

## FRACTIONAL INTEGRAL MODEL IN CELLULAR AUTOMATA EMBEDDED FOR BIOMETRIC RECOGNITION

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Received October 2017; accepted January 2018

*ABSTRACT.* In this paper, a proposed model of cellular automata is studied by means of fractional integral function. From point of view of fractional calculus especially its applications in physics, we briefly introduce intermediate processes and critical phenomena in cellular automata. A cellular automaton is a decentralized computing model providing an excellent platform for performing complex computation with the help of only local information. The paper discusses how to use fractional integral function for representing cellular automata model and cell information memory vector. The architecture of computing and learning model for face and fingerprint recognition system will be given and the results of calibrating of approach are also given.

**Keywords:** Fractional integral, Memory, Cellular automata, Learning, Biometric, Mobile

1. **Introduction.** Cellular automata model as a biologically inspired distributed computing paradigm was used as universal computing machine. Cellular automata model is simple and useful for many applications [1-5]. Each cell defines an automaton and the state of cell each instant of time depends on the state of its neighbors and on its own previous state. The learning [6] in the cell's state is given through the transition rule that is used to update and change the state of every cell in the model of cellular automata. So, by iterating the state transition process in time, the objective of the system is achieved. No research paper as we assume concerns with the structure of the memory of the cell to store the cell's information. In addition, where the transition rule is stored and how the learning is done with this transition rule. Here, we will present a model that uses fractional integral [7,8] to store the state of every cell in the cellular automata model and we will discuss how the learning will be done through the transition rule.

Smartphone is a ubiquitous and Internet-access device used to facilitate daily activities. Mobile devices include phones, tablets and similar smart devices with incipient context awareness and personalized services supported by Internet and cloud computing. The widespread use of smartphone devices has resulted that mobile users tend to store all personal information on their devices. Therefore, it needs to perform reliable user authentication on mobile device with one of biometric systems [9-11]. Biometric identification has been widely utilized to authenticate users' identities with biometric image including fingerprint, irises, facial patterns, etc. [2,12]. Biometrics recognition research is intended to find biometric image features and use them to identify and validate an individual. Fingerprint identification is one of the most nature biometric technologies and then mobile devices have integrated it now. Fingerprints are the oldest and most widely used form of biometric identification. Fingerprint sensors in mobile devices are becoming an attractive alternative to the traditional authentication methods of identification. A fingerprint is formed as an impression of the pattern of ridges on a finger. A ridge is defined as a single

curved segment and a valley is a region that lies in between ridges. In this paper, we propose a biometric identification scheme, which has prominent level of efficiency based on cellular automata integrated with fractional integral function.

The research in [8] presents model with fractional order differentiation that generalizes power law dynamics to describe complex temporal voltage dynamics. The main characteristic of this model is that it depends on all past values of the voltage that causes long-term memory. The work showed that the subthreshold voltage and the firing rate of the fractional-order model make transitions from exponential to power law dynamics when the fractional order decreases from one to smaller values. The voltage-memory trace that represents the long-term memory has a feedback regulatory mechanism and affects spiking activity. The results suggested that fractional-order models might be appropriate for understanding multiple time scale neuronal dynamics.

The work in [2] proposed two-dimensional cellular automata memory model that controls a robot swarm when undertaking the foraging task in a previously known environment with nests. Besides, a short-term memory inspired by search is applied to enabling robots to remember their last positions and to avoid useless explorations. Moreover, a dynamic information was used to avoid queues of robots and bottlenecks next to the nests. The model was implemented and submitted to several simulations to evaluate its resultant behavior.

The structure of this paper is as follows. Section 2 will explain the proposed model and state how the cellular automata model combines with fractional integral function. Section 3 presents the simulation of the proposed model of cellular automata and fractional integral function to biometric recognition system. Section 4 shows the experimental results. Section 5 provides some conclusions of work.

**2. Fractional Integral Cellular Automata Model.** One way to model memory behavior is through fractionally integral function processes [7,8]. Fractionally integral processes have autocorrelation functions that decay very slowly and so exhibit long memory. Therefore, by using fractional integral techniques, we can establish a new model of cellular automata. With this model, one can reproduce cellular automata transition system of any given parameters.

Let  $X$  be a finite set describing the possible states of any cell in cellular automata model. A configuration is a snapshot of the states of cells in the cellular automata. Given a configuration  $S_t$ , we denote by  $x(i, j) \in X$  the state of the cell in position  $(i, j)$ . In Lebesgue integrals space  $L_1$ , the variability of cell's state can be explained as the integral response to neighbors' excitation by brief period learning disturbances in terms of:

$$\frac{dS_t}{dt} = M(t) \quad (1)$$

where  $M$  represents the model excitations (transition factor), and  $S_t$  stands for cellular automata configuration. It has been noticed that calculations are simply based on one-order integration, especially when modeling it with different fractal properties or strengths. By applying the fractional integral formula with  $k$  order, the configuration model can be written explicitly as,

$$S_t = \frac{1}{\Gamma(k)} \int_0^t \frac{M(u)}{(u-t)^{1-k}} du \quad (2)$$

where  $u$  stands for a historical time point,  $M$  represents the model excitations (transition factor),  $k$  is the maximum state for cells, and  $\Gamma$  denotes the well-known gamma function. The fractional integration starts from  $u =$  zero, which stands for the beginning of iterations. By adjusting the integral-order  $t$ , one can simulate the configuration  $S_t$ . Different  $k$ -values determine different transition rates of  $S_t$ .  $k =$  zero means there are no

integration procedures, and the cellular automata configuration state  $S$  is died. While  $k > 0$ , it means the configuration is changed by applying  $t$ -order integration excitation  $M(u)$ , which will simultaneously introduce the configuration  $S_t$ .

Suppose we want to know the next configuration as  $S_{t+1}$ , theoretically the system state can be calculated from the historical state using,

$$S_{t+1} = S_{t-1} \oplus S_t \tag{3}$$

$$S_t = \frac{1}{\Gamma(k)} \int_{u=0}^{t-d} \frac{M(u)}{(t-u)^{1-k}} du + \varepsilon(t) \tag{4}$$

where  $S_0, S_d, \dots, S_{t-d}$  are the historical configuration state and  $d$  is the sampling intervals of the observed time. The difference between Equation (4) and Equation (2) is that we only use the historical factor  $M(u)$  for the integrations from  $(t-d)$  to  $t$ , considering the sampling interval  $d$  and the singular nature of Equation (2), the estimated integrating factor for  $M(t)$  as

$$M(t) = \sum_{t=d1}^{d-d1} \frac{d}{\Gamma(K)(d-t)^{1-K}} \tag{5}$$

$M(t)$  is in line with our above understandings, that any state time  $t$  is composed of historical cumulated influence  $M(t)$  to find the present configuration state  $S_t$ . Therefore, transition equation can be further written as,

$$S_t = M(k) \otimes \Psi_0^{t-\delta} + S_t \tag{6}$$

$\Psi = \{M(0), M(d), \dots, M(t-u), \dots, M(t-\delta)\}$  represents the integrating factor of every step and also represents the state at each historical time point where  $\delta$  is the time distances between historical time points. Since the integrating factor  $k$  describes to what extent the historical state  $M(u)$  then it should affect the present state  $S_i$ . A general idea is that we may derive transition rule reversely and obtain the configuration state by time point from the historical observations  $\{S_0, S_d, \dots, S_{i-u}, \dots, S_{i-\delta}\}$ . With the estimated historical state, one thus can calculate the transition rate simply. We shall investigate the kind of transition rule for cellular automata cells as

$$x(t, i, j) = x(0, i, j) \oplus \frac{1}{\Gamma(k)} \int_0^t \frac{M(i, j, u, x(i, j))}{(u-t)^{1-k}} du \tag{7}$$

The concept of emergence is an integral part of self-adaptive systems as cellular automata. It can be explained as emergent behavior based on interactions of model's parts or cells. One main interesting results of research as in [Hassan] is how self-organization models follow simple rule as transition rule in cellular automata model that results in an emergent pattern. Any emergent pattern that results from local interactions among the system cells should be adaptive to its operating target. We have discovered from our model that using fractional integral rule among cell's neighborhood results in an emergent pattern that meets the system objective.

**3. Biometric Using Fractional Integral Cellular Automata.** Biometric is the science of recognizing individuals based on their biological traits [9-12]. These traits include face, fingerprints, iris, periocular region, gait, voice, etc. Biometric recognition has been developed as a reliable alternative to traditional authentication system based on passwords or PINs. The use of new proposed model placed the emphasis on the processing of the biometric characteristics found in different pixels with geometric relationships in the image. Therefore, the biometric images are taken close enough to subject (face, finger, etc.) to obtain a relatively detailed view allowing for the use of the geometric layout of the pixel sets to provide an adequate identification of the person present [13-15].

A two-step algorithm is presented here in which the first step makes use of cellular automata to create spectrally representative relatively regions of biometric image. To become a valid user and hope to access services of mobile devices for example, the registration process should begin. The proposed model uses local visual features of user image in cellular automata representation, which is proved useful in biometric recognition, and his image does not require high quality images. The proposed model can overcome some influences brought about by the unfavorable factors such as blurred fingerprint images and impact in facial illumination. The biometric model in second step of algorithm will use fractional integral cellular automata to store this authoritative image in the model with evolving cellular automata based system.

The cell in our proposed model is placed over each pixel of biometric image with state given by RGB-band vector. Consequently, the state space is continuous and corresponds to the positive vector space  $\mathfrak{R}^3$ . The image is a bi-dimensional array  $Z$  of  $(M \times M)$  pixels. Each pixel can be characterized by the triplet  $(i; j; c)$  where  $(i; j)$  represents its position in the array and  $c$  the associated value. The image will be considered as a configuration state of a cellular automaton that has cellular space. The model describes the simulating of the pixel associated value by cell's state. The transition rule for configuration's value in the system (state change) is constructed as:

$$S_t = M(u) \cdot S_{i-1} + \varepsilon(u),$$

$$M(u) = \frac{1}{\Gamma(K)} \int_{t=0}^u \frac{S(t)}{(t - \varepsilon)^{1-k}} dt \quad (8)$$

The initial configuration state will be notated by  $S_0$  where the original image is mapped into cellular automata space. In this way, one can extract the future/historical state point by point exactly. When making calculations, cutoffs are usually selected per the calculation accuracy. The referencing ending image will be designed to provide prior knowledge and it has the same image size of original one.

The de-noising algorithm in [1] is employed for the red, green and blue components of the biometric image. After denoising, all image's components are geometrically with the same size of original image. By using the segmentation function in [3] with eight cell neighborhoods together with a distance measure that is independent from the dimensionality of the data, the biometric system can identify the login user by biometric image. The segmentation algorithm uses the pixels and is interacted with their neighbors to determine if pixels belong to the same segment or not (see Figure 1). Use the object in saved biometric image and with coming image it compares to authenticate to the system.

The classification of new user biometric image is done based on comparisons of new image at  $S_u$  with history set of configurations  $S_t$  that can be induced by fractional integral model. To be able to compare the images, a distance measure should be defined here. For image  $a$ , the distance measure  $d$  with respect to image  $a'$  is defined as

$$d_{aa'} = \cos \frac{2}{\pi} \left( \frac{\sum S_a S_{a'}}{\sum_i S_a^2 \sum_j S_{a'}^2} \right) \quad (9)$$

This value is normalized between zero and one. Thus, for both the history configuration (image) and new biometric image, a vector  $D$  is made up of  $t$  values.

$$D = (d_{a1} d_{a2} \cdots d_{at}) \quad (10)$$

We have two threshold distances  $\alpha$  and  $\beta$  where  $0 < \alpha < d_{aa'} < \beta < 1$  to give authorized matching. These thresholds represent the acceptance range for biometric image authentication and they are determined experimentally. The matching vector  $C$  will be assigned

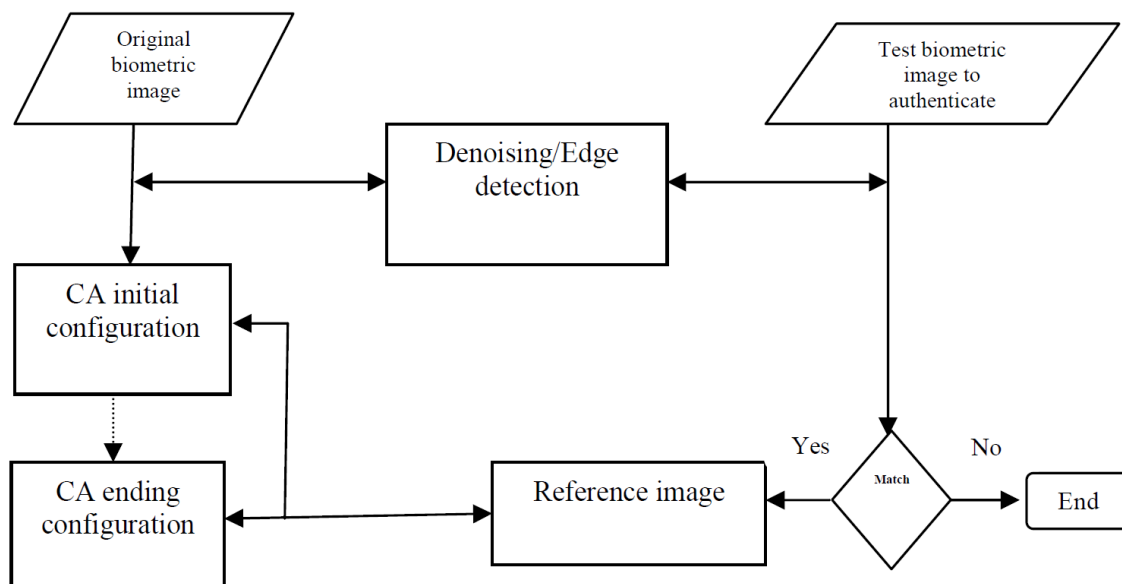


FIGURE 1. Biometric images system

values as

$$\begin{aligned}
 d_{aa'} < \alpha &\rightarrow C_{aa'} = -1 \\
 \alpha < d_{aa'} < \beta &\rightarrow C_{aa'} = 0 \\
 d_{aa'} > \beta &\rightarrow C_{aa'} = +1
 \end{aligned} \tag{11}$$

Thus, the proposed model selects the input biometric image whose  $C$  values are most like zeros.

**4. Experimental Results.** The main operation in this implementation is biometric image recognition. All images will be used here with an  $80 \times 80$  size. In the experiment, we select  $N$  ( $N = 2, 3, 4, 5$ ) images for each person. The object space in an image is modeled as a finite space of the Euclidean space. Once the object's image is captured via an imaging system, the model of underlying space is a finite subset of the Cartesian coordinates. The object at the intensity coming image is the pixel  $(i, j)$  and if it is above a certain threshold, the corresponding pixel  $(i, j)$  is assumed to be present; otherwise it is assumed to be absent. A pixel color is represented by a state in  $\{0, 1, \dots, h\}$  where  $h = 2$  for monochromatic image;  $h = 16$  for an image with 16 colors, 0' state is the quiescent state associated to the cell (absent pixel). Since the processing within a local window around the pixel is not recursive, it may be organized simultaneously for all the pixels, independently on each other.

Figures 2 and 3 show a cellular automata gray scale version of biometric image obtained from the sensor (first row) and output (last row). The image (f) is blank because system fails to find the reference image for it. One image of row number two is shown in Figure 4 as topology configuration.

The proposed model is applied to 390 biometric images. For our experiments, we manually establish biometric dataset, which consists of public face dataset, and public fingerprint dataset. The dataset contains 240 face images (5 images for 48 persons) and 150 fingerprint images (5 images for 30 persons). We select randomly five-face image for each person to train and choose the left images for each person as the testing samples.

The experiment results are shown in Table 1, where the performance denotes the recognition accuracy. Table 2 shows the comparison of recognition accuracy of new proposed

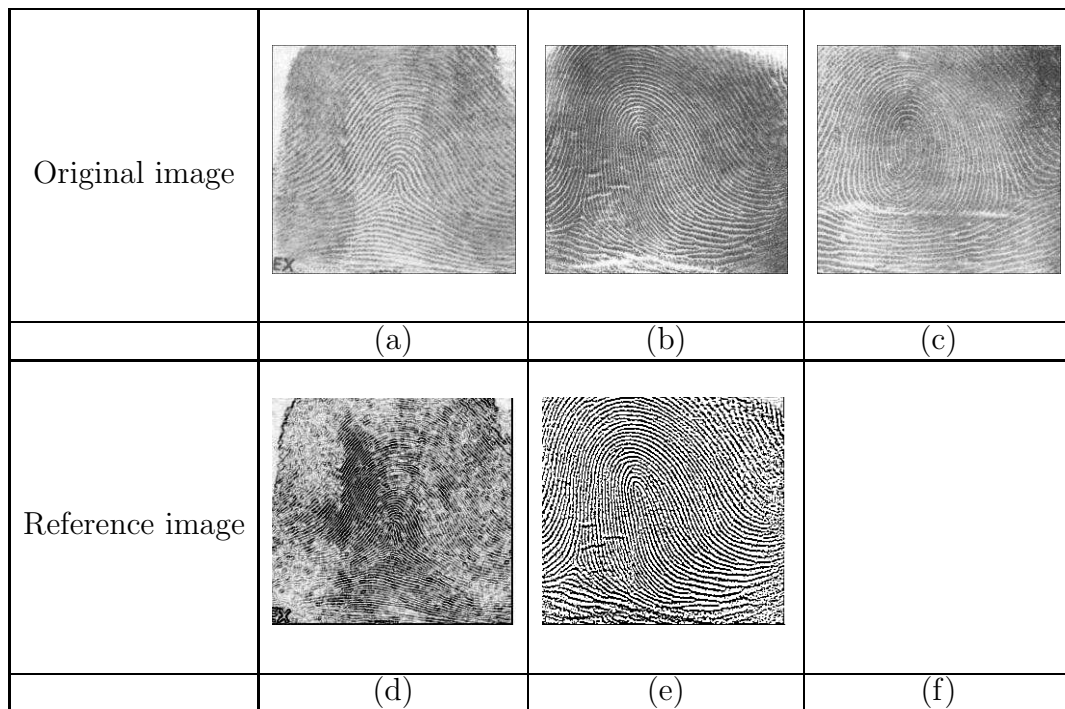


FIGURE 2. Finger print images

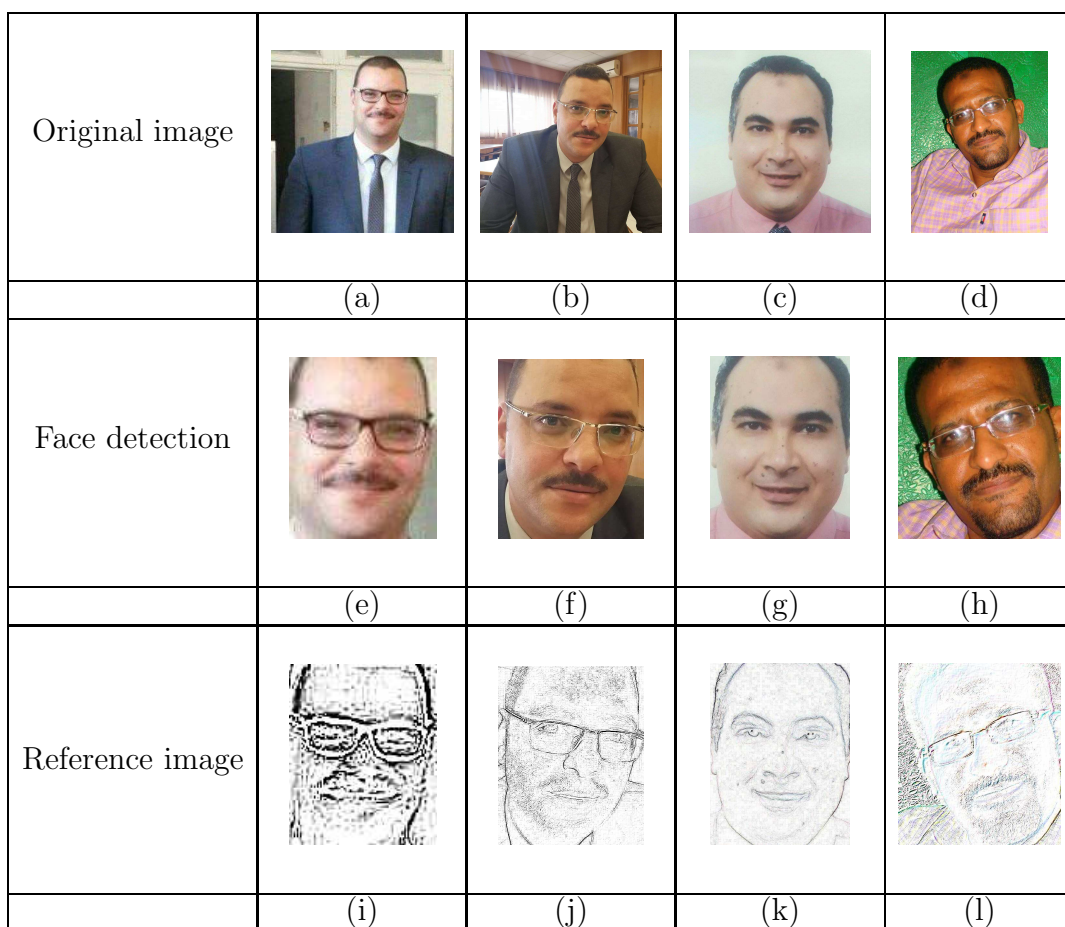


FIGURE 3. Face biometric for different persons

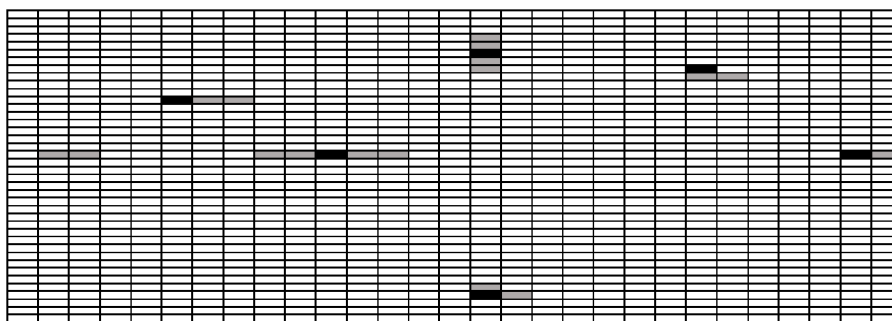


FIGURE 4. Cellular topology configuration (black is reference cell and gray are neighbor cells)

TABLE 1. Accuracy of various datasets

Datasets	Neural network classifiers %	Euclidean distance similarity measure %
Data1 (Face images)	96.66	92.10
Data2 (Fingerprint images)	92.00	88.14

TABLE 2. Accuracy of various datasets with comparisons

Dataset	Classifier	Accuracy %
Data1	KNN	65.70
	SVM	86.39
	Neural networks	84.17
	Proposed method	92.10
Data2	KNN	76.74
	SVM	78.14
	Neural networks	87.36
	Proposed method	88.14

TABLE 3. Time consumption of various datasets with comparisons

Dataset	Classifier	Time (second)
Data1	KNN	7.12
	SVM	41.58
	Neural networks	31.44
	Proposed method	6.13
Data2	KNN	7.78
	SVM	49.07
	Neural networks	28.16
	Proposed method	6.14

method with existing biometric recognition methods with different classifiers. We compare the recognition performance with SVM as in [14], KNN as in [14] and artificial neural networks as in [15]. The experimental results show that the proposed model presents the highest recognition rate in two datasets compared with KNN, SVM and neural networks.

The comparison time results in Table 3 show that SVM takes 47.95 second (in average among all images in dataset), KNN takes 7.71 seconds, neural networks take 32.59 seconds in average and the proposed model takes 6.143 seconds in average. The proposed method has the fastest time in training and recognition. It has been shown in the experiments

that our new method succeeds reasonably in recognizing fingerprint/face image in the presence of noise.

**5. Conclusions.** This paper introduced a proposed model of cellular automata based on fractional integral function. This innovative approach combines image analysis and matching approach as an integrated fashion. The cellular automata model introduced in this paper is significantly different from traditional cellular automata in which the traditional cellular automata has not clearly memory to store the cell's information and proposed model presents the new mathematical learning model. The present hybrid approach was applied to biometric problem. The results of experiments show that the proposed approach can generate more concise and more accurate rules. We need to extend this work to be fractional integral learning system and then give some frameworks for it and mathematically explain how we can apply it to different machine learning methods.

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