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COALITIONAL GAME-THEORETIC MODEL FOR INVENTORY MANAGEMENT

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Abstract. *In this study, we use an optimization model to solve production planning problems. A model in coalitional game theory uses a characteristic function defined under the framework of risk measure, precisely conditional value-at-risk. Through the model, managers can analyze the degree of risk for each period through a parameter, which indicates a penalty. This information allows them to employ consistent strategies to prevent lost revenue.*
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1. **Introduction.** In production planning, managers pursue on using efficient policies to overcome the loss. An important element to consider during the process is the risk. This element includes both the uncertainty of outcomes and as well their benefits [1]. Outcomes are related to the profit and loss statement, and the uncertainty in profits defined by the distribution function, which points the probable benefits or loss. Profits, unfortunately may be high or low sometimes [2, 3]. Concerning risk measures, several innovations have been presented, among which value-at-risk (VaR), conditional value-at-risk (CVaR) also known as tail conditional expectation, and shortfall expectation (SE) as well lead the list of accepted methodologies by practitioners.

Within the business world, there may exist scenarios where a group of companies tries to reach atypical agreement to pursue a common goal. This kind of interaction requires a fair agreement to avoid disruption of some members of the coalition from the business treat. While they keep this interaction efficiently they may experiment great benefits, i.e., the cost to order specific item by a single company would be less while that company maintains its position on the group coalition of corporations, while the opposite would be evident when the company decides to act solely. To that extent, strategies chosen by decision-makers (DMs) need to be efficient and consistent. However, this is not a simple task to perform due to some factors, take for instance the uncertainty related to

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demand or even necessary policies as regards to inventory management. Among others, stochastic models can be applied to responding such call. However, because of the existing interaction among the companies, a typical problem arising would be how the agents should (companies, different periods for production, etc.) divide the tasks or benefits equitably after between them. This situation leaves a gap to the game theory usage.

This work attempts to solve production planning problems mainly under the framework of coalitional game theory with transferable utility (TU games) and risk management as well. For the sake of fairness and uniqueness in the solution set, we consider Shapley value [4] defined as a least square value (LS) [5] as the representative of TU games and combine this approach to risk management [6, 7]. Moreover, it is important to mention that several studies related to the application of coalitional games in inventory management have been performed in the last years proving how this theory is efficient for solving this class of problems [8, 9].

After this introduction, the following section presents an overview of risk and risk measures with the focus on (CVaR) defined as the characteristic function of the game. Section 3 briefly describes inventory managements and main policies used within the process. In Section 4, starting with basic ideas with regards to coalitional games, theories on inventory games and production planning are also considered. Aiming to support DMs through mathematical modeling, a quadratic optimization model to solve production management problems is introduced in Section 5 followed by a numerical example in Section 6, and then some remarks conclude the study.

2. **Risk.** This section briefly introduces some ideas about risk. Particular interest is given CVaR a risk measure for the reason of offering efficient results when dealing with optimization problem, and here defined as a characteristic function.

Risk is a common element for almost every human activity. According to Holton in [1], the concept is extended to several different fields. Thus, it has distinct meanings and can be found in relations such as risk versus probability, risk versus threat, and all payoffs versus negative payoffs. Usually, risk involves two facts, namely: uncertainty and possible outcome, which can be positive, i.e., the one with benefit or negative when referring to a less expected result or in the worst case a total loss of revenue.

2.1. **Measures of risk.** Risk management is used to analyze and quantify probable losses, followed by practical actions as regard to pre-established objectives [6, 7]. Some of the techniques used to measure risk are listed in [13]: value-at-risk (VaR), conditional value-at-risk (CVaR), expected regret (ER), expected shortfall (ES), tail conditional expectation (TCE), tail mean (TM), worst conditional expectation (WCE) and spectral risk measures. Due to the objectives of this article, particular attention is given to CVaR, which will be described in the following subsection.

2.2. **Conditional value-at-risk (CVaR).** CVaR is the extension of VaR. Both techniques are popular functions for measuring risk. However, the former is more efficient when applied to optimization problems [6, 12, 14]. CVaR denotes the mean of the generalized α-tail distribution. More elaborated definition of conditional value-at-risk (CVaR) for random variables with a possibly discontinuous distribution function can be found in [6, 14].

Consider X a random variable normally distributed with the cumulative distribution function $F_x(y) = P\{X \geq y\}$. The CVaR of X with confidence level $\alpha \in [0, 1]$ is formally given as follows:

$$\mathrm{CVaR}_{1-\alpha}(X) = \int_{-\infty}^{\infty} y dF_x^{\alpha}(y) \tag{1}$$

with

$$F_x^{\alpha}(y) = \begin{cases} 0, & \text{if} \quad y < \mathrm{VaR}_{1-\alpha}(X) \\ \dfrac{F_x(y) - \alpha}{1 - \alpha}, & \text{if} \quad y \geq \mathrm{VaR}_{1-\alpha}(X) \end{cases} \tag{2}$$

While VaR with the confidence level $\alpha \in [0, 1]$ can be formally defined as follows:

$$\mathrm{VaR}_{1-\alpha}(X) = \min\{y \colon F_x(y) \geq \alpha\} \tag{3}$$

Hence, $\mathrm{CVaR}_{1-\alpha}(X)$ denotes the conditional expectation of X, subject to $X \geq \mathrm{VaR}_{1-\alpha}(X)$:

$$\mathrm{CVaR}_{1-\alpha}(X) = \{\mathrm{E}[X], \text{ s. t. } X \geq \mathrm{VaR}_{1-\alpha}(X)\} \tag{4}$$

3. **Inventory Management.** This section deals with inventory management. It describes the types of inventory policies and ends with the graphical representation of the so-called economic order quantity.

The field of inventory management was developed in the 20th century with the rapid growth of engineering and manufacturing industries. According to Fiestras-Janeiro et al. [16], mathematical models for inventory management were first proposed by Harris [15]. These models depend upon the choice of inventory policy adopted by those in the position of such a responsibility. Usually, an inventory graph as a function of time is used to implement policies. In a nutshell, the main objective of inventory management is to achieve the minimum cost per time unit based on a predetermined minimum level. Inventory policies can be classified into three types as stated by Vrat in [18].

1) Economic Order Quantity (EOQ)-Reorder Point (ROP) Policy: requires that the inventory status is observed in a regular base, that is to say, stock levels must constantly be supervised. A replenishment order of fixed quantity (EOQ) is used when an inventory level falls to a predetermined level (ROP). Two decision variables, (Q) for economic order quantity and (R) for reorder point, are used to evaluate the cost delivery and quantity delivery. It is expected from these decisions an efficient minimization of total costs in investments of inventories [17]. Figure 1 represents the dynamic of this type of policy graphically.

   In this case, the order quantity (Q) is equal to the difference between the high inventory level (HIL) and the low inventory level (LIL), i.e., $Q = HIL - LIL$ [17, 18].

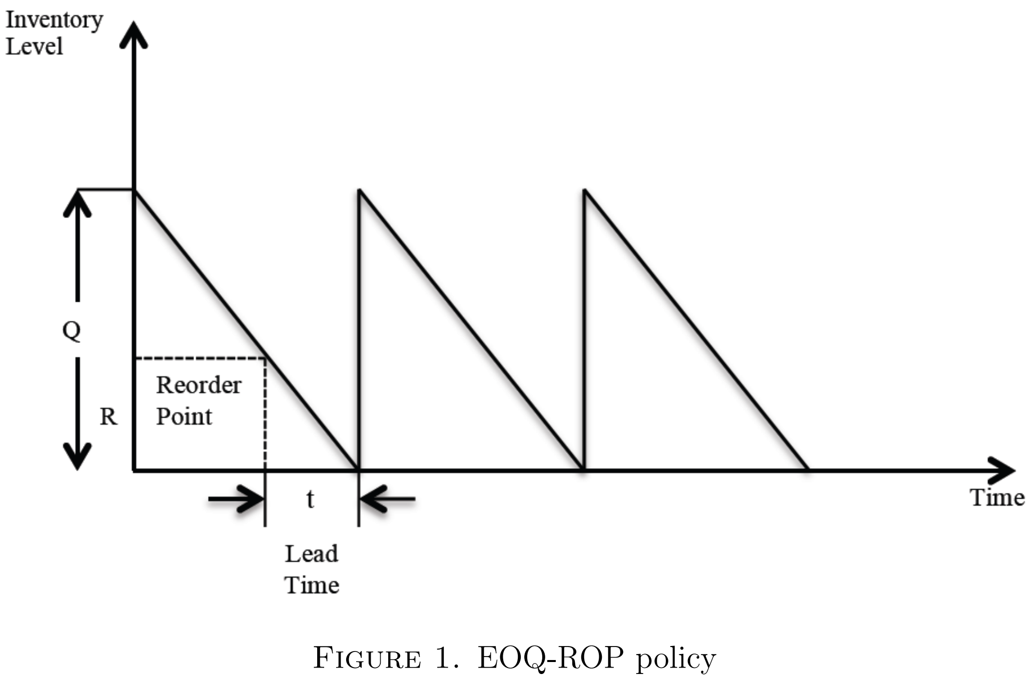

FIGURE 1. EOQ-ROP policy

2) Periodic Review Inventory Policy: after a specific time interval (T) a revision for the stock status is set, and then the order is placed.
3) Optional Replenishment Policy: this policy has two levels of inventories: maximum level (S) and minimum level (s). Although being similar to the previous policy, if at the time of review s has high value the replenishment happens only in the next period of review, and thus no order is placed for the reason that the stock is sufficient for that time. In the case where the stock level (x) is less than or equal to (s), the inventory policy is performed to raise the stocks to S level. Formally,

$$\begin{aligned} Q &= S - X \quad \text{if} \quad X \leq s \\ &= 0 \qquad\quad\ \text{if} \quad X \geq s \end{aligned} \tag{5}$$

Minimum-maximum stock level policy or just (s, S) policy is the other name for the optional replenishment policy with s, S, and T as decision variables. This policy seems to be better compared to the previous as long as the decision variables are defined.

4. **Coalitional Game.** This section deals with coalitional game theory. It starts by defining the formal representation of this kind of games and then two important properties are presented without proof. The two subsections describe inventory games briefly and as well elements related to production planning under demand uncertainty, respectively.

Cooperation helps companies to save on inventory cost. Ordering simultaneously as a group instead of individually, for instance, appears to be more beneficial to avoid paying much ordering costs when there exists a value per order settled. In this situation, DMs have to find efficient and consistent answers to solve the following problem: how should the total minimal inventory costs of the grand coalition be divided among the companies?

Attempting to solve the problem described above, a state of agreement and an assignment of actions is required to lead with the interactions among companies' decisions. Hence, game theory, as proved by an ever-increasing number of papers applying its techniques, and models, can be used to analyze the interactions between agents (players) in inventory field. In this paper, we focus on one range of game theory, i.e., cooperative (coalitional) game theory with transferable utility (TU games).

A coalitional game (TU game) on a finite set of players is defined as a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is the set of players and $v : 2^n \to \mathbb{R}$ a real-valued called characteristic function mapping with $v(\emptyset) = 0$. Any nonempty subset of N (including N itself and all the one-element subsets) is called a coalition [4, 5]. The characteristic function $v(\mathcal{S})$, i.e., the worth of coalition $\mathcal{S}$, represents the total amount of transferable utility that members of $\mathcal{S}$ could earn without any help from the players outside of $\mathcal{S}$. Which is equivalent to say, it is the maximum sum utility payoffs that the members of coalition $\mathcal{S}$ can guarantee themselves against the best offensive threat by the complementary coalition $N \backslash \mathcal{S}$.

Definition 4.1 (**Superadditivity**). *Given $\mathcal{S}, \mathcal{T} \subset N$ with $\mathcal{S} \cap \mathcal{T} = \emptyset$, game (N, v) is said to be superadditive if*

$$v(\mathcal{S} \cup \mathcal{T}) \geq v(\mathcal{S}) + v(\mathcal{T}) \tag{6}$$

Definition 4.2 (**Subadditivity**). *Given two disjoint coalitions $\mathcal{S}$ and $\mathcal{T}$, a game (N, v) is said to be subadditive when*

$$v(\mathcal{S} \cup \mathcal{T}) \leq v(\mathcal{S}) + v(\mathcal{T}), \ \forall \ \mathcal{S}, \mathcal{T} \subset N \tag{7}$$

Two important properties deserving to be pointed out for the sake of the aim of this study are the group rationality and the individual rationality. The satisfaction of the former about production planning is essential. They are described as follows.

Property 4.1. Efficiency *(Group rationality): players distribute among themselves the resources available to the grand coalition:*

$$\sum_{j=1}^{n} \Phi_j(N, v) = v(N) \tag{8}$$

Property 4.2. Individual fairness *(Individual rationality): every player gets at least as much as he would have received without cooperation:*

$$\Phi_j(N, v) \geq v(\{j\}), \quad \forall\, j = \{1, 2, \ldots, n\} \tag{9}$$

Inventory Games. Meca et al. [19] analyze a set of games (inventory games) for an n-company inventory case. Their study extends solutions of inventory management for a single company to a set of companies which minimizes their joint inventory cost through cooperation by analyzing within the framework of coalitional game theory. For further details on inventory games and other types of games the interested reader is referred to [9, 19, 20] and references therein.

Production Planning. Consider $D = d_1 + d_2 + \cdots + d_n$ as the cumulative demand, with d_j, $(j = 1, 2, \ldots, n)$, which expresses the demand for order at certain period i of planning. Demand $d = [d_1, d_2, \ldots, d_n]$ follows normal distribution, i.e., $d \sim N\left(\bar{d}, \Sigma\right)$, with $\bar{d}$ denoting the expected demand. The inventory for period j is given by S_j, $(j = 1, 2, \ldots, n)$ and calculated through Equation (10). Initially, inventory S_0 is assumed to be known.

$$S_j = S_0 + \sum_{t=1}^{j} \hat{x}_t - \sum_{t=1}^{j} \bar{d}_t \tag{10}$$

with $\hat{x}_j$ denoting the production level set for each period j, $(j = 1, 2, \ldots, n)$ which is computed by applying Equation (11).

$$\mathbf{X} = \mathbf{Qz} + \tilde{\mathbf{d}} \tag{11}$$

$1 - \alpha$ is the confidence level, often set to 0.95 or 0.99, and Σ is the variance-covariance matrix (12), with σ_{ij} elements.

$$\mathbf{\Sigma} = \begin{bmatrix} \omega_1^2 & \omega_1^2 & \cdots & \omega_1^2 \\ \omega_1^2 & \omega_1^2 + \omega_2^2 & \cdots & \omega_1^2 + \omega_1^2 \\ \vdots & \vdots & \ddots & \vdots \\ \omega_1^2 & \omega_1^2 + \omega_2^2 & \cdots & \omega_1^2 + \omega_2^2 + \cdots + \omega_n^2 \end{bmatrix} \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \tag{12}$$

with ω_i^2 – variance and $\sigma_{ij} = \omega_1^2 + \omega_2^2 + \cdots + \omega_i^2$, $(i < j)$.

5. **Model.** This section describes a quadratic model representing Shapley value, and then extends this model to a new one which combines elements of game theory and risk measures. Among other features, the model makes use of a constant called weighting factor through which a DM can evaluate the dynamic of production planning during a set of periods.

As a solution concept in coalitional games, the Shapley value [4] has several alternative models. In this study, we work on the one proposed in [5], and proved to be a value in the family of least square (LS) techniques, with a weight function $M_{n,s}$ defined as

$$M_{n,s} = \frac{1}{n-1} \binom{n-2}{s-1}^{-1} \tag{13}$$

where s indicates the cardinality of any coalition S, and n the number of players in the game. Formally the least square Shapley value is obtained by finding the optimal solution of the following problem:

$$\begin{aligned} &\min \sum_{s\subset N} \big(v(\mathcal{S}) - x(\mathcal{S})\big)^2 m(s) \\ &\text{s.t.} \ \sum_{i\in N} x_i = v(N) \\ &\qquad x_i \geq v(i), \ (\forall\, i \in N) \end{aligned} \tag{14}$$

Here $m(s) = M_{n,s}$ is the weight function, and x is a payoff vector with $x(\mathcal{S}) = \sum_{i\in\mathcal{S}} x_i$ for any coalition $\mathcal{S}$.

We transformed the previous program into a new one, Equation (15), within the framework of production planning. This model includes order quantity and grand coalition as constraints, and a weighting factor [11] to support managers while forecasting different production periods.

$$\begin{aligned} &\min_z \sum_i \sum_{|\mathcal{S}|} w_i M \left(v(\mathcal{S}) - \sum_{i\in\mathcal{S}} z_i(N, v) \right)^2 \\ &\text{s.t.} \ \sum_{i\in\mathcal{S}} z_i = v(N) \\ &\qquad z_i \geq \beta, \ (\exists\, i \in N) \end{aligned} \tag{15}$$

Notation: w_i – weighting factor at period i; z_i – Shapley values; Each value from Equation (13) constitutes the elements of a diagonal matrix M, i.e.,

$$\boldsymbol{M}_{n,s} = \begin{bmatrix} M_{n,1} & & & & \\ & M_{n,1} & \mathbf{0} & & \\ & & \ddots & & \\ & \mathbf{0} & & \ddots & \\ & & & & M_{n,n-1} \end{bmatrix} \tag{16}$$

Characteristic Function. The characteristic function for coalition $\mathcal{S}$ corresponds to the CVaR and is computed through Equation (17) as follows.

$$v(\mathcal{S}) = \mathrm{CVaR}(\mathcal{S})_{(1-\alpha)} = \sum_{i\in\mathcal{S}} d_i + \sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{S}} \sigma_{ij} \frac{\varphi(z_{1-\alpha})}{1 - \Phi(z_{1-\alpha})} \tag{17}$$

with,

$$z_{1-\alpha} = \frac{\mathrm{VaR}_D(1-\alpha) - \sum\limits_{i\in\mathcal{S}} d_i}{\sqrt{\sum\limits_{i\in\mathcal{S}} \sum\limits_{j\in\mathcal{S}} \sigma_{ij}}} \tag{18}$$

Here, φ is the standard normal density, Φ denotes the cumulative function and σ_{ij} are the element of the variance-covariance matrix Σ shown in (12).

6. Numerical Example.

Suppose we intent to analyze a production planning for three periods, i.e., $N = \{1, 2, 3\}$, with $S_0 = 10$, the estimated demand is given by $d = [10, 20, 24, 6, 12]$, while $w = 3$ and significance level given by $\alpha = 0.01$. The covariance-variance matrix (12) is then given by

$$\boldsymbol{\Sigma} = \begin{bmatrix} 9 & 9 & 9 & 9 & 9 \\ 9 & 18 & 18 & 18 & 18 \\ 9 & 18 & 27 & 27 & 27 \\ 9 & 18 & 27 & 36 & 36 \\ 9 & 18 & 27 & 36 & 45 \end{bmatrix}$$

Table 1. Characteristic function

Coalitions	$v(\mathcal{S})$
$v\{1\}$	0.2
$v\{2\}$	0.17
$v\{3\}$	0.31
$v\{12\}$	0.5
$v\{13\}$	0.64
$v\{23\}$	0.55
$v\{123\}$	1

Using Equation (13), we have $M_{3,1} = M_{3,2} = 0.5$. Table 1 shows the characteristic functions of the 3-periods game. We tested the model with different scenarios as an hipotetical case in the mind of a DM while forecasting production described as follows.

- Case A1: $z_3 \geq 0.7$, $w_2 = 100$
- Case A2: $z_3 \geq 0.7$, $w_1 = 100$
- Case A3: $z_3 \geq 0.6$, $w_2 = 100$
- Case A4: $z_3 \geq 0.6$, $w_1 = 100$
- Case A5: $z_3 \geq 0.5$, $w_1 = 100$
- Case A6: $z_3 \geq 0.5$, $w_2 = 100$
- Case A7: $z_2 + z_3 \geq 0.8$, $w_1 = 100$
- Case A8: $z_1 + z_3 \geq 0.8$, $w_2 = 100$
- Case A9: $z_1 + z_3 \geq 0.8$, $w_1 = 100$

z_i and w_i are constraints and penalties imposed to those periods, respectively.

Results and Discussion. The above cases were studied (see Table 2) employing Equation (14) in the first row, whose optimal solutions represent Shapley value. Since there exists not any new constraint inserted into the model, the corresponding space in the table is left blank. Optimal solutions for all other cases were attained through Equation (15) with new constraints as described in the second column, respectively. The last two columns show how much cost to minimize (badness regarding risk) and the total penalty the process suffers in each period by considering weighting factor and the constraints. Table 3 summarizes the single penalty in each period.

Table 2. Loss distribution

Cases	**Constraints**	**Period 1**	**Period 2**	**Period 3**	**O. f. (badness)**	**T. Penalty**
1		0.327	0.267	0.407	0.034	0.000
A1	$z_3 \geq 0.7$, $w_2 = 100$	0.130	0.160	0.700	0.168	-0.080
A2	$z_3 \geq 0.7$, $w_1 = 100$	0.190	0.100	0.700	0.164	-0.080
A3	$z_3 \geq 0.6$, $w_2 = 100$	0.230	0.170	0.600	0.090	0.000
A4	$z_3 \geq 0.6$, $w_1 = 100$	0.200	0.190	0.600	0.090	0.000
A5	$z_3 \geq 0.5$, $w_1 = 100$	0.201	0.299	0.500	0.074	0.000
A6	$z_3 \geq 0.5$, $w_2 = 100$	0.329	0.171	0.500	0.074	0.000
A7	$z_2 + z_3 \geq 0.8$, $w_1 = 100$	0.200	0.330	0.470	0.073	0.000
A8	$z_1 + z_3 \geq 0.8$, $w_2 = 100$	0.200	0.173	0.627	0.098	0.000
A9	$z_1 + z_3 \geq 0.8$, $w_1 = 100$	0.203	0.200	0.597	0.089	0.000

Table 3. Penalty over individual rationality

Cases	**Period 1**	**Period 2**	**Period 3**
1	0.127	0.097	0.097
A1	−0.070	−0.010	0.390
A2	−0.010	−0.070	0.390
A3	0.030	0.000	0.290
A4	1.000	0.020	0.290
A5	0.001	0.020	0.190
A6	0.129	0.129	0.190
A7	0.000	0.001	0.160
A8	0.000	0.160	0.190
A9	0.003	0.003	0.317

- All cases observe the group rationality (Property 4.1), while individual rationality (Property 4.2) is not satisfied. On the production planning viewpoint is preferable that both or the former property be satisfied rather than only the latter being satisfied.
- The weighting factor (w_i) and other constraints allow the DM to predict the dynamics of the production at each period in different scenarios.
- Shapley value in the first row does not have an overall penalty. Notice that the decision-maker does not need to impose any particular rule while forecasting the production for each period.
- Imposing weighting factor to the first and second periods and constraints to the last period (cases A1 and A2), the level of badness is relatively small, while the total penalty resulted from such policies is negative and equal for both cases.
- The weighting factor affects the total penalty to be null, from case A3 to A9. Major gain is obtained in periods in which this factor is employed.
- In the three last rows of periods A7 to A9, DM combines two periods to predict the total penalty his\her policies might suffer as to be null allowing his\her to proceed with strategies chosen.
- All cases with null penalty present a relatively small amount of badness.

The relationship between Table 2 and Table 3 can be summed as follows.

- While the former shows the total penalty for each case in its last column the later presents the single penalty for each case in all period.
- Cases A1 and A2 present the worst scenarios during the first two periods. Periods 1 (cases A7 and A8) and period 2 (case A3) do not present any penalty.

7. **Concluding Remarks.** In this study, we presented a model based on TU games to analyze production, inventory management. A weighting factor was introduced to support the forecasting process, and a set of constraints chosen by the decision-maker to control risk at each period was considered as well. A comparison study to analyze the performance of the model using the traditional economic order quantity would be an interesting direction for future studies.

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