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A NOVEL WAVELET THRESHOLD DE-NOISING METHOD APPLIED TO ECG SIGNAL BY USING UNFIXED THRESHOLDING

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Abstract. *This paper mainly focuses on the concept of unfixed thresholding and its wavelet de-noising method. This thresholding has overcome the difficulties of thresholding determination. The unfixed thresholding reserves certain width, on both sides of the D. L. Donoho thresholding, to form a critical inter-area. It can achieve the process of de-noising by expanding or shrinking the interval. The scope of application is wide for the concept. Moreover, we apply the method into Electrocardiogram (ECG) signals. On the basis of simulation results of MATLAB, the method has shown its advantages in Signal to Noise Ratio (SNR), Root Mean Square Error (RMSE) and approximation to the real signal compared with the traditional wavelet thresholding de-noising methods.*
Keywords: Wavelet de-noising, Unfixed threshloding, Critical inter-area, Adaptive threshold function, ECG signal

1. **Introduction.** Noise is unavoidable in our real world, and many circumstances need to decrease the noise, such as system control [1], and signal processing. The arrival of information age accompanied by the rapid development of science and technology has made signal processing increasingly important. However, the process of obtaining and transmitting signal will be disturbed inevitably by noise, which results in the adverse effects on processing subsequent signal. In order to deal with the signal at a higher level, it is necessary to ensure that the signal after the noise reduction is proper and more scientistic. Since D. L. Donoho and I. M. Johnstone proposed the concept of adaptive wavelet threshold de-noising on the basis of wavelet transform in 1994 [2], this method has been widely applied and studied in de-noising process [3,4]. An adaptive threshold is assigned to each dyadic resolution level to minimize the Stein's Unbiased Risk Estimate (SURE) for threshold estimates [5]. [6] proposed adaptive de-noising method based on SURE so as to gain the best parameter and threshold for achieving well de-noising effect. Nowadays, more and more researchers have paid attention to the issue of the de-noising in signals and images [7,8]. Traditional threshold function can embrace hard threshold function and soft threshold function [9]. However, the hard threshold function is not continuous. Meanwhile, the soft threshold has a fixed deviation. To improve these shortcomings, a large number of experts and scholars had made great efforts and also achieved good results on improving the threshold function or determining the threshold. More complex parameter was introduced to work on the improvement of threshold functions in [10,11]. [12] is applying particle swarm optimization to wavelet de-noising, which has found a new way of threshold function construction and thresholding determination.

The wavelet threshold de-noising method is constantly upgraded, which makes its application wide. For example, it is used in medical signals. The T wave signal in the ECG

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signal is very vulnerable to be polluted by noise. Noise reduction has become a difficult problem to restrict the research progress of ECG signal. In order to solve this problem, [13] used the dyadic stationary wavelet transform in the Wiener filter and estimated the noise-free signal. It reduced the broadband myopotentials in ECG signals. For de-noising ECG signal, a novel modified S-median thresholding technique is proposed in [14]. Besides, determined threshold function and various thresholding evaluation conditions also limited its wider applications. Therefore, this paper proposed a new wavelet threshold de-noising method.

This method proposes the concept of unfixed thresholding and it establishes the adjustable critical inter-area at the same time. What can be controlled are the size of the shrinking range, the noise removal ratio and the signal details reservetion ratio, i.e., the method can achieve the target of reducing noise and reserve signal details composition as much as possible. We process the wavelet coefficients in different intervals through the corresponding threshold function. This method proves to be successful in practical operations. The simulation results indicate that this method shows simple and effective performance, producing a good de-noising effect on various noisy signals.

2. **Threshold Function.** Suppose the original signal $x(k)$ with the length as N is corrupted by white noise $e(k)$ which has the standard normal distribution $N(0, \sigma^2)$. The noise signal $f(k)$ is shown as follows:

$$f(k) = x(k) + e(k), \quad 1 \leq k \leq N \tag{1}$$

The decomposition layer of signal is J. Corresponding signal and noise wavelet coefficients will apparently be distinguished if the signal energy is obviously greater than noise energy. However, there is an interval called critical inter-area where the noise wavelet coefficients are close to the signal wavelet coefficients. As mentioned above, useful signal will be lost. And also if the threshold got too low, noise would be remained.

2.1. **Traditional threshold function.** The traditional function is hard threshold function and soft threshold function:

Hard threshold function

$$\hat{w}_{j,k} = \begin{cases} w_{j,k} & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases} \tag{2}$$

Soft threshold function

$$\hat{w}_{j,k} = \begin{cases} sgn(w_{j,k})(|w_{j,k}| - \lambda) & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| < \lambda \end{cases} \tag{3}$$

where $w_{j,k}$ is referring to the wavelet coefficient of noisy signal, and $\hat{w}_{j,k}$ is representing the wavelet coefficient after thresholding. λ is the threshold.

Hard threshold function is discontinuous, which may generate some artificial "noise points" in the restoration signal and cause some vibration in the reconstruction signal. Soft threshold function sometimes will lose useful high frequency information.

2.2. **An improved threshold function.** In view of the shortcomings of the traditional threshold function, we have proposed many improved threshold functions, such as following the establishment of the new threshold function:

$$\hat{w}_{j,k} = \begin{cases} sgn(w_{j,k}) \left(|w_{j,k}| - \dfrac{\ln(\beta\lambda)}{\ln(\beta|w_{j,k}|)} \right) & |w_{j,k}| \geq \lambda \\ 0 & |w_{j,k}| \leq \lambda \end{cases} \tag{4}$$

Xia et al. [15] took β a moderate value 5. The threshold function has good adaptive characteristics, but the threshold function on the threshold is not smooth.

The success of those methods can be summarized as following two points: for one thing, they established continuous threshold functions without constant deviation; for the other thing, they determined the exact thresholding. In order to achieve the two points, various parameters, algorithm and constraint condition were introduced to the wavelet de-noising method, which may drive the method to become complicated, unstable and need large amount of calculation. It made the wavelet de-noising process lose the original feature of simple and easy implement.

2.3. **Adaptive threshold function in this paper.** Threshold function was the key point for wavelet de-noising. Concretely, wavelet coefficients larger than λ_1 were retained, and those wavelet coefficients between $[\lambda_2, \lambda_1]$ were shrank and those ones smaller than λ_2 were treated as zero.

$$\hat{w}_{j,k} = \begin{cases} w_{j,k} & |w_{j,k}| \geq \lambda_1 \\ sgn(w_{j,k})\left(|w_{j,k}| - \dfrac{\lambda_2|\ln(\lambda_1 - \lambda_2)|}{|\ln(\lambda_1 - |w_{j,k}|)|}\right) & \lambda_2 < |w_{j,k}| < \lambda_1 \\ 0 & |w_{j,k}| \leq \lambda_2 \end{cases} \tag{5}$$

When $|w_{j,k}| = \lambda_2$, $\hat{w}_{j,k} = 0$. As $|w_{j,k}| \to \lambda_2$, $\hat{w}_{j,k} \to 0$. When $|w_{j,k}| = \lambda_1$, $\hat{w}_{j,k} = w_{j,k}$. As $|w_{j,k}| \to \lambda_1$, $\hat{w}_{j,k} \to sgn(w_{j,k})$, $|w_{j,k}| = w_{j,k}$. Namely it is continuous at λ_1 and λ_2. This could effectively avoid the problems of the hard threshold method. If the $|w_{j,k}|$ increases, then the $\frac{\lambda_2|\ln(\lambda_1-\lambda_2)|}{|\ln(\lambda_1-|w_{j,k}|)|}$ decreases continuously. So, the drawbacks of the soft threshold method could be effectively avoided, which has fixed deviation and produces a constant attenuation. The attenuation of greater absolute value of wavelet coefficients decreases with the absolute value increase through $\hat{w}_{j,k}$ measuring the attenuation. It could avoid loss of useful high frequency information and improve signal to noise ratio.

The previous approach is to find the confirmed value about λ_1 and λ_2; however, this is complicated and needs to take a long time to calculate. This problem is solved by the unfixed thresholding proposed in this paper.

3. **Selection of Thresholding.** The selection of thresholding is crucially important in wavelet de-noising effect. If the threshold value was too large, reconstruction of the signal would be too smooth to lose the useful information; on the contrary, if the threshold value was too low, it would retain numerous noise in the process of reconstructing signal, thus losing the significance of de-noising.

3.1. **D. L. Donoho thresholding.** It was critically important to select a precise threshold. However, the determination of threshold value needed the variance of the noise, which could not be a reality. It can only be estimated by the method proposed by D. L. Donoho. The most popular thresholding is proposed by D. L. Donoho and it is expressed as follows [1]:

$$\lambda = \sigma\sqrt{2\ln N}, \quad \sigma = median(|c|)/0.6475 \tag{6}$$

where N is the length of noisy signal, and σ is the hard deviation of zero-mean additive white Gaussian noise estimated by D. L. Donoho and I. M. Johnstone. c is referring to the detail coefficients of wavelet transform. However, the estimation of σ is directly affecting the accuracy of λ, which has further impacted the effectiveness of the de-noising.

3.2. **Unfixed thresholding.** Different from existing wavelet threshold ways, this method is not centering its attention on overcoming the disadvantages of hard threshold and soft threshold function, but on the wavelet coefficients which the noisy signal is decomposed. From the thought of wavelet threshold de-noising, noisy signal was decomposed into wavelet coefficients by wavelet basis. Then the appropriate threshold was selected to distinguish whether these coefficients were signal's or noise's. However, there existed a critical interval that mixed the wavelet coefficients of signal and noise.

Obviously, the above thresholding would affect the accuracy of the thresholding as well as the de-noising effect. Thus, we proposed the concept of unfixed thresholding, which could be used to deal with the shrinkage about wavelet coefficients of the critical interval by the corresponding threshold function. In this way, the wavelet coefficients of the signal were retained. Meanwhile, the wavelet coefficients of the noise were successfully suppressed. The expression of the unfixed thresholding is presented as follows:

$$\lambda_1 = \frac{b+1}{b}\lambda, \quad \lambda_2 = \frac{b}{b+1}\lambda \tag{7}$$

where $\lambda = \sigma\sqrt{2\ln N}$. In addition, signal singularity and detail features could be maintained by reserving large wavelet coefficients and shrinking wavelet coefficients in critical interval. Purpose of doing this can be traced as follows. To start with, it could avoid concussion caused by the discontinuous threshold function in the process of signal reconstruction. The adjustable interval ($[\lambda_2, \lambda_1]$) was set up by adaptive threshold. Through adjusting the parameter b, the size of critical interval can be determined to control the noise removal ratio. It reserves certain widths, $\frac{1}{b}\lambda$ and $\frac{1}{b+1}\lambda$, which are on both sides of the λ. Two requirements of threshold function were listed as follows. Firstly, the function was continuous on the threshold. Secondly, the wavelet coefficients in different districts were zeroed, shrank, and remained by threshold function.

4. **Simulation Result.** In order to illustrate the validity of the new wavelet threshold de-noising method, we test it in comparison with the soft and hard thresholding. In this paper, we apply the unfixed thresholding to the adaptive threshold function. We select the blocks signals including Gaussian white noise signal and corresponding chose the db1 wavelets. The largest decomposition scale is $J = 5$ and $b = [1, 10]$. We used the median value method to estimate the standard deviation of the noise.

The de-noising effect is shown in Figure 1, respectively. There are many noise's coefficients which are not removed by the hard threshold. The soft threshold method reduces a large number of useful information. By method in [15], certain inflection points are smoothed, too. From Figure 1 we can see the method in this paper retains details signals and removes the vast majority of noise at the same time. From Figure 2, we can draw the conclusion that when b ranges from about two to four, signal has the highest SNR and the lowest RMSE. Obviously, the application of the new method improves the de-noising effect a lot.

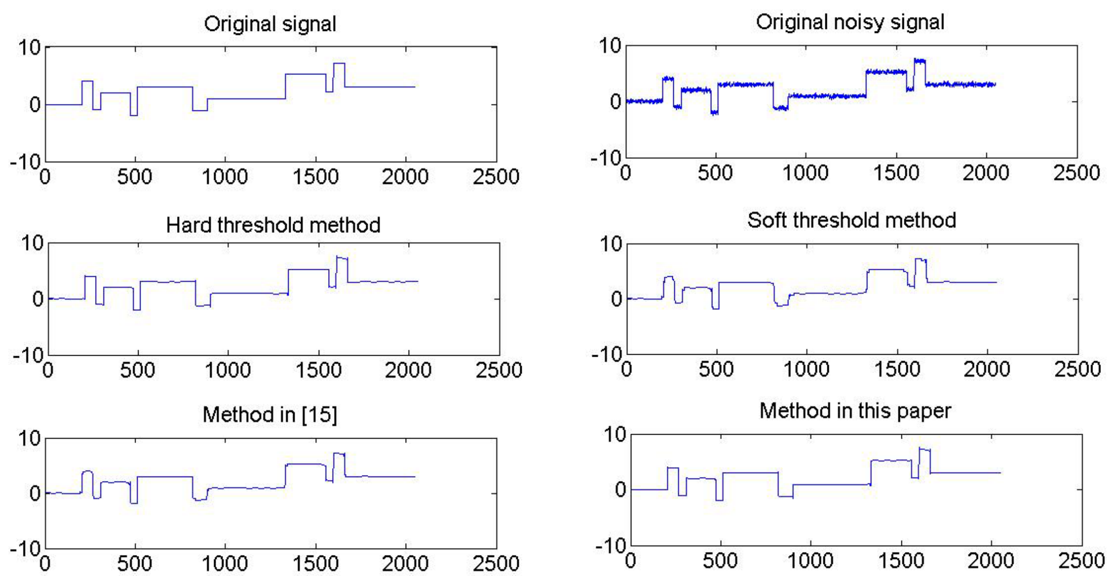

FIGURE 1. Blocks signal and denoising results of different methods

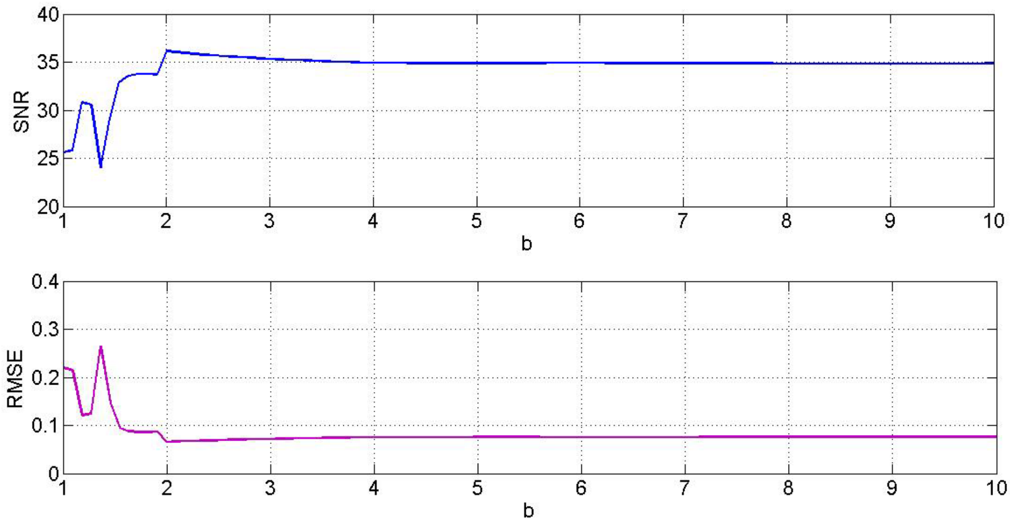

Figure 2. Blocks signal's SNR and RMSE in this paper

Table 1. Comparative performance of different de-noising methods about blocks

Methods	RMSE	SNR
Hard threshold method	0.0703	35.5232
Soft threshold method	0.2239	25.3715
Method in [15]	0.1337	29.8918
Best in this paper	0.0626	36.5311

The experiment results are shown in Table 1. In the best situation, the SNR value to the blocks signal in this paper is 36.7358, while the hard threshold method is 35.6391, the soft threshold method is 24.4005, the method in [15] is 28.5579, as the RMSE is. Consequently, the method in this paper is better than others.

5. **Application to ECG Signal.** In this paper, we try to choose the data numbering 114 in MIT-BIH ECG signal with the new thresholding method. Muscle Artifacts (MA) is a high frequency interference signal, and its spectrum is close to white noise. The cause of MA is the muscle excitation and contraction. After lots of simulation experiments, we choose the best wavelet basis bior5.5 which is more suitable for ECG signal. In order to illustrate the validity of the new wavelet threshold de-noising method, we test it in comparison with the soft, hard threshold and method in [15]. The largest decomposition scale is $J = 5$ and $b = [1, 10]$.

It is the same as Figure 1. Within the range of 1200 to 1400, the effect of unfixed threshold method is better than hard threshold method and method in [15]. However, the soft threshold method decreases quite a lot details signals. From Figure 3, the method in this paper has better performance compared with the others. Different from blocks signal, the SNR values are under 20 during four regions from Figure 4. So it needs to continue to study on de-noising stability.

From Table 2, the SNR value to the ECG signal in this paper is 36.6324 in the best situation, while the hard threshold method is 36.5904, the soft threshold method is 36.5873, and the method in [15] is 36.6157. As we can see, the method in this paper is better than others. Above all, the unfixed thresholding is effective for ECG signals.

6. **Conclusions.** Simulation results of MATLAB indicate that this method is superior than de-noising method of the soft and hard threshold in index of SNR and RMSE. This method does not need to determine thresholding and the selection of thresholding

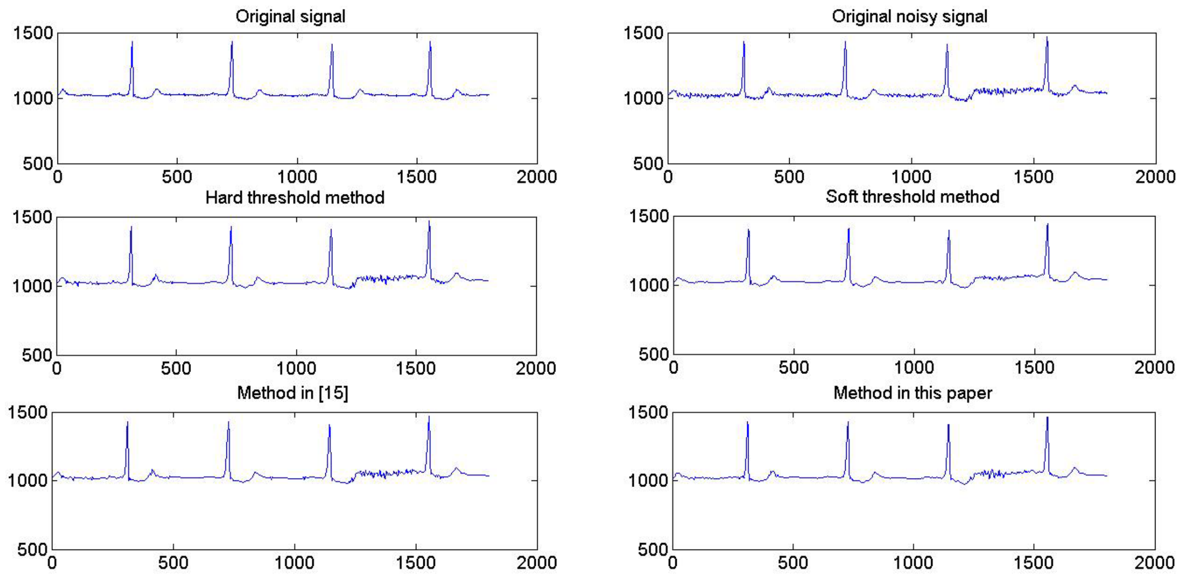

FIGURE 3. ECG signal and denoising results of different methods

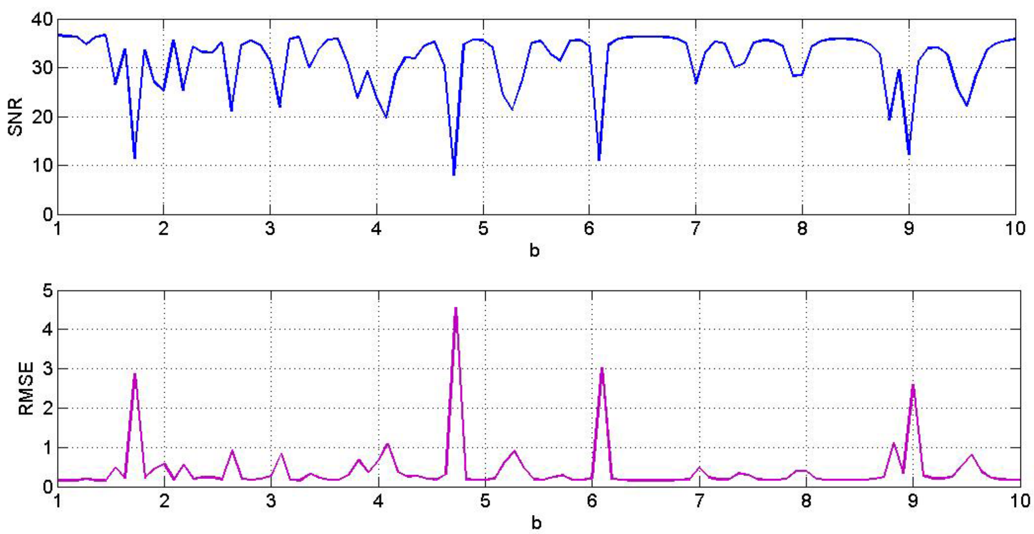

FIGURE 4. ECG signal's SNR and RMSE in this paper

TABLE 2. Comparative performance of different de-noising methods about ECG

Methods	RMSE	SNR
Hard threshold method	0.1537	36.4660
Soft threshold method	0.1537	36.5904
Method in [15]	0.1533	36.5870
Best in this paper	0.1529	36.6157

is flexible, which provides a wider application. Furthermore, the de-noising effect of the method when applied to ECG signals is marked. However, this method has no satisfactory results on ECG signals compared with blocks signal. Our future research will focus on finding more accommodate threshold functions and thresholding for ECG signals.

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