$. According to Equations (4)-(6), $e_1 = e_2 = \cdots = e_m = 0$, furthermore, $w_1 = w_2 = \cdots = w_m = \frac{1}{m}$.

On the basis of the above, we can get $y^* = y_{i_1 i_2 \ldots i_m}$.

If we put

$$f(x_1, x_2, \ldots, x_m) \triangleq \sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_m=1}^{n_m} \phi_{i_1 i_2 \ldots i_m}(x_1, x_2, \ldots, x_m) y_{i_1 i_2 \ldots i_m},$$

then (7) is obtained.

Remark 4.1. *From Theorem 4.1, $w_j = 0$ when there exists $j \in \{1, 2, \ldots, m\}$ such that $A_{i_j j}\left(x_j^*\right)$ are equal for all $i_j = 1, 2, \ldots, n_j$, and the jth antecedent component has no effect on reasoning.*

5. **Conclusion.** By analyzing aggregation problem of antecedent components in MISO fuzzy systems, we used entropy to reduce the state values of antecedent components of fuzzy rules. The method not only shows the relative importance of every antecedent component in MISO fuzzy systems, but also can dynamically adjust the weights of antecedent components according to various input values. In addition, the improved MISO fuzzy systems still have the interpolation property in function approximation. Therefore, the proposed premise reduction method has theoretical and practical significance. In view of approximation properties of MISO fuzzy systems to unknown multivariate functions, we will study the approximation accuracy of MISO fuzzy systems based on entropy weights in future.

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