ICIC Express Letters
Volume **12**, Number **6**, June **2018**

ICIC International ©2018 ISSN 1881-803X
pp. 519–527

DIFFERENTIAL EVOLUTION AND GENETIC ALGORITHM METHODS FOR PARAMETER ESTIMATION OF THE GENERALIZED HALF NORMAL DISTRIBUTION WITH HYBRID CENSORING

Hua Xin[1], Jianping Zhu[1,2] and Tzong-Ru Tsai[3,*]

[1]School of Mathematics and Statistics
Northeast Petroleum University
No. 99, Xuefu Street, Daqing 163318, P. R. China

[2]School of Management and Data-Mining Research Center
Xiamen University
No. 422, Siming South Road, Xiamen 361005, P. R. China

[3]Department of Statistics
Tamkang University
Tamsui Dist., New Taipei City 25137, Taiwan
[*]Corresponding author: tzongru@gms.tku.edu.tw

Received November 2017; accepted February 2018

Abstract. *Hybrid censoring is an efficient scheme for saving test time and cost. However, a hybrid censoring life test often induces a complicated likelihood function and makes the search of maximum likelihood estimates (MLEs) of the model parameters difficult. In this paper, we use the differential evolution (DE) and genetic algorithm (GA) methods to obtain the MLEs of generalized half normal distribution (GHN) parameters, based on hybrid censoring samples, through Monte Carlo simulations. Simulation results show that the DE method outperforms the GA method with smaller bias and mean squared error. The DE method has great potential instead of gradient computation methods to obtain the MLEs of GHN parameters from a hybrid censoring life test.*

Keywords: Differential evolution method, Genetic algorithm, Maximum likelihood estimation method, Quasi-Newton method

1. **Introduction.** For saving test cost and time, censoring methods have been widely applied to shortening the duration of life test. The type I and type II censoring schemes are two most widely used censoring methods for implementing life tests. When implementing a life test with type I censoring scheme, the life test is terminated at a predetermined testing time and the surviving products are treated as censoring. A life test with type II censoring is implemented to stop the life test when a predetermined number of failed products are obtained. It is easier to implement statistical inference based on type II censoring samples than that based on type I censoring samples. However, the experimental time of a type I censoring test is predetermined and the experimental time of a type II censoring test is random. The type I and type II censoring schemes have merits and also contains shortcomings for practical applications. Reliability inferences using type I or type II censoring schemes have been widely studied. Comprehensive introduction for using type I and type II censoring schemes can be found in [1,2].

Hybrid censoring scheme can be an alternative from the type I and type II censoring schemes. The hybrid censoring scheme is a mixture of the type I and type II censoring schemes, which was initially studied by [3]. A hybrid censoring test is terminated when either reaching a predetermined time (say T) or obtaining a predetermined number of failed products (say r). Hence, the hybrid censoring test can be terminated at a random time of $\min(X_{r:n}, T)$, where $X_{i:n}$ is the ith shortest lifetime of products in the life test for $1 \leq i \leq n$. This censoring scheme is known as hybrid type I censoring. It is noted that the type I and type II censoring schemes are special cases of hybrid type I censoring scheme. The working data for statistical inference can be either the $\{X_{1:n}, X_{2:n}, \ldots, X_{r:n}\}$ if $X_{r:n} < T$ or $\{X_{1:n}, X_{2:n}, \ldots, X_{m:n}\}$ if $X_{m+1:n} > T$ and $m < r$. Some reliability studies about using the hybrid censoring scheme for life test are [4-12], in which Ahmadi et al. [4] proposed estimation methods for the GHN distribution based on progressive type II censoring samples. Asgharzadeh et al. [5] suggested methods to predict future failures in Weibull distribution with hybrid censoring. Balakrishnan and Kundu [6] studied hybrid censoring methods and their applications. Singh and Tripathi [12] proposed Bayesian estimation and prediction methods for a hybrid censored lognormal distribution.

On the basis of the findings in this study, we would like to recommend potential parameter estimation methods other than the gradient methods for practitioners to obtain reliable MLEs of the GHN distribution parameters from hybrid censoring samples. The quasi-Newton method is a popular gradient method in reliability applications to search the MLEs of the model parameters from censoring samples. However, the quasi-Newton method often fails to obtain reliable MLEs of the model parameters if the target function is complicated. Complicated censoring schemes are useful for users to save test time, but such schemes often bring a complicated target function and then make the quasi-Newton method fail to obtain reliable estimates of the model parameters to optimize the target function.

Because the likelihood function based on the hybrid type I censoring samples is very complicated, it is difficult to obtain reliable MLEs of the distribution parameters. In this paper, we would like to study the performance of using the methods of DE and GA to obtain the MLEs of the GHN distribution parameters. Other soft computing methods, for example, the particle swarm optimization method, could also have potential to improve the estimation performance of gradient methods. However, the DE and GA methods have been well studied and many free packages are available for users to implement the DE and GA methods. Hence, the DE and GA methods are considered in this study for obtaining the MLEs of the GHN distribution parameters.

The rest of this paper is organized as follows. The GHN distribution will be addressed and reviewed in Section 2. Moreover, the DE and GA methods will be reviewed in Section 2. In Section 3, intensive Monte Carlo simulations are conducted to evaluate the performance of the DE and GA methods to obtain the MLEs of the GHN distribution parameters. Finally, some conclusions are given in Section 4.

2. **Statistical Model and Parameter Estimation Methods.** Let X denote the lifetime of product that follows the GHN. The probability density function (pdf) of GHN can be presented by

$$f(x;\alpha,\beta) = \sqrt{\frac{2}{\pi}}\left(\frac{\alpha}{x}\right)\left(\frac{x}{\beta}\right)^{\alpha}\exp\left\{-\frac{1}{2}\left(\frac{x}{\beta}\right)^{2\alpha}\right\},\ x>0,\ \alpha>0,\ \beta>0, \tag{1}$$

where α is shape parameter and β is scale parameter (see [13]). Implementing reparameterization for the pdf in Equation (1) through using $\theta = \beta^{-2\alpha}$, we obtain

$$f(x;\alpha,\theta) = \sqrt{\frac{2}{\pi}}\alpha\sqrt{\theta}x^{\alpha-1}\exp\left\{-\frac{1}{2}\theta x^{2\alpha}\right\},\ x>0,\ \alpha>0,\ \theta>0. \tag{2}$$

The cumulative density function based on Equation (2) can be obtained as

$$F(x;\alpha,\theta) = 2\Phi\left(\sqrt{\theta}x^{\alpha}\right) - 1 = 1 - 2\Phi\left(-\sqrt{\theta}x^{\alpha}\right),\ x>0,\ \alpha>0,\ \theta>0, \tag{3}$$

where $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{t^2}{2}\right\} dt$ is the cumulative density function of the standard normal distribution. The survival function of the GHN can be obtained by

$$S(x;\alpha,\theta) = 1 - F(x;\alpha,\theta) = 2\left[1 - \Phi\left(\sqrt{\theta}x^{\alpha}\right)\right] = 2\Phi\left(-\sqrt{\theta}x^{\alpha}\right). \quad (4)$$

The hybrid type I censoring sample consists of the order statistics $X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{a:n}$ where $a = r$ if $X_{r:n} < T$ and $a = m$ if $m < r$ and $X_{(m+1):n} > T$. We can obtain the likelihood function for the hybrid type I censoring sample by

$$\begin{aligned} L(\alpha,\theta) &= \prod_{i=1}^{a} f(x_{i:n};\alpha,\theta)[S(x_{a:n};\alpha,\theta)]^{n-a} \\ &= 2^{n-a/2}\pi^{-a/2}\alpha^{a}\theta^{a/2}\prod_{i=1}^{a} x_{i:n}^{\alpha-1}\exp\left\{-\frac{1}{2}\theta x_{i:n}^{2\alpha}\right\}\left[\Phi\left(-\sqrt{\theta}x_{a:n}^{\alpha}\right)\right]^{n-a}. \end{aligned} \quad (5)$$

In particular, we can present the log-likelihood function based on the $L(\alpha,\theta)$ in Equation (5) by

$$\ell(\alpha,\theta) \propto a\ln\alpha + \frac{a}{2}\ln\theta + (\alpha-1)\sum_{i=1}^{a}\ln x_{i:n} - \frac{\theta}{2}\sum_{i=1}^{a} x_{i:n}^{2\alpha} + (n-a)\ln\left(\Phi\left(-\sqrt{\theta}x_{a:n}^{\alpha}\right)\right). \quad (6)$$

The MLEs of α and θ, denoted by $\hat{\alpha}$ and $\hat{\theta}$, are the solutions of the following two likelihood equations:

$$\frac{a}{\alpha} + \sum_{i=1}^{a}\ln x_{i:n} - \theta\sum_{i=1}^{a}\ln x_{i:n}^{2\alpha}\ln x_{i:n} - (n-a)\frac{\sqrt{\theta}x_{a:n}^{\alpha}(\ln x_{a:n})\exp\left\{-\frac{1}{2}\theta x_{a:n}^{2\alpha}\right\}}{\sqrt{2\pi}\Phi\left(-\sqrt{\theta}x_{a:n}^{\alpha}\right)} = 0, \quad (7)$$

and

$$\frac{a}{\theta} - \sum_{i=1}^{a} x_{i:n}^{2\alpha} - (n-a)\frac{x_{a:n}^{\alpha}\exp\left\{-\frac{1}{2}\theta x_{a:n}^{2\alpha}\right\}}{\sqrt{2\pi\theta}\Phi\left(-\sqrt{\theta}x_{a:n}^{\alpha}\right)} = 0. \quad (8)$$

It is very difficult to obtain $\hat{\alpha}$ and $\hat{\theta}$ by simultaneously solving Equations (7) and (8) through using gradient computation methods, for example, the quasi-Newton method, see [14]. The soft computing methods of DE and GA are hence employed in this study to search $\hat{\alpha}$ and $\hat{\theta}$ to maximize the $L(\alpha,\theta)$ in Equation (5). The GA was firstly studied by Holland in 1975, see [15]. The GA is an evolutionary algorithm, which uses the technique inspired by natural evolution to generate solutions to optimize the target function. The DE is another potential heuristic method to improve the estimation performance of the quasi-Newton method. DE also uses evolutionary computation algorithm to optimize the target function through iteratively trying to improve a candidate solution utilizing a specific measure of quality. The DE method uses actual real number, and the ideas of mutation and crossover in DE are substantially different from that in the GA. The mutation and crossover operations in the DE method create a new vector through using the difference between two or more vectors in the population such that the DE could have higher opportunities to reach optimal solutions than the GA method in many instances. Both GA and DE do not use gradient function to search optimal solutions. Some recent studies regarding using GA and DE methods for optimization can be found in [16-20].

The principal of GA (see [18]) is briefly given as follows.

1) Choose an initial population.
2) Determine the fitness of each individual.
3) Perform selection.
4) Perform crossover and mutation.
5) Determine the fitness of each individual.

6) Perform selection.
7) Repeat the above processes until reaching a termination condition.

Common termination condition(s) can be one of the following conditions, or combinations of them.

1) Reach a solution to meet the specific criteria.
2) Reach a fixed number of iterations.
3) Reach the allocated budget.
4) Reach the highest ranking solution's fitness, or cannot improve the solutions by successive iterations.

The principal of DE (see [20]) is briefly given as follows.

1) Choose an initial population.
2) Determine the fitness of each individual.
3) Perform mutation.
4) Perform recombination.
5) Perform selection.
6) Determine the fitness of each individual.
7) Repeat the above process until reaching a termination condition.

Common termination condition(s) can be one of the following conditions, or combinations of them.

1) Reach a solution to meet the specific criteria.
2) Reach the fixed number of iterations.
3) Reach the allocated budget.
4) Reach the highest ranking solution's fitness, or cannot improve the solutions by successive iterations.

3. **Simulations.** For implementing the GA method, we consider the parameters of population size PS = 100, crossover probability CP = 0.7 and 0.8, mutation probability MP = 0.1 and 0.15 and maximum number of iterations MI = 100 to search the MLEs $\hat{\alpha}$ and $\hat{\theta}$ to maximize the target function in Equation (5). The parameters of crossover probability CP = 0.5, weighting factor WF = 0.6 and 0.8 and the maximum number of iterations MI = 150 are used to implement the DE method for searching the MLEs $\hat{\alpha}$ and $\hat{\theta}$ to maximize the target function in Equation (5). The GHN parameters are set up as that in Table 1, where we consider T as the 75th percentile of the GHN(α, θ).

TABLE 1. The selected parameters of GHN(α, θ) and T for simulations

α	β	θ	T
1	2	0.2500	2.3006
1	3	0.1111	3.4510
1	4	0.0625	4.6014
2	2	0.0625	2.1451
2	3	0.0123	3.2176
2	4	0.0039	4.2902

Based on $F(x; \alpha, \theta) = 2\Phi\left(\sqrt{\theta}x^{\alpha}\right) - 1$ in Equation (3), the pth quantile can be obtained through using the equation of $x_p = \left[\Phi^{-1}\left(\frac{p+1}{2}\right)/\sqrt{\theta}\right]^{1/\alpha}$ for $0 < p < 1$. Hence, we can generate random variables of GHN from the $\left(\frac{p+1}{2}\right)$th quantile of standard normal distribution. Algorithm 1 can be used to generate a hybrid type I censoring sample.

Algorithm 1: Generating hybrid type I samples

Step 1: Given the values of α, β, θ, n, r and T.
Step 2: Generate r order statistics from GHN(α, θ) and denote them by $x_{1:n}, x_{2:n}, \ldots, x_{r:n}$.
Step 3: If $x_{r:n} < T$, the hybrid type I censoring sample is $x_{1:n}, x_{2:n}, \ldots, x_{r:n}$; otherwise, the hybrid type I censoring sample can be $x_{1:n}, x_{2:n}, \ldots, x_{m:n}$, where $x_{m:n}$ is the number to satisfy the conditions $x_{m+1:n} > T$ and $m < r$.

The bias and mean squared error (MSE) of an estimator, through using N generated hybrid type I censoring samples, can be evaluated through using Algorithm 2, where N is a big positive integer.

Algorithm 2: Evaluating the bias and MSEs of estimators

Step 1: Implement Algorithm 1 N times and denote the obtained hybrid type I censoring samples by $Y_1, Y_2, \ldots, Y_N$.
Step 2: For Y_i, search the estimates to maximum $L(\alpha, \theta|Y_i)$ in Equation (5) through using the DE method and denote the obtained estimates by $\hat{\alpha}_{D,i}$ and $\hat{\theta}_{D,i}$ for $i = 1, 2, \ldots, N$.
Step 3: For Y_i, search the estimates to maximum $L(\alpha, \theta|Y_i)$ in Equation (5) via using the GA method. Denote the obtained MLEs by $\hat{\alpha}_{G,i}$ and $\hat{\theta}_{G,i}$ for $i = 1, 2, \ldots, N$.
Step 4: The bias can be evaluated by $Bias\left(\hat{\alpha}_j\right) = \bar{\hat{\alpha}}_j - \alpha$ for $j = D$ and G, where $\bar{\hat{\alpha}}_j$ is the sample mean from estimates of $\hat{\alpha}_{j,i}$ for $i = 1, 2, \ldots, N$, and by $Bias\left(\hat{\theta}_j\right) = \bar{\hat{\theta}}_j - \theta$ for $j = D$ and G, where $\bar{\hat{\theta}}_j$ is the sample mean from estimates of $\hat{\theta}_{j,i}$ for $i = 1, 2, \ldots, N$. The MSEs can be evaluated by $\mathrm{MSE}(\hat{\alpha}_j) = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\alpha}_{j,i} - \alpha\right)^2$ for $j = D$ and G, and by $\mathrm{MSE}\left(\hat{\theta}_j\right) = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\theta}_{j,i} - \theta\right)^2$ for $j = D$ and G.

First, we would like to search better parameters for implementing the GA and DE methods. In this study, we consider $n = 50$, $r = n/2$, $N = 2000$ and all simulation results are reported in Table 2 and Table 3. From Table 2 we find that the designs of (CP, MP) = (0.7, 0.1), (0.7, 0.15), (0.8, 0.1) and (0.8, 0.15) are competitive to implement the GA method in terms of the bias and MSE. The combination of (CP, MP) = (0.7, 0.1) works slightly better to obtain a reliable estimate of α, and the combination of (CP, MP) = (0.7, 0.15) or (0.8, 0.15) works slightly better to obtain a reliable estimate of θ. From Table 3 we find that the design of WF = 0.8 works better than that of WF = 0.6 to implement the DE method in terms of the bias and MSE. Hence, we select the designs of (PS, CP, MP, MI) = (100, 0.7, 0.10, 100), (100, 0.7, 0.15, 100) and (100, 0.8, 0.15, 100) to implement the GA method and select the design of (PS, CP, WF, MI) = (100, 0.5, 0.8, 150) to implement the DE method. These four designs of GA and DE methods are used to search the MLEs of the GHN parameters. Two thousand simulation runs are used for the parallel comparison of estimation performance when $\alpha = 2$, and $\beta = 3$ (or $\theta = 0.0123$). Because the patterns of the simulation results of (PS, CP, MP, MI) = (100, 0.7, 0.15, 100) and (100, 0.8, 0.15, 100) are similar, this study only reports the results of (PS, CP, MP, MI) = (100, 0.7, 0.10, 100) and (100, 0.7, 0.15, 100) for the GA method and the simulation results of (PS, CP, WF, MI) = (100, 0.5, 0.8, 150) for the DE method to save pages. All simulation results are displayed in Figure 1 and Figure 2.

From Figures 1 and 2 we can find that the DE method performs more reliable than the two GA methods with smaller bias and MSE. In particular, Figure 2 shows that the GA method has a higher probability to overestimate θ with wider dispersion compared with that based on the DE method. Based on all simulation results, we recommend using the DE method to obtain reliable MLEs of the GHN parameters.

TABLE 2. The bias and MSE (in parentheses) of the GA based MLEs with PS = 100 and MI = 100

α	β	θ	T	CP	MP	$\hat{\alpha}_G$	$\hat{\theta}_G$
1	2	0.2500	2.3006	0.7	0.1	0.0727 (0.0548)	0.0253 (0.0121)
1	2	0.2500	2.3006	0.7	0.15	0.0793 (0.0600)	0.0260 (0.0128)
1	2	0.2500	2.3006	0.8	0.1	0.0778 (0.0530)	0.0260 (0.0134)
1	2	0.2500	2.3006	0.8	0.15	0.0748 (0.0557)	0.0255 (0.0130)
1	3	0.1111	3.4510	0.7	0.1	0.0234 (0.0391)	0.0189 (0.0028)
1	3	0.1111	3.4510	0.7	0.15	0.0403 (0.0438)	0.0157 (0.0027)
1	3	0.1111	3.4510	0.8	0.1	0.0286 (0.0432)	0.0203 (0.0031)
1	3	0.1111	3.4510	0.8	0.15	0.0434 (0.0459)	0.0157 (0.0028)
1	4	0.0625	4.6014	0.7	0.1	−0.0532 (0.0352)	0.0278 (0.0020)
1	4	0.0625	4.6014	0.7	0.15	−0.0390 (0.0354)	0.0211 (0.0015)
1	4	0.0625	4.6014	0.8	0.1	−0.0502 (0.0422)	0.0287 (0.0026)
1	4	0.0625	4.6014	0.8	0.15	−0.0179 (0.0408)	0.0219 (0.0018)
2	2	0.0625	2.1451	0.7	0.1	−0.0550 (0.1301)	0.0217 (0.0015)
2	2	0.0625	2.1451	0.7	0.15	−0.0248 (0.1309)	0.0170 (0.0012)
2	2	0.0625	2.1451	0.8	0.1	−0.0455 (0.1340)	0.0215 (0.0015)
2	2	0.0625	2.1451	0.8	0.15	0.0119 (0.1457)	0.0149 (0.0012)
2	3	0.0123	3.2176	0.7	0.1	−0.6042 (0.4398)	0.0438 (0.0028)
2	3	0.0123	3.2176	0.7	0.15	−0.5268 (0.3494)	0.0339 (0.0017)
2	3	0.0123	3.2176	0.8	0.1	−0.5868 (0.4259)	0.0435 (0.0029)
2	3	0.0123	3.2176	0.8	0.15	−0.5157 (0.3486)	0.0355 (0.0021)
2	4	0.0039	4.2902	0.7	0.1	−0.9023 (0.8603)	0.0506 (0.0036)
2	4	0.0039	4.2902	0.7	0.15	−0.8493 (0.7691)	0.0428 (0.0027)
2	4	0.0039	4.2902	0.8	0.1	−0.9054 (0.8743)	0.0557 (0.0050)
2	4	0.0039	4.2902	0.8	0.15	−0.8649 (0.8094)	0.0491 (0.0039)

TABLE 3. The bias and MSE (in parentheses) of the DE based MLEs with PS = 100, CP = 0.5, and MI = 150

α	β	θ	T	WF	$\hat{\alpha}_D$	$\hat{\theta}_D$
1	2	0.2500	2.3006	0.6	0.0915 (0.0597)	0.0240 (0.0220)
1	2	0.2500	2.3006	0.8	0.0748 (0.0535)	0.0223 (0.0115)
1	3	0.1111	3.4510	0.6	0.0729 (0.0547)	0.0047 (0.0026)
1	3	0.1111	3.4510	0.8	0.0849 (0.0570)	0.0049 (0.0026)
1	4	0.0625	4.6014	0.6	0.0894 (0.0563)	0.0009 (0.0010)
1	4	0.0625	4.6014	0.8	0.0767 (0.0552)	0.0030 (0.0012)
2	2	0.0625	2.1451	0.6	0.1562 (0.2139)	0.0019 (0.0010)
2	2	0.0625	2.1451	0.8	0.1565 (0.2195)	0.0024 (0.0011)
2	3	0.0123	3.2176	0.6	−0.0088 (0.0650)	0.0030 (0.0001)
2	3	0.0123	3.2176	0.8	−0.0044 (0.0633)	0.0028 (0.0001)
2	4	0.0039	4.2902	0.6	−0.3372 (0.1295)	0.0065 (< 0.0001)
2	4	0.0039	4.2902	0.8	−0.3348 (0.1284)	0.0065 (< 0.0001)

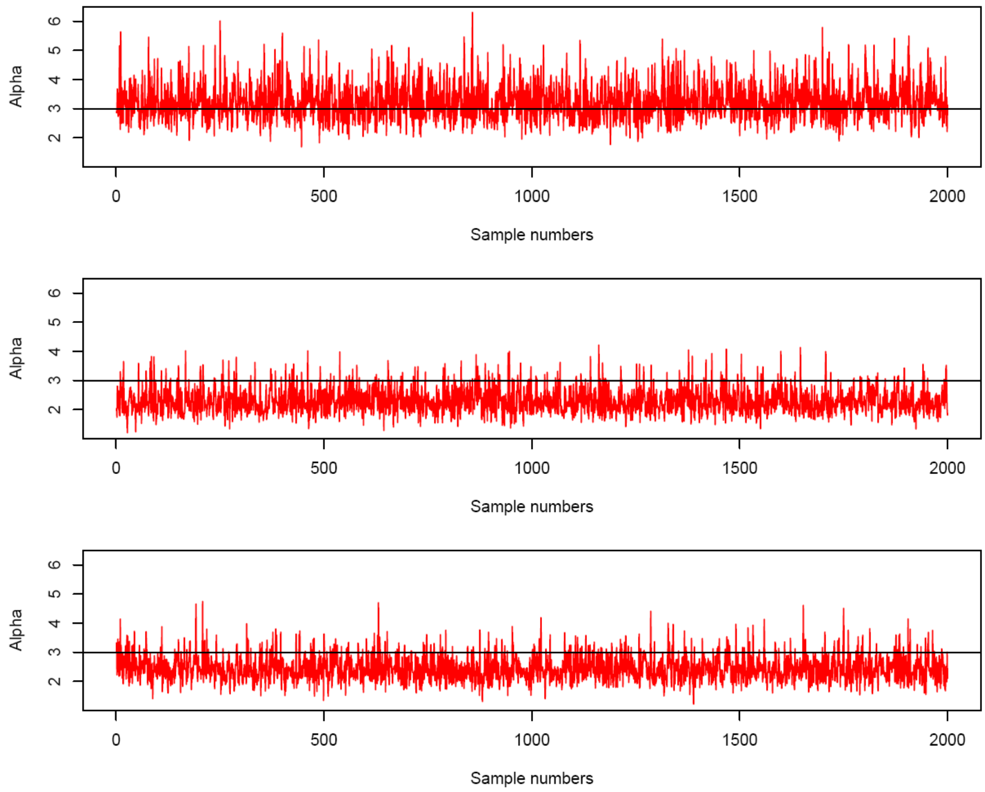

FIGURE 1. Two thousand MLEs of α by using the DE (top), GA (CP = 0.7, MP = 0.1; middle), and GA (CP = 0.7, MP = 0.15; bottom) methods

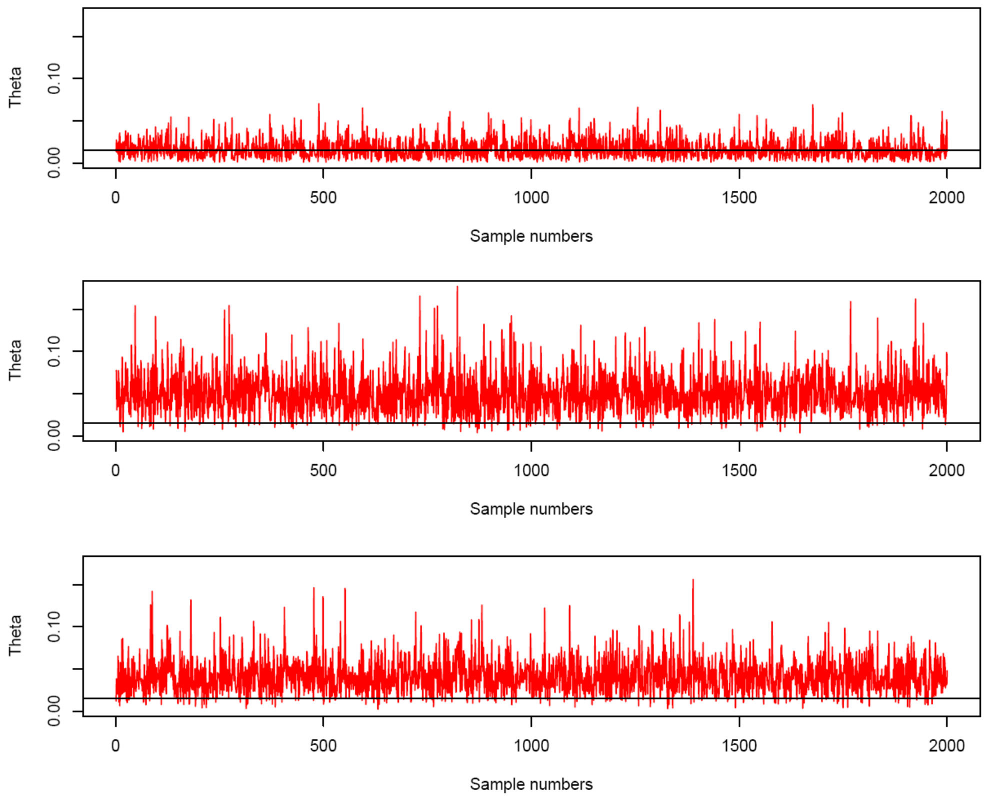

FIGURE 2. Two thousand MLEs of θ by using the DE (top), GA (CP = 0.7, MP = 0.1; middle), and GA (CP = 0.7, MP = 0.15; bottom) methods

4. **Conclusions.** In this paper, we evaluate the performance of using the DE and GA methods to search the MSE of the GHN parameters based on hybrid type I censoring samples. An intensive simulation study is conducted for parallel comparisons. In the first step of simulation, we did a small simulation study to identify better parameter designs for using the DE and GA methods based on the hybrid type I censoring samples from the GHN distribution. Then, we compare the performance of DE and GA methods in terms of the spread, bias and MSE from 2000 obtained DE and GA based MLEs, respectively. We found that the DE method performs better than the two GA methods with smaller bias and MSE. Overall, we conclude that the DE method performs better than the GA method in obtaining reliable MLES of the GHN parameters.

Soft computation algorithms other than the DE and GA could have potential to obtain reliable MLEs of the GHN distribution parameters or other lifetime distribution parameters from hybrid-censoring life tests. Studying the performance of other soft computation methods on searching reliable MLEs of the model parameters from a broad of lifetime distributions with different censoring schemes is interesting. We will study such issue in the future.

Acknowledgment. This work is supported by the grant of Ministry of Science and Technology, Taiwan MOST 106-2221-E-032-038-MY2. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] J. F. Lawless, *Statistical Models and Methods for Lifetime Data*, Wiley, New York, 1982.
[2] W. Q. Meeker and L. A. Escobar, *Statistical Models for Reliability Data*, Wiley, New York, 1998.
[3] B. Epstein, Truncated life tests in the exponential case, *Annals of Mathematical Statistics*, vol.25, pp.555-564, 1954.
[4] K. Ahmadi, M. Rezaei and F. Yousefzadeh, Estimation for the generalized half-normal distribution based on progressive type-II censoring, *Journal of Statistical Computation and Simulation*, vol.85, pp.1128-1150, 2015.
[5] A. Asgharzadeh, R. Valiollahi and D. Kundu, Prediction for future failures in Weibull distribution under hybrid censoring, *Journal of Statistical Computation and Simulation*, vol.85, pp.824-838, 2015.
[6] N. Balakrishnan and D. Kundu, Hybrid censoring: Models, inferential results and applications, *Computational Statistics and Data Analysis*, vol.57, pp.166-209, 2013.
[7] S. Dube, B. Pradhan and D. Kundu, Parameter estimation of the hybrid censored lognormal distribution, *Journal of Statistical Computation and Simulation*, vol.81, pp.275-287, 2011.
[8] D. Kundu, On hybrid censored Weibull distribution, *Journal of Statistical Planning and Inference*, vol.137, pp.2127-2142, 2007.
[9] D. Kundu and B. Pradhan, Estimating the parameters of the generalized exponential distribution in presence of hybrid censoring, *Communication in Statistics – Theory and Methods*, vol.38, pp.2030-2041, 2009.
[10] M. K. Rastogi and Y. M. Tripathi, Inference on unknown parameters of a Burr distribution under hybrid censoring, *Statistical Papers*, vol.54, pp.619-643, 2013.
[11] M. K. Rastogi and Y. M. Tripathi, Estimation using hybrid censored data from a two-parameter distribution with bathtub shape, *Computational Statistics and Data Analysis*, vol.67, pp.268-281, 2013.
[12] S. Singh and Y. M. Tripathi, Bayesian estimation and prediction for a hybrid censored lognormal distribution, *IEEE Trans. Reliability*, vol.65, pp.782-795, 2016.
[13] K. Cooray and M. M. A. Ananda, A generalization of the half-normal distribution with applications to lifetime data, *Communication in Statistics – Theory and Methods*, vol.37, pp.1323-1337, 2008.
[14] R. H. Byrd, P. Lu, J. Nocedal and C. Zhu, A limited memory algorithm for bound constrained optimization, *SIAM Journal of Scientific Computing*, vol.16, pp.1190-1208, 1995.
[15] J. H. Holland, *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*, 2nd Edition, MIT Press, 1992.
[16] T.-R. Tsai, J.-Y. Jiang, Y. L. Lio, N. Jiang and Y.-Y. Fan, Parameter estimation of the Burr type XII distribution with a progressively interval-censored scheme using genetic algorithm, *Proc. of the 3rd International Conference on Industrial Application Engineering*, pp.189-194, 2015.

[17] Z. Guo, X. Yue, S. Wang, H. Yang and K. Li, Self-adaptive differential evolution with elite opposition-based learning, *ICIC Express Letters*, vol.10, no.2, pp.405-410, 2016.
[18] J. Y. Chiang, Y. L. Lio and T.-R. Tsai, Empirical Bayesian strategy for sampling plans with warranty under truncated censoring, *International Journal of Reliability, Quality and Safety Engineering*, vol.23, no.5, 2016.
[19] F.-T. Lin, Application of differential evolution for fuzzy linear programming, *ICIC Express Letters*, vol.5, no.6, pp.1851-1856, 2011.
[20] J. Zhu, H. Xin, J. Sun, Y.-Y. Fan, C. Zheng and T.-R. Tsai, Parameter estimation for Burr type XII distribution with differential evolution and quasi-Newton approaches based on progressively type I interval-censored samples, *ICIC Express Letters, Part B: Applications*, vol.8, no.9, pp.1299-1306, 2017.