ICIC Express Letters ICIC International ©2018 ISSN 1881-803X
Volume **12**, Number **6**, June **2018** pp. 567–574

DESIGN OF DISCRETE-TIME PIDA CONTROLLER USING KITTI'S METHOD WITH FIRST-ORDER HOLD DISCRETIZATION

Pittaya Pannil, Teenapong Sungsorn, Thanit Trisuwannawat and Prapart Ukakimaparn

Faculty of Engineering
King Mongkut's Institute of Technology Ladkrabang
Chalongkrung Rd., Ladkrabang, Bangkok 10520, Thailand
{ pittaya.pa; thanit.tr; prapart.uk }@kmitl.ac.th; nex_teenapong@hotmail.com

Received November 2017; accepted February 2018

Abstract. *This paper proposes a successful technique to design a discrete-time (DT) proportional-integral-derivative-acceleration (PIDA) controller for third-order plants. The proposed technique consists of three major steps: discretization of the controlled plant using the first-order hold (FOH) method, discretization of the continuous-time (CT) PIDA controller structure using the delayed first-order hold (DFOH) method to achieve the DT PIDA controller structure, and design of the target controller using the Kitti's method. Simulation results confirm that the controller designed by utilizing the proposed technique provides equivalent performances compared to a conventional CT controller and a DT controller discretized by utilizing the Tustin's method.*
Keywords: PIDA controller, Discrete-time, Continuous-time, First-order hold, Delayed first-order hold, Kitti's method, Third-order plant

1. **Introduction.** According to the type and order, most industrial process plants are type "0" and comprise three to five first-order lags or one first-order lag plus dead time [1]. The controller commonly used in process industry is a proportional-integral-derivative (PID) controller, because it can be simply designed for second-order plants. However, the design of PID controller suitable for third-order plants is quite difficult, because the plant order is greater than the number of zeros provided by the PID controller. Therefore, a proportional-integral-derivative-accelerate (PIDA) controller has been proposed [2]. The PIDA controller can be easily designed to satisfy any desired specifications for third-order plants. Alternatively, a simple method to design a proportional-derivative (PD) controller incorporated with PID controller for the n-th order plants has been introduced [3]. This proposed method later referred to as "Kitti's method" is suitable to apply in the design of PIDA controllers because of its advantages [4,5]. The Kitti's method is based on placing almost zeros provided by the controller (except one zero) to close with the poles. The remained zero is required to find its location along with the controller gain that satisfies system stability as desired specifications. The discrete-time (DT) PIDA controller proposed in [4] is designed in the z-domain using zero-order hold (ZOH) for controlled plant discretization, while the DT PIDA controller proposed in [5] is designed in the z-domain using the Tustin's method for designed controller discretization. Recently, applying the Kitti's method to designing the PID$\times(n-2)$ stage PD cascade controllers has been suggested [6]. This design approach is based on the employment of first-order hold (FOH) and delayed first-order hold (DFOH) for discretizing the controlled plant and the proposed controller, respectively. Because of the effectiveness of the proposed technique in [6], we develop this approach in the different way for designing the DT PIDA in this paper. The newly proposed controller is designed in the z-domain by using the Kitti's method with DFOH discretization. Comparison between simulation results obtained from

the controller proposed in this paper and from the controllers previously proposed in [4,5] are also presented.

This paper is organized as follows. Section 2 describes a concept of digital controller design. Section 3 explains the proposed technique for DT PIDA controller design. Section 4 demonstrates the performances of the proposed technique by comparing simulation results obtained from the continuous-time (CT) system as well as from previously proposed methods in [4,5] with that of the proposed method in this paper. Section 5 gives the conclusions and possible future work.

2. **Concept of Digital Controller Design.** Figure 1 shows the design procedures for digital control system [7]. The controlled plant is CT system, while the controller is DT system. Thus, it is required to converse from CT to DT in the plant modeling stage or the controller design stage. There are three possible ways to design the digital controller: analog controller discretization, sampled-data design from analog model, and DT-based design from digital model.

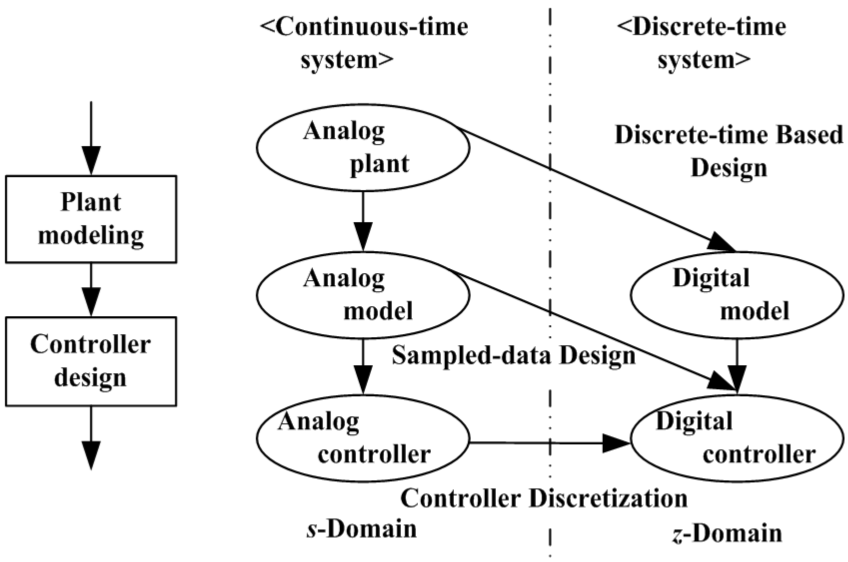

FIGURE 1. Design procedures of digital control system [7]

3. **Proposed Design of DT PIDA Controller.** Figure 2 shows a general architecture of control system used to describe the proposed design method of DT PIDA controller. The control system design is concerned with an arrangement of the system structure as well as a selection of the suitable components and parameters. For example, if a set of performance measures is specified to be less than some specified values, conflicting requirements possibly occur. Therefore, if the controller $K(s)$ suitable for the plant $G(s)$ is required to obtain, its specifications such as percent overshoot (PO) and settling time must be identified.

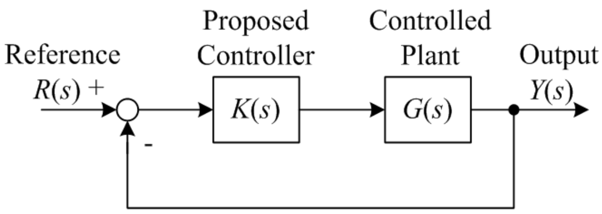

FIGURE 2. General structure of control system

The proposed controller design technique to obtain DT PIDA controller for third-order plants, the transfer function of the required PIDA controller can be stated as [6]

$$K(s) = K_p + \frac{K_i}{s} + K_d s + K_a s^2, \tag{1}$$

where K_p, K_i, K_d, and K_a denote a proportional gain, an integral gain, a derivative gain, and an acceleration gain, respectively [8]. Three major steps to design the DT PIDA controller can be discussed as follows.

3.1. **Plant discretization by using FOH method.** Suppose that the third-order plant controlled by the PIDA controller in this paper has the transfer function [2], $G(s)$, as follows

$$G(s) = \frac{1}{s(s+1)(s+7)}, \tag{2}$$

where $G(s)$ is the continuous-time transfer function. Assume that $G(s)$ is discretized by FOH with sampling period T. The DT plant transfer function can be given by [9]

$$G(z) = \frac{(z-1)^2}{Tz}\mathscr{Z}\left\{\frac{1}{s^2}G(s)\right\}. \tag{3}$$

Hence, from (2) and (3), the plant transfer function can be written as

$$G(z) = \frac{(z-1)^2}{Tz}\mathscr{Z}\overbrace{\left\{\frac{1}{s^2 \cdot s(s+1)(s+7)}\right\}}^{F(s)}, \tag{4}$$

where

$$F(s) \equiv \frac{a}{s^3} + \frac{b}{s^2} + \frac{c}{s} + \frac{d}{(s+1)} + \frac{e}{(s+7)} = \frac{\left(\frac{1}{7}\right)}{s^3} + \frac{\left(\frac{-8}{49}\right)}{s^2} + \frac{\left(\frac{57}{343}\right)}{s} + \frac{\left(\frac{-1}{6}\right)}{(s+1)} + \frac{\left(\frac{1}{2058}\right)}{(s+7)}. \tag{5}$$

Then,

$$\begin{cases} \mathscr{Z}\{F(s)\} = \left\{\left(\frac{1}{14}\right)\left[\frac{T^2 z(z+1)}{(z-1)^3}\right] + \left(\frac{-8}{49}\right)\left[\frac{Tz}{(z-1)^2}\right] + \left(\frac{57}{343}\right)\left[\frac{z}{(z-1)}\right] \right. \\ \left. \qquad + \left(\frac{-1}{6}\right)\left[\frac{z}{(z-e^{-T})}\right] + \left(\frac{1}{2058}\right)\left[\frac{z}{(z-e^{-7T})}\right]\right\}, \end{cases} \tag{6}$$

and

$$\begin{cases} G(z) = \left\{\left(\frac{1}{14}\right)\left[\frac{T(z+1)}{(z-1)}\right] + \left(\frac{-8}{49}\right) + \left(\frac{57}{343}\right)\frac{(z-1)}{T} \right. \\ \left. \qquad + \left(\frac{-1}{6}\right)\left[\frac{(z-1)^2}{T(z-e^{-T})}\right] + \left(\frac{1}{2058}\right)\left[\frac{(z-1)^2}{T(z-e^{-7T})}\right]\right\}. \end{cases} \tag{7}$$

For $T = 1/500$ sec/samples, the DT transfer function of the controlled plant can be stated as

$$G(z) = \frac{\begin{bmatrix} \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix}^T \begin{bmatrix} z^3 \\ z^2 \\ z \\ 1 \end{bmatrix}}{(z-1)\left(z-e^{-T}\right)\left(z-e^{-7T}\right)}, \tag{8}$$

where β_3, β_2, β_1, and β_0 are the discrete-time coefficients, and their values can be defined as follows:

$$\begin{cases} \beta_3 = 3.323 \times 10^{-10}, & \\ \beta_2 = 3.643 \times 10^{-9}, & e^{-T} = 0.998, \\ \beta_1 = 3.632 \times 10^{-9}, & e^{-7T} = 0.986, \\ \beta_0 = 3.291 \times 10^{-10}. & \end{cases} \tag{9}$$

Thus, the transfer function of the third-order plant that is required to be controlled can be expressed as

$$G(z) = 10^{-8}\frac{(z+9.867)(z+0.997)(z+0.101)}{(z-1)(z-0.9980)(z-0.9861)}. \tag{10}$$

3.2. **Controller discretization by using DFOH method.** The DT PIDA controller is obtained by using DFOH [10], and its desired DT transfer function can be written as:

$$K(z) = \left(1-2z^{-1}+z^{-2}\right)\mathscr{Z}\left\{\mathscr{L}^{-1}\left(\frac{K(s)}{Ts^2}\right)\right\}, \tag{11}$$

$$\begin{cases} \dfrac{K(s)}{Ts^2} = \dfrac{1}{Ts^2}\left(K_p + \dfrac{K_i}{s} + K_d s + K_a s^2\right), \\ \mathscr{L}^{-1}\left(\dfrac{K(s)}{Ts^2}\right) = \left\{\dfrac{K_p}{T}\left(\dfrac{Tz}{(z-1)^2}\right) + \dfrac{K_i}{T}\left(\dfrac{T^2z(z+1)}{2(z-1)^3}\right) + \dfrac{K_d}{T}\left(\dfrac{z}{z-1}\right) + \dfrac{K_a}{T}\right\}, \\ K(z) = \left(\dfrac{(z-1)^2}{z^2}\right)\left\{\dfrac{K_p}{T}\left(\dfrac{Tz}{(z-1)^2}\right) + \dfrac{K_i}{T}\left(\dfrac{T^2z(z+1)}{2(z-1)^3}\right) + \dfrac{K_d}{T}\left(\dfrac{z}{z-1}\right) + \dfrac{K_a}{T}\right\}. \end{cases} \tag{12}$$

Then, the controller transfer function in discrete time can be stated as

$$\begin{cases} K(z) = \dfrac{\beta_3 z^3 + \beta_2 z^2 + \beta_1 z + \beta_0}{z^2(z-1)}, \quad \begin{bmatrix} \beta_3 \\ \beta_2 \\ \beta_1 \\ \beta_0 \end{bmatrix} = \dfrac{1}{2T}\begin{bmatrix} 0 & 0 & 2 & 2 \\ 2T & T^2 & -4 & -6 \\ -2T & T^2 & 2 & 6 \\ 0 & 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} K_p \\ K_i \\ K_d \\ K_a \end{bmatrix}, \\ K(z) \equiv K\dfrac{(z-z_a)(z-z_b)(z-z_c)}{z^2(z-1)}, \end{cases} \tag{13}$$

where z_a, z_b, and z_c are the zeros of the PIDA control on z-plane.

3.3. **Controller design by using the Kitti's method.** From (10) and (13), the open-loop transfer function used to design the DT PIDA controller can be written as

$$K(z)G(z) = K\frac{(z-z_a)(z-z_b)(z-z_c)}{z^2(z-1)} \times 10^{-8}\frac{(z+9.867)(z+0.997)(z+0.101)}{(z-1)(z-0.998)(z-0.9861)}. \tag{14}$$

Based on the Kitti's method to design the controller $K(z)$, let $z_a = 0.997$ and $z_b = 0.9851$. Then, the open-loop transfer function without $(z - z_c)$ can be given by

$$KGwoz_c(z) = K\frac{(z-0.997)(z-0.9851)}{z^2(z-1)} \times 10^{-8}\frac{(z+9.867)(z+0.997)(z+0.101)}{(z-1)(z-0.998)(z-0.9861)}. \tag{15}$$

The desired specifications are usually specified in terms of transient and steady state response characteristics of the control system to a unit-step input, exhibited by a pair of complex-conjugate dominant closed-loop poles, $s_{d\pm} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$, as follows:

$$\begin{cases} \text{Percent Overshoot } (PO) = e^{\left(-\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100\%, \\ \text{Settling Time } (t_s) = -\ln\left(0.02\sqrt{1-\zeta^2}\right)\Big/\zeta\omega_n, \ (\pm 2\%), \end{cases} \tag{16}$$

where ζ is a damping ratio, and ω_n is a natural frequency. If the desired specifications in (16) are set to $PO \le 5\%$ and $t_s \le 2$ sec ($\pm 2\%$). Then, one of the dominant closed-loop poles is located at

$$s_d = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} = -2.118 + j2.221. \tag{17}$$

From the desired specifications and dominant closed-loop pole location in (17), then the dominant closed-loop pole on z-plane can be stated as

$$z_d = e^{T\cdot s_d} = 0.996 + j4.423 \times 10^{-3}, \quad T = 1/500 \text{ sec/sample}. \tag{18}$$

The necessary angle of the open-loop transfer function without the zero $(z - z_c)$ at the dominant closed-loop pole z_d is:

$$\begin{cases} KGwoz_c(z_d) = 1.184 \times 10^{-3} + j6.26 \times 10^{-3}, \\ \arg\left[KGwoz_c(z_d)\right] = 79.292^\circ. \end{cases} \tag{19}$$

From the angle condition of the root locus method, the angle from z_c to z_d is

$$\arg\left[z_c\right] = \pi - \arg\left[KGwoz_c(z_d)\right] = 100.708^\circ. \tag{20}$$

Since the angle of the zero $(z - z_c)$ is greater than 90°, then the z_c is located at the right-hand side of z_d as

$$z_c = |\mathrm{Re}(z_d)| + \frac{|\mathrm{Im}(z_d)|}{\tan\left(\pi - \arg\left[z_c\right]\right)} = 0.997. \tag{21}$$

The remained parameter is the controller gain K, which can be found from the magnitude condition of the root locus method as

$$K = 1/|K(z_d)G(z_d)| = 3.487 \times 10^4. \tag{22}$$

Finally, the controller transfer function can be stated as

$$K(z) = K\frac{(z - z_a)(z - z_b)(z - z_c)}{z^2(z - 1)} = 3.487 \times 10^4 \frac{(z - 0.997)(z - 0.9851)(z - 0.997)}{z^2(z - 1)}. \tag{23}$$

4. **Simulation Results.** The performances of the proposed DT PIDA controller were studied through the MATLAB simulation results. Its performances were also compared with not only the CT controller but also two conventional DT PIDA controllers discretized by using the ZOH technique [4] and the Tustin's method [5]. The transfer functions of the DT PIDA controller based on ZOH discretization and the controlled plant, respectively, [4] can be given as:

$$\begin{cases} K(z) = \overbrace{7.512 \times 10^4}^{\text{Find}} \dfrac{\overbrace{(z - 0.9761)(z - 0.997)}^{\text{Pre-set}}\overbrace{(z - 0.9975)}^{\text{Find}}}{z^2(z - 1)}, \\ G(z) = 10^{-8}\dfrac{(z + 3.7172)(z + 0.2669)}{(z - 1)(z - 0.9980)(z - 0.9861)}. \end{cases} \tag{24}$$

The transfer functions of the DT PIDA controller based on the Tustin's discretization and the controlled plant, respectively, [5] can be expressed as:

$$\begin{cases} K(z) = \overbrace{4.15 \times 10^5}^{\text{Find}} \dfrac{\overbrace{(z - 0.997)(z - 0.9851)}^{\text{Pre-set}}\overbrace{(z - 0.997)}^{\text{Find}}}{(z - 1)(z + 1)(z + 1)}, \\ G(z) = 10^{-8}\dfrac{(z + 1.0127)(z + 0.9937 \pm j0.0109)}{(z - 1)(z - 0.9980)(z - 0.9861)}. \end{cases} \tag{25}$$

From (10) and (23), the transfer functions of the proposed DT PIDA controller discretized by using DFOH method and the controlled plant discretized by using FOH method, respectively, can be rewritten as:

$$\begin{cases} K(z) = \overbrace{3.487 \times 10^4}^{\text{Find}} \dfrac{\overbrace{(z - 0.997)(z - 0.9851)}^{\text{Pre-set}}\overbrace{(z - 0.997)}^{\text{Find}}}{z^2(z - 1)}, \\ G(z) = 10^{-8}\dfrac{(z + 9.867)(z + 0.997)(z + 0.101)}{(z - 1)(z - 0.9980)(z - 0.9861)}. \end{cases} \tag{26}$$

It should be noted that the controlled plant transfer functions always have the same three poles for all three design methods, whereas the controller transfer functions also have three zeros and three poles. It is implied that the Kitti's method can be applied for various methods of discretization. Figure 3 shows the unit step responses of all four controller design approaches: CT system, using ZOH discretization [4], using Tustin's method [5], and using FOH discretization proposed in this paper, which are displayed as white dotted line, red solid line, green solid line, and blue solid line, respectively. It can be seen that the proposed controller design method by using FOH offers the same performances of the DT controller discretized by the Tustin's method and the CT controller. The original application of the Kitti's method is to design the forward controller for decreasing the percent overshoot in response to step input. The proposed controller structure then has two degree-of-freedom. However, the desired specification can be achieved by increasing the loop gain to be 10 times. The circular shapes of the root loci in Figures 4 and 5 are good explanations of loop gain increment. Figure 6 illustrates the DT unit step responses before and after increasing the loop gains.

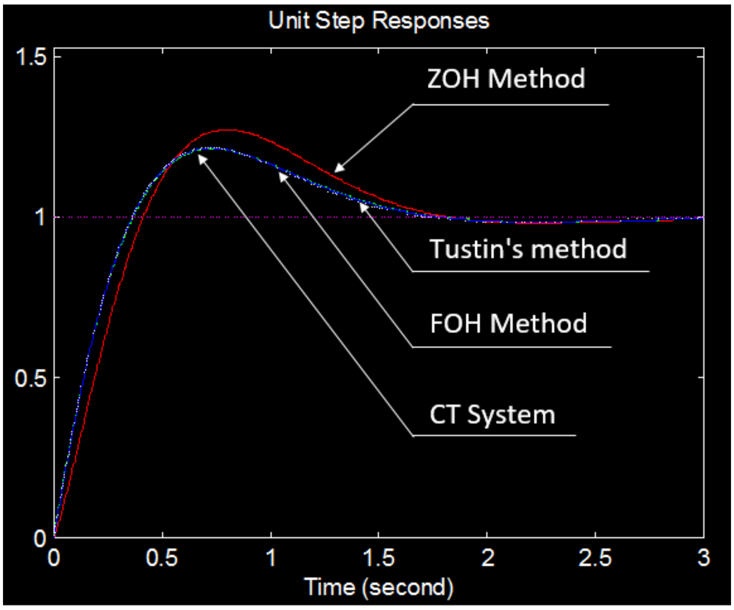

FIGURE 3. Unit step responses

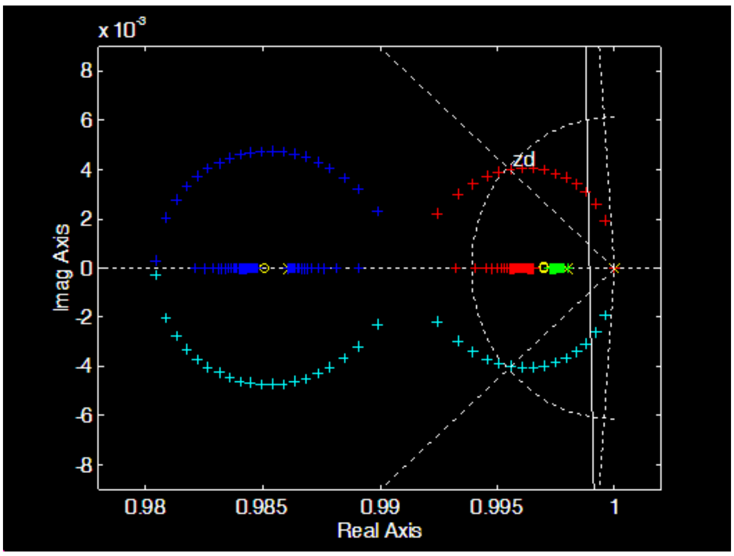

FIGURE 4. Root locus plot in z-plane

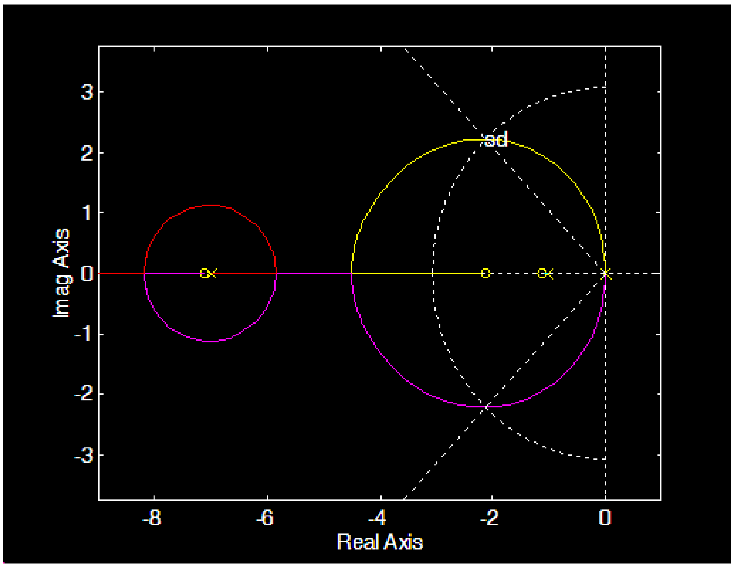

FIGURE 5. Root locus plot in s-plane

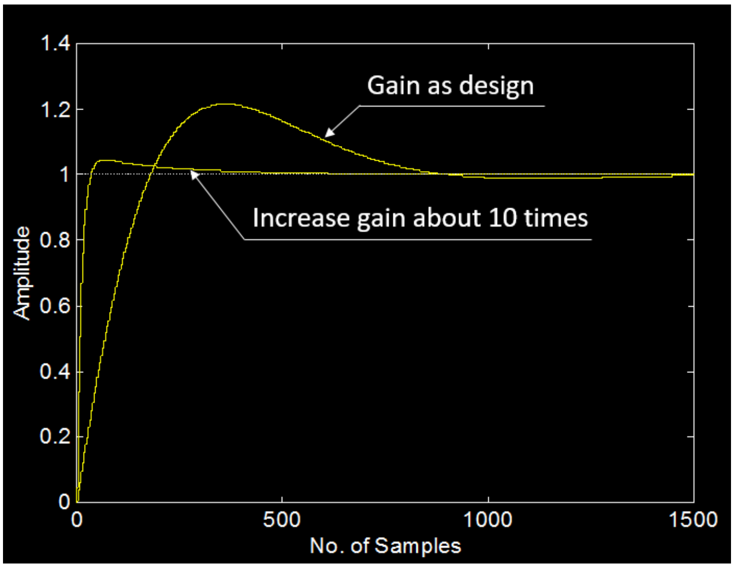

FIGURE 6. Discrete-time unit step responses

5. **Conclusions.** The technique to design the digital PIDA controller in discrete time has been introduced in this paper. The first step is the discretization of the controlled plant in continuous time by using the FOH method. The second step is the discretization of the desired controller structure by using the DFOH method. The last step is to design the proper structure of the DT PIDA controller by using the Kitti's method in z-plane, which can be done in similar way with the design in s-plane. To verify the validation of the proposed design technique, the MATLAB simulations were conducted by concerning the proposed controller and the conventional controllers. Simulation results confirm that the proposed DT PIDA controller provides the satisfied performances. Moreover, the simulation results also demonstrate the effectiveness of the Kitti's method for controller design. The Kitti's method will be applied to a second-order hold discretization in the future work.

REFERENCES

[1] D. W. Pessen, A new look at PID controller tuning, *Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control*, vol.116, pp.553-557, 1994.

[2] S. Jung and R. C. Dorf, Analytic PIDA controller design technique for a third order system, *Proc. of the 35th Conference on Decision and Control*, Japan, pp.2513-2518, 1996.

[3] T. Trisuwannawat, K. Tirasesth, J. Ngamwiwit and M. Iida, PID$\times(n-2)$ stage PD cascade controller for SISO systems, *Proc of the 37th SICE Annual Conference*, Chiba, Japan, pp.965-968, 1998.

[4] P. Ukakimaparn, P. Pannil, P. Boonchuay and T. Trisuwannawat, PIDA controller designed by Kitti's method, *Proc. of ICROS-SICE International Joint Conference*, Fukuoka, Japan, pp.1547-1550, 2009.

[5] K. Smerpitak, P. Ukakimaparn, T. Trisuwannawat and P. La-orsri, Discrete-time PIDA controller designed by Kitti's method with bilinear transform, *Proc. of the 12th International Conference on Control, Automation and Systems*, Jeju, Korea, pp.1585-1590, 2012.

[6] C. Chiengtee, P. Pannil, P. Ukakimaparn and T. Trisuwannawat, Discrete-time PID$\times(n-2)$ stage PD cascade controllers with first order hold and delayed first order hold discretization, *Proc. of the International MutiConference of Engineers and Computer Scientists*, vol.1, pp.251-255, 2017.

[7] H. Fujimoto, *General Framework of Multirate Sampling Control and Applications to Motion Control Systems*, Ph.D. Thesis, The University of Tokyo, 2001.

[8] N. Narkvitul, P. Ukakimaparn, P. Pannil and T. Trisuwannawat, Closed-form formulas for continuous/discrete-time PIDA controllers' parameters, *Proc. of the 15th International Conference on Control, Automation and Systems*, Busan, Korea, pp.326-329, 2015.

[9] J. B. Hoagg, S. L. Lacy, R. S. Erwin and D. S. Bernstein, First-order-hold sampling of positive real systems and subspace identification of positive real models, *Proc. of the 2004 American Control Conference*, Boston, MA, pp.861-866, 2004.

[10] M. Tomizuka, T. Tsao and K. Chew, Analysis and synthesis of discrete-time repetitive controllers, *Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control*, vol.111, pp.353-358, 1989.