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THE BROADBAND DIGITAL PHASED ARRAY ANTENNA BASED ON FARROW STRUCTURE DELAY FILTER

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Abstract. *Spatial dispersion and time dispersion are not necessary to be considered in the conventional narrowband phased array antenna, but they generate huge challenge for the implement of broadband phased array antenna. To solve the problem of dispersion in broadband phased array antenna, digital delay filter based on Farrow structure is proposed to complete the beamforming instead of the phase weighting as narrow band phased array antenna does. In this paper, the effects of spatial dispersion and time dispersion to phased antenna are introduced firstly. Then the design method of Farrow filter with symmetric coefficients is deduced when applied to broadband phased array. We also analyze the relationship between filter order and the precision of delay correction. Through the simulation of antenna pattern, it is proved that the problem of spatial dispersion and time dispersion is solved effectively, since the main lobe has no deviation in the case of large bandwidth.*
Keywords: Broadband phased array, Beamforming, Spatial dispersion, Time dispersion, Digital time delay filter

1. **Introduction.** In modern society, phased array antenna is widely used in various field including communication, radar, deep space exploration, etc. [1-3]. High amplification, electronically scanning and low side lobe are typical advantages for phased array. At present, the beamforming in phased array is often implemented through phase shifter and power splitter. With the development of wireless technique, broadband systems are a tendency in the future. However, when phased array antenna is applied to the broadband systems, spatial dispersion and time dispersion will produce huge challenge [4]. Traditional phase shifting in narrow band phased array cannot solve these problems. Therefore, method of delay for beamforming must be employed. Analog delay line is the fundamental device as illustrated in [5]. However, the accuracy suffers from the noncontinuity of analog delay line, since the length of delay line is unchangeable. On the other hand, digital delay filter can improve the delay accuracy significantly. At the initial time, digital delay filter was mainly used for software radio receivers with high speed sampling [6]. And [7] introduces an application of digital delayer in phased array, but it does not give the design method of delay filter. [8] introduces the design method of delay filter in time domain and frequency domain. Unfortunately, this procedure is not suited to digital phased array, because the desired delay time is varying very quickly with scanning angle and it is impossible to redesign the delay filter in real time. The delay filter based on Farrow structure is a nice strategy to overcome the shortcoming of

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time domain and frequency domain design methodology, because the Farrow filter can adjust the delay time without changing the filter coefficients. Generally, Farrow filter is widely used in multiple rate signal processing [9]. [10,11] introduce how to compute the Farrow filter coefficients, but these methods are not suited for the broadband phased array due to the bandwidth. In this paper, a weighted filter coefficient design methodology is adopted under the assumption that Farrow filter coefficients are symmetric. The new approach can support the phased array even in broadband system. To be more specific, this design scheme has the following advantages. Firstly, it can solve the problem of spatial dispersion and time dispersion caused by broadband. Secondly, delay value can be adjusted according to actual situation constantly benefited from Farrow structure filter. Thirdly, with filter order increasing, the precision of delay correction is also improved. Absolutely this approach has a better prospect of application.

The paper is organized as follows. Background information is given in this section. Spatial dispersion and time dispersion are discussed in Section 2. Farrow filter design and performance analysis are detailed in Section 3. The simulation work is presented in Section 4. Finally, the paper is concluded in Section 5.

2. **Spatial Dispersion and Time Dispersion.** Considering a uniform linear array with L elements which are all ideal omnidirectional antennas, the distance d between the two adjacent elements is equal to half wavelength of the carrier at the central frequency ω_0. Given incident angle θ_0 of radio signal, we calculate the direction vector $\boldsymbol{v}(\theta_0, \omega_0)$ with frequency ω_0 as

$$\boldsymbol{v}(\theta_0, \omega_0) = \begin{bmatrix} 1 & \dots & e^{j\pi(L-1)\sin\theta_0} \end{bmatrix}^T \tag{1}$$

Then in the application of narrowband phased array, we define the weight vector $\boldsymbol{w}$ equal to the direction vector, which is

$$\boldsymbol{w} = \boldsymbol{v}(\theta_0, \omega_0) \tag{2}$$

However, when the frequency ω of the signal which impinges from angle θ_0 is not equal to center frequency, the actual direction angle of the main lobe $\theta_{\max}$ is not equal to θ_0 according to the definition of radiation pattern. The error between $\theta_{\max}$ and expected direction θ_0 is

$$\Delta\theta = \theta_0 - \theta_{\max} = \frac{\omega_0 - \omega}{\omega_0} tg\theta_0 \tag{3}$$

The deviation of the main lobe caused by expansion of the frequency is called as spatial dispersion in phased array [4]. According to the equation above it is clear that the direction deviation is proportional to the relative bandwidth. Of course, the angle offset will result in degradation for array gain. This is why broadband phased array is unable to do beamforming through phase shifter as narrowband phased array does.

On the other hand, assuming that the signal at the original point of the coordination system is

$$s(t) = m(t)e^{j\omega_0 t} \tag{4}$$

In the above formula, $m(t)$ represents complex baseband signal. The delay between received signal $s_L(t)$ of element L and $s(t)$ is presented as τ_L, which is named as aperture fill time. For narrowband signal, the aperture fill time τ_L is significantly small compared with the pulse width in radar or compared with the symbol period in communication. Therefore, the delay of the signal envelope is negligible, i.e.,

$$s_L(t) = m(t - \tau_L)e^{j\omega_0(t-\tau_L)} \approx m(t)e^{j\omega_0(t-\tau_L)} \tag{5}$$

When the broadband signal is in discussion, the pulse width in radar is narrow or the symbol period in communication is small. Then the delay of signal envelope caused by aperture fill time is non-ignorable [12]. If the phased array antenna uses phase shifter

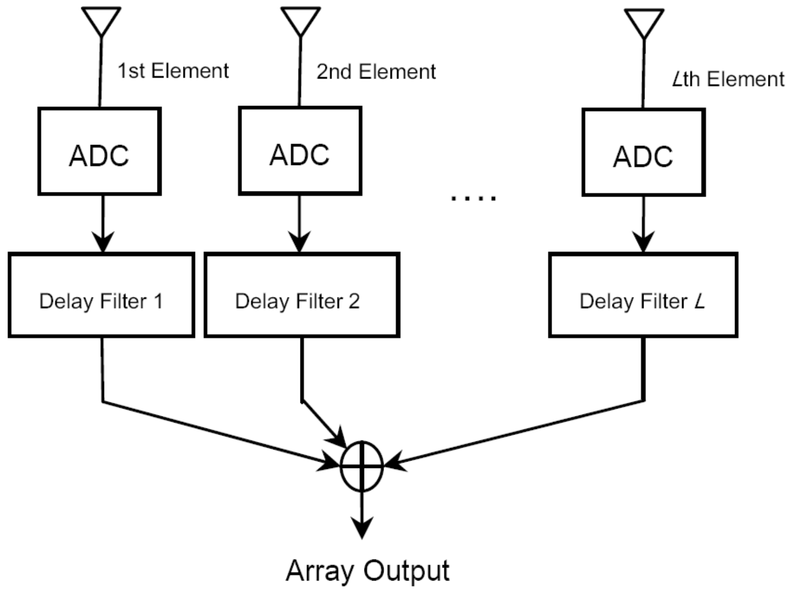


Figure 1. The structure of the broadband receiving phased array antenna with delay filter

to finish beamforming, radar pulse will be expanded, or inter-symbol interference will be very severe, which is called time dispersion.

In conclusion, broadband phased array cannot use phase shifter to scan the direction of pattern due to spatial dispersion and time dispersion. So delay devices must be employed. In digital domain, delay filter is a Finite Impulse Response (FIR) filter whose different coefficients correspond to different delay values [13]. Figure 1 shows the structure of the broadband receiving phased array antenna with delay filter. In fact, all kinds of digital array almost have the similar structure as Figure 1 [14].

3. Methodology of Digital Delay Filter Design.

3.1. **Farrow filter model.** Digital delay filter is an FIR filter. Assuming that the order of the filter is N, the coefficient vector can be expressed as

$$\boldsymbol{h} = [h_0 \ \dots \ h_N]^T \tag{6}$$

while delay is

$$\tau = nT_s + T_l \tag{7}$$

Namely the delay includes n-time sampling period T_s and a fractional term T_l which is shorter than the sampling period T_s. For digital signal processing, the first term can be generated in a simple way through clock control. Hence, the main task is to design variable fractional delay filters to implement the second term T_l. We define a parameter as

$$D = \frac{T_l}{T_s} \in [-0.5, 0.5] \tag{8}$$

Although certain method can be used to calculate $\boldsymbol{h}$ in time domain or frequency domain, the $\boldsymbol{h}$ will change with parameter τ during phased array working. When Field Programmable Gate Array (FPGA) or Digital Signal Processing (DSP) is taken to implement filter design, these two means cannot satisfy the requirement of real time in broadband phased array [15].

A delay filter with Farrow structure is a good choice to overcome the disadvantage mentioned above. For Farrow filter, each element of $\boldsymbol{h}$ is presented as M-order polynomial

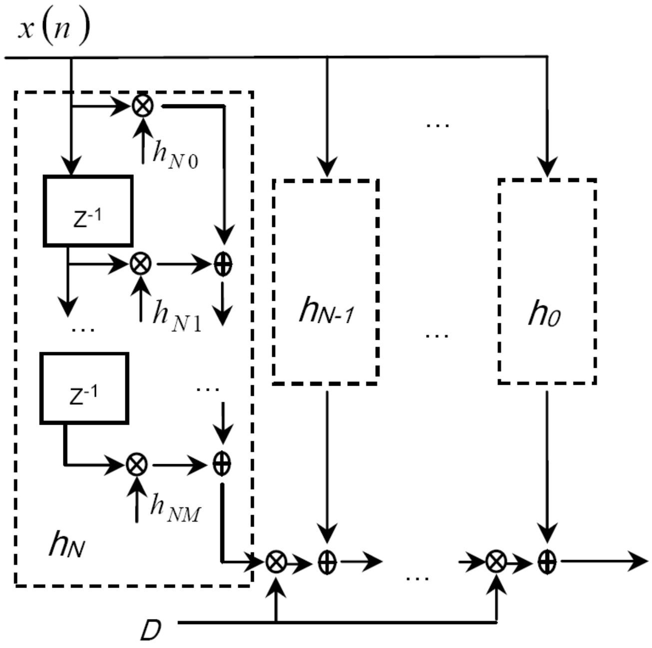


FIGURE 2. The structure of Farrow filter

of delay parameter D, namely

$$h_n = \sum_{m=0}^{M} h_{nm} D^m \quad n = 0, 1, \ldots, N \tag{9}$$

Then the structure of Farrow filter can be drawn as Figure 2 [16]. Clearly, we only need to adjust the input parameter D to generate any desired delay value T_l without updating h_{nm}.

In order to compute h_{nm}, we assume the coefficient vector $\boldsymbol{h}$ is symmetric since it is FIR filter. For the sake of convenience, we redefine the coefficient vector of Farrow filter as

$$\boldsymbol{h} = \boldsymbol{c} = [c_{-N} \ \ldots \ c_N]^T \tag{10}$$

Obviously, according to the definition of Farrow structure, we have

$$c_n = \sum_{m=0}^{M} c(n,m) D^m \qquad n = -N, \ldots, 0, \ldots, N \tag{11}$$

Meanwhile, assuming that received signal of the array is low pass signal, the delay filter is a low pass filter, so pass band is $[0, \alpha\pi]$, in which

$$0 < \alpha < 1 \tag{12}$$

3.2. **Object of Farrow filter design.** Firstly, the system function of the ideal delay filter is

$$H(\omega, D) = e^{-j\omega D} \tag{13}$$

And the amplitude-frequency characteristics of the Farrow filter is

$$C(\omega, D) = \sum_{n=-N}^{N} \sum_{m=0}^{M} c(n,m) D^m e^{-jn\omega} \tag{14}$$

Now we define a weighted error function by

$$J(\boldsymbol{c}) = \int_0^{\alpha\pi} \int_{-0.5}^{0.5} W(\omega, D) |E(\omega, D)|^2 d\omega dD \tag{15}$$

The error function in the equation above is

$$E(\omega, D) = C(\omega, D) - H(\omega, D) \tag{16}$$

At the same time, $W(\omega, D)$ is a non-negative function, which has two features, i.e.,

$$W(\omega, D) = W_1(\omega)W_2(D) \tag{17}$$

$$W(\omega, -D) = W(\omega, D) \tag{18}$$

So the object of Farrow filter design is to find out suitable coefficient set c, namely $c(n, m)$, and minimize $J(\boldsymbol{c})$ according to band width $\alpha\pi$ and delay parameter D.

3.3. **Coefficient symmetric Farrow filter design.** With the supposition that the filter coefficient is symmetric, we get that

$$c(-n, m) = \begin{cases} c(n, m) & \text{even } m \\ -c(n, m) & \text{odd } m \end{cases} \tag{19}$$

$$c(0, m) = 0 \quad \text{odd } m \tag{20}$$

By substituting the conclusion into the amplitude-frequency characteristic function of the filter, it gives that

$$C(\omega, D) = \boldsymbol{a}^T \boldsymbol{B}_e \boldsymbol{p}_e - j\boldsymbol{b}^T \boldsymbol{B}_o \boldsymbol{p}_o \tag{21}$$

in which

$$\boldsymbol{a} = [1 \;\; \cos(\omega) \;\; \cdots \;\; \cos(N\omega)]^T \tag{22}$$

$$\boldsymbol{b} = [1 \;\; \sin(\omega) \;\; \cdots \;\; \sin(N\omega)]^T \tag{23}$$

$$\boldsymbol{B}_e = \begin{bmatrix} \beta(0,0) & \beta(0,2) & \beta(0,4) & \cdots & \beta(0,M-1) \\ \beta(1,0) & \beta(1,2) & \beta(1,4) & \cdots & \beta(1,M-1) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta(N,0) & \beta(N,2) & \beta(N,4) & \cdots & \beta(N,M-1) \end{bmatrix} \tag{24}$$

$$\boldsymbol{B}_o = \begin{bmatrix} \beta(1,1) & \beta(1,3) & \beta(1,5) & \cdots & \beta(1,M) \\ \beta(2,1) & \beta(2,3) & \beta(2,5) & \cdots & \beta(2,M) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta(N,1) & \beta(N,3) & \beta(N,5) & \cdots & \beta(N,M) \end{bmatrix} \tag{25}$$

The element of the matrix is

$$\begin{cases} \beta(0, 2m') = c(0, 2m') & n = 0 \quad m' = 0, \ldots, M' \\ \beta(n, 2m') = 2c(n, 2m') & n > 0 \quad m' = 0, \ldots, M' \\ \beta(n, 2m'+1) = 2c(n, 2m'+1) & n > 0 \quad m' = 0, \ldots, M' \end{cases} \tag{26}$$

Substituting the result above into error function and taking the conjugate property of the weighted function into consideration, we get the weighted error function

$$J(\boldsymbol{c}) = J(\boldsymbol{B}_e, \boldsymbol{B}_o) = \int_0^{\alpha\pi} \int_0^{0.5} W(\omega, D)|E(\omega, D)|^2 d\omega dD \tag{27}$$

Compared to Equation (15), the integration of delay parameter D is defined in $[0, 0.5]$. Hence, the process to solve $c(n, m)$ can be converted into the process to solve the matrix $(\boldsymbol{B}_e, \boldsymbol{B}_o)$ which minimizes $J(\boldsymbol{B}_e, \boldsymbol{B}_o)$.

The weighted error function can be expressed as

$$\begin{aligned} J(\boldsymbol{B}_e, \boldsymbol{B}_o) = & -2tr[\boldsymbol{B}_e \boldsymbol{A}_1] + tr\left[\boldsymbol{B}_e \boldsymbol{A}_2 \boldsymbol{B}_e^T \boldsymbol{A}_3\right] + tr\left[\boldsymbol{B}_o \boldsymbol{A}_4 \boldsymbol{B}_o^T \boldsymbol{A}_5\right] \\ & - 2tr[\boldsymbol{B}_o \boldsymbol{A}_6] + constant \end{aligned} \tag{28}$$

in which

$$\boldsymbol{A}_1 = \int_0^{\alpha\pi} \int_0^{0.5} W_1(\omega)W_2(D)\cos(\omega D)\boldsymbol{p}_e \boldsymbol{a}^T d\omega dD \tag{29}$$

$$\boldsymbol{A}_2 = \int_0^{0.5} W_2(D)\boldsymbol{p}_e \boldsymbol{p}_e^T dD \tag{30}$$

$$\boldsymbol{A}_3 = \int_0^{\alpha\pi} W_1(\omega)\boldsymbol{a}\boldsymbol{a}^T d\omega \tag{31}$$

$$\boldsymbol{A}_4 = \int_0^{0.5} W_2(D)\boldsymbol{p}_o\boldsymbol{p}_o^T dD \tag{32}$$

$$\boldsymbol{A}_5 = \int_0^{\alpha\pi} W_1(\omega)\boldsymbol{b}\boldsymbol{b}^T d\omega \tag{33}$$

$$\boldsymbol{A}_6 = \int_0^{\alpha\pi}\int_0^{0.5} W_1(\omega)W_2(D)\sin(\omega D)\boldsymbol{p}_o\boldsymbol{b}^T d\omega dD \tag{34}$$

while

$$\boldsymbol{p}_e = \begin{bmatrix} D^0 & D^2 & \cdots & D^{M-1}\end{bmatrix}^T \tag{35}$$

$$\boldsymbol{p}_o = \begin{bmatrix} D^1 & D^3 & \cdots & D^{M}\end{bmatrix}^T \tag{36}$$

As long as the weight is known, all the equations above can be solved. Considering that $\boldsymbol{A}_2$, $\boldsymbol{A}_3$, $\boldsymbol{A}_4$ and $\boldsymbol{A}_5$ are symmetric and positive definite, they can be factorized into $\boldsymbol{U}_2$, $\boldsymbol{U}_3$, $\boldsymbol{U}_4$ and $\boldsymbol{U}_5$ through Cholesky factorization [17].

Finally, according to Lagrangian multiplier method, computing the derivative of $J(\boldsymbol{B}_e, \boldsymbol{B}_o)$ and letting it be zero, we get

$$\begin{cases} \boldsymbol{B}_e = \boldsymbol{U}_3^{-1}\left(\boldsymbol{U}_3^T\boldsymbol{A}_1^T\boldsymbol{U}_2^{-1}\right)\boldsymbol{U}_2^{-T} \\ \boldsymbol{B}_o = \boldsymbol{U}_5^{-1}\left(\boldsymbol{U}_5^T\boldsymbol{A}_6^T\boldsymbol{U}_4^{-1}\right)\boldsymbol{U}_4^{-T} \end{cases} \tag{37}$$

Following the steps above, $C(n, m)$ can be calculated and the design of the Farrow filter is completed.

3.4. **Weight value determination.** To evaluate the error between Farrow filter $C(\omega, D)$ we designed and ideal one $H(\omega, D)$, we define three parameters as follows

$$\varepsilon_{A\max} = \max\{20\lg|E(\omega, p)|\},\ \omega \in [0, \varepsilon\pi],\ D \in [-0.5, 0.5] \tag{38}$$

$$\varepsilon_{D\max} = \max\{|\tau(\omega, D) - D|\},\ \omega \in [0, \varepsilon\pi],\ D \in [-0.5, 0.5] \tag{39}$$

$$\varepsilon_e = \left[\frac{\int_0^{\alpha\pi}\int_{-0.5}^{0.5}|E(\omega, D)|^2 d\omega dD}{\int_0^{\alpha\pi}\int_{-0.5}^{0.5}|H(\omega, D)|^2 d\omega dD}\right]^{1/2} \times 100\% \tag{40}$$

And $\tau(\omega, D)$ denotes the group delay of the designed filter $C(\omega, D)$. These errors describe the performance in amplitude and phase respectively. And the allowable errors should be given before starting the filter design as Section 3.3 mentioned. Generally, the weight is set as

$$W_1(\omega) = W_2(D) = 1 \tag{41}$$

at the first time. After the filter coefficient is generated, we compute the errors as Equations (38) to (40) and compare them with the previous values given before design. If the requirements for error cannot be satisfied, we modify the weight and define weight as

$$W_1(\omega) = \begin{cases} s_1 & \omega \in [0, \omega_0] \\ s_2 & \omega \in [\omega_0, \alpha\omega] \end{cases} \tag{42}$$

$$W_2(D) = \begin{cases} s_3 & D \in [0, D_0] \\ s_4 & D \in [D_0, 0.5] \end{cases} \tag{43}$$

In detail, for the frequency interval where the errors are larger than the desired value, the weight should be increased. Otherwise, the weight keeps small. Now, we need to design the filter following the methodology in Section 3.3 and evaluate the errors again, until the errors meet the requirement.

4. Simulation Result.

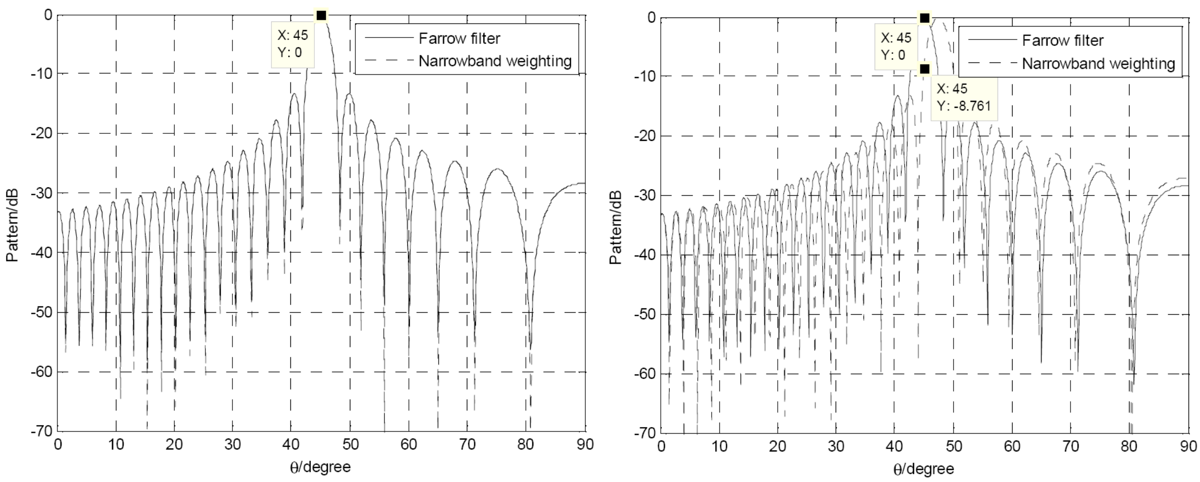
4.1. Beamforming with phase shifter and delay filter. In the simulation, a linear array with 50 elements is employed. Center frequency of the transmitting array is 30.2GHz with bandwidth 2.4GHz. The direction of beam was set to 45 degrees. DAC conversion rate is 8GHz. The orders of Farrow delay filter are $N = 34$, $M = 7$. The frequency band is separated into two intervals $[0, 0.9]$ and $[0.9, 1]$ for weight. Delay is separated into $[0, 0.4]$ and $[0.4, 0.5]$. The corresponding weight function is

$$W_1(\omega) = \begin{cases} 1 & \omega \in [0, 0.9] \\ 3700 & \omega \in [0.9, 1] \end{cases} \tag{44}$$

$$W_2(D) = \begin{cases} 1 & D \in [0, 0.4] \\ 47 & D \in [0.4, 0.5] \end{cases} \tag{45}$$

We performed the simulations at two input frequencies, 29GHz and 30.2GHz, respectively. For the reason that digital delay filter is implemented in digital domain, the inputs should be IF signal. Therefore, the frequencies of inputs in this simulation are set to 1.5GHz and 0.3GHz which correspond to carrier wave 30.2GHz and 29GHz respectively.

The radiation pattern of central frequency at 1.5GHz is shown as Figure 3(a). The solid line denotes the simulation result with Farrow filter, while the dotted line denotes that with traditional narrowband phased array weighting. These two lines overlap each other very well, and the directions of main lobe are both correct. At the central frequency, there is no any dispersion, so that even narrowband phased array can work perfectly. At the same time, the plot also proves that the Farrow filter plays the role of phase shifting and the proposed methodology is correct.



(a) Pattern at 1.5GHz (b) Pattern at 0.3GHz

FIGURE 3. Patterns of narrowband and broadband phased array

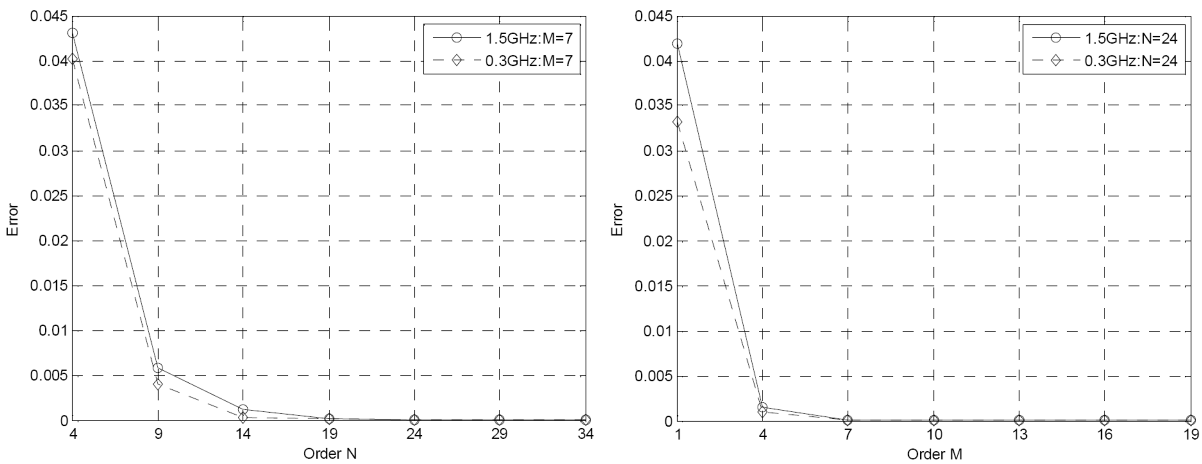
The radiation pattern of low side frequency at 0.3GHz is shown as Figure 3(b). The main lobe with Farrow filter is pointing to the desired direction. That means Farrow filter is still effective for broadband array. Whereas the narrow band methodology results in direction deviation and the gain is reduced with more than 8dB. In fact, the same phenomenon emerges at high side frequency at 2.7GHz. So the phase shifting used in narrow band array is not suited for the broadband array.

4.2. Performance of Farrow filter with order. The delay accuracy is the key quantity to evaluate the performance of Farrow filter, which is decided by the filter order, namely N and M. Supposing the input signal $s(t)$ is continuous wave with frequency 1.5GHz and 0.3GHz respectively, and the delay $D = 0.3$, the output signal of Farrow filters with different orders is $y(t)$. Then we define the error by

$$E(t) = |y(t) - s(t - \tau)| \tag{46}$$

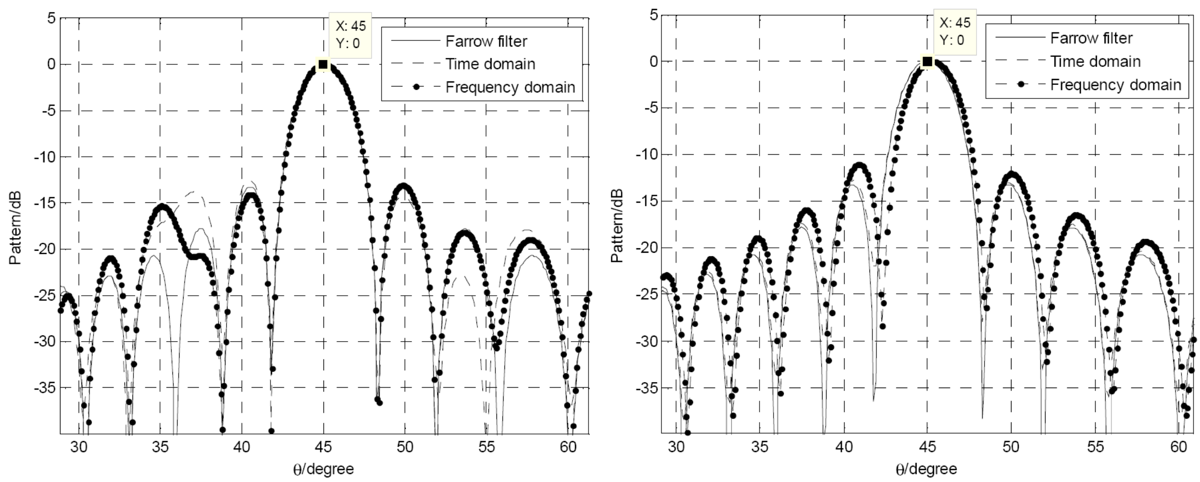
Finally, the simulation patterns are shown in Figures 4(a) and 4(b) for different frequencies. From the picture, we can see that larger order N and M can both reduce the error. So high order Farrow filter can improve the performance of delay accuracy. Nevertheless, when the orders increase to a certain value, the error will not change anymore. Moreover, with the same orders, the low frequency signal has less error than the high frequency signal. This indicates that the delay error is related to the bandwidth.

4.3. **Comparison of delay filter design methods in phased array.** Taking the same simulation parameters as in Section 4.1, we design the delay filters through three strategies, which are Farrow filter we proposed, time domain method and frequency domain method illustrated in [8]. Then we get the pattern as Figure 5, where 5(a) and 5(b) correspond to signal at 1.5GHz and 0.3GHz respectively. For the central frequency, all the methods play very well in Figure 5(a). However, the main lobe direction has slight deviation for the time domain and frequency domain methods in Figure 5(b), when we simulate



(a) Error at center frequency 1.5GHz (b) Error at side frequency 0.3GHz

FIGURE 4. Relationship between delay accuracy of Farrow filter and its orders



(a) Pattern at 1.5GHz (b) Pattern at 0.3GHz

FIGURE 5. Comparison of different filter design methods

the broadband signal at 0.3GHz. Although the three methods have similar performance, the advantages of the new design methodology are obvious. Firstly, the coefficient does not need to compute when the direction of the array is changing. In addition, we can get less delay error with the same filter order according to the performance of sidelobe.

5. **Conclusion.** The proposed broadband phased array antenna with the Farrow structure could overcome the space dispersion and time dispersion problems. However, how to choose the weight value to simplify the design is the future work to do. In addition, how to reduce the delay error of the Farrow filter is also needed to research.

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REFERENCES

[1] N. N. Alotaibi and K. A. Hamdi, Switched phased-array transmission architecture for secure millimeter-wave wireless communication, *IEEE Trans. Communications*, vol.64, no.3, pp.1303-1312, 2016.
[2] S. Yang, T. Cheng and Z. He, A colocated mimo radar transmit beamforming approach for radio frequency stealth, *ICIC Express Letters*, vol.9, no.8, pp.2169-2176, 2015.
[3] K. Sarabandi and F. N. Wang, Phased array of large reflectors for deep-space communication, *IEEE Trans. Aerospace and Electronic Systems*, vol.43, no.1, pp.251-261, 2007.
[4] M. R. Patel and R. K. Arora, Dispersion effects in a circular phased array, *IEEE Trans. Aerospace and Electronic Systems*, vol.20, no.6, pp.817-821, 1984.
[5] R. P. Hsia, W. M. Zhang, C. W. Domier et al., A hybrid nonlinear delay line-based broad-band phased antenna array system, *IEEE Microwave and Guided Wave Letters*, vol.8, no.5, pp.182-184, 1998.
[6] M. D. Meyer and D. P. Agrawal, A high sampling rate delayed LMS filter architecture, *IEEE Trans. Circuits and Systems II: Analog and Digital Signal Processing*, vol.40, no.11, pp.727-729, 1993.
[7] P. K. Bailleul, A new era in elemental digital beamforming for spaceborne communications phased arrays, *Proc. of the IEEE*, vol.104, no.3, pp.1-10, 2016.
[8] H. H. Dam, Variable fractional delay FIR filter design with a bicriteria and coefficient relationship, *IEEE Trans. Circuits and Systems II: Express Briefs*, vol.61, no.1, pp.36-40, 2014.
[9] H. Johansson and O. Gustafsson, Linear-phase FIR interpolation, decimation, and mth-band filters utilizing the Farrow structure, *IEEE Trans. Circuits and Systems I: Regular Papers*, vol.52, no.10, pp.2197-2207, 2005.
[10] M. Abbas, O. Gustafsson and H. Johansson, On the fixed-point implementation of fractional-delay filters based on the Farrow structure, *IEEE Trans. Circuits and Systems I: Regular Papers*, vol.60, no.4, pp.926-937, 2013.
[11] H. Johansson, Farrow-structure-based reconfigurable bandpass linear-phase FIR filters for integer sampling rate conversion, *IEEE Trans. Circuits and Systems II: Express Briefs*, vol.58, no.1, pp.46-50, 2011.
[12] V. Szortyka, K. Raczkowski, M. Kuijk et al., A wideband beamforming lowpass filter for 60 GHz phased-array receivers, *IEEE Trans. Circuits and Systems I: Regular Papers*, vol.62, no.9, pp.2324-2333, 2015.
[13] X. Mu and Y. Zhang, A second-order cone programming method for the design of FIR filters with discrete coefficients, *ICIC Express Letters*, vol.5, no.4, pp.1183-1189, 2011.
[14] S. H. Talisa, K. W. O'Haver and T. M. Comberiate, Benefits of digital phased array radars, *Proc. of the IEEE*, vol.104, no.3, pp.530-543, 2016.
[15] J. Gao, An improved delayed structure of least mean square adaptive filter and its field programmable gate array implementation, *ICIC Express Letters, Part B: Applications*, vol.4, no.1, pp.69-73, 2013.
[16] C. K. S. Pun, Y. C. Wu, S. C. Chan et al. On the design and efficient implementation of the Farrow structure, *IEEE Signal Processing Letters*, vol.10, no.7, pp.189-192, 2003.
[17] J. H. Choi and C. C. Lim, A cholesky factorization based approach for blind FIR channel identification, *IEEE Trans. Signal Processing*, vol.56, no.4, pp.1730-1735, 2008.