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AN ALTERNATIVE KALMAN FILTERING BASED UNKNOWN INPUT IDENTIFICATION SCHEME

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Abstract. *In this paper, an alternative Kalman filtering based unknown input identification scheme is proposed. The unknown input is augmented as a state term and then estimated by state estimation filtering. The unknown input noise covariance is considered as a useful design parameter to adjust finely the proposed scheme's performance. Through computer simulations, the proposed scheme is verified by computer simulations for the speed-sensorless DC motor system. Simulation results show that the unknown input noise covariance can make the tradeoff between the noise suppression and the tracking speed of the state estimation as well as the unknown input identification.*

Keywords: Kalman filter, Unknown input estimation, Noise covariance, Noise suppression, Tracking speed

1. **Introduction.** Faults and model uncertainties can be represented as unknown inputs [1-3]. The unknown input identification or estimation arises in many areas such as fault detection and diagnosis (FDD) for various systems and maneuver detection and target tracking (MDTT) of flying objects. The Kalman filtering based unknown input identification scheme has been developed for statistically optimal estimates of the state and unknown inputs for stochastic systems with noise [2-4]. Due to the compact representation and the efficient manner, the Kalman filter has been applied successfully for various areas including an unknown input identification [5]. However, the Kalman filter has an infinite memory structure that utilizes all observations accomplished by equaling weighting and has a recursive formulation. Thus, the Kalman filter tends to accumulate the filtering error as time goes and can show even divergence phenomenon for abrupt unknown input and temporary modeling uncertainty [6]. In addition, actually, long past measurements are not useful for detection or identification of signal with unknown times of occurrence. Moreover, it is also known that the increase of the number of measurements for a detection decision will increase detection latency in a system for detecting a signal with unknown time of occurrence. Therefore, as an alternative to the Kalman filter with infinite memory structure, the finite memory structure filter has been developed and also applied successfully for unknown input identification areas [7-9]. However, in case of too large window length, the real-time application of the finite memory structure filter is somewhat difficult because the computational load depends on the window length.

Therefore, in this paper, an alternative Kalman filtering based unknown input identification scheme is proposed to resolve above issues. For the proposed scheme, the unknown input is considered as a state term and then estimated by the Kalman filtering. In order to adjust finely the performance of the proposed scheme, the unknown input noise covariance is considered as a useful design parameter. The proposed scheme is verified by computer simulations for the DC motor system. Computer simulations show that the

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unknown input noise covariance can make the tradeoff between the noise suppression and the tracking speed of the state estimation as well as the unknown input identification.

This paper is organized as follows. In Section 2, the Kalman filtering for state estimation and the unknown input identification is described. In Section 3, the unknown input noise covariance is considered as useful design parameter. In Section 4, extensive computer simulations are performed to verify the proposed scheme. Finally, conclusions are presented in Section 5.

2. **Kalman Filtering for State Estimation and Unknown Input Identification.**

In general, the unknown input model can be represented by the following discrete-time state space model with unknown input as well as noises:

$$\begin{aligned} x(i+1) &= \Phi x(i) + Du(i) + \Delta p(i) + Ew(i), \\ y(i) &= Cx(i) + v(i), \end{aligned} \tag{1}$$

where $x(i) \in \Re^n$ is the state vector, and $u(i) \in \Re^l$ and $y(i) \in \Re^q$ are the input vector and the measurement vector. The covariances of the system noise $w(i) \in \Re^p$ and the measurement noise $v(i) \in \Re^q$ are Q_w and R, respectively. The unknown input vector $p(i) \in \Re^q$ in the system under consideration is to be represented by random-walk processes as

$$p(i+1) = p(i) + \delta(i),$$

where the unknown input $p(i) \equiv [p_1(i) \quad p_2(i) \quad \cdots \quad p_s(i)]^T$ and the unknown input noise $\delta(i) \in \Re^s$ is a zero-mean white Gaussian random process with covariance Q_δ. It is noted that the random-walk process provides a general and useful tool for the analysis of unknown time-varying parameters and has been widely used in the detection and identification area.

The unknown input can be treated as auxiliary states and then the state-space model (1) can be rewritten as an augmented state-space model as

$$\begin{aligned} \begin{bmatrix} x(i+1) \\ p(i+1) \end{bmatrix} &= A \begin{bmatrix} x(i) \\ p(i) \end{bmatrix} + Bu(i) + G \begin{bmatrix} w(i) \\ \delta(i) \end{bmatrix}, \\ y(i) &= Cx(i) + v(i), \end{aligned} \tag{2}$$

where

$$A = \begin{bmatrix} \Phi & \Delta \\ 0 & I \end{bmatrix}, \quad B = \begin{bmatrix} D \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix},$$

and the system noise and the unknown input noise term $[w^T(i) \quad \delta^T(i)]^T$ is a zero-mean white Gaussian random process with covariance $Q = diag\,([Q_w \quad Q_\delta])$.

This paper considers the estimation filtering for both the state estimates and the unknown input identification. The unknown input is augmented as a state term and then estimated by state estimation filtering. For the estimation filtering, the well-known Kalman filter with the infinite memory structure is used. The Kalman filter provides the state estimate $\hat{x}(i)$ and the unknown input estimate $\hat{p}(i)$ for the system state $x(i)$ and the unknown input $p(i)$ as follows:

$$\begin{aligned} \begin{bmatrix} \hat{x}(i+1) \\ \hat{p}(i+1) \end{bmatrix} &= A \left(I + \Sigma(i)C^T R^{-1} C\right)^{-1} \left(\begin{bmatrix} \hat{x}(i) \\ \hat{p}(i) \end{bmatrix} + \Sigma(i) C^T R^{-1} y(i) \right) + Bu(i), \\ \Sigma(i+1) &= A \left(I + \Sigma(i)C^T R^{-1} C\right)^{-1} \Sigma(i) A^T + GQG^T, \\ Q &= diag\,([Q_w \quad Q_\delta]), \end{aligned} \tag{3}$$

where $\hat{x}(i_0) = \bar{x}(i_0)$ and $\Sigma(i)$ is the error covariance of the estimate $\left[x^T(i) \quad p^T(i)\right]^T$ with initial value $\Sigma(i_0)$. The Kalman filter has been used generally and widely for the optimal state estimation using all past measurements and thus applied successfully for various areas. However, due to the infinite memory structure and the recursive formulation, the Kalman filter tends to accumulate the filtering error as time goes and can show even divergence phenomenon for abrupt unknown input and temporary modeling uncertainty.

3. **Unknown Input Noise Covariance as Useful Design Parameter.** The important issue here is how to choose an appropriate covariance Q_δ of unknown input noise $\delta(i)$ that makes the filtering performance as good as possible. Intuitively, a reasonable criterion for the choice of the unknown input noise covariance should be how much information about the current state of the system the older data and the new data contain. If the newly coming data bring enough information about the current state, or the older data contain less information on the present data, the unknown input noise covariance Q_δ should be smaller. When the exact information about the unknown input noise covariance cannot be obtained but some rough information about the unknown input noise can be obtained, there are some choices of Q_δ. If the covariance Q_δ of unknown input noise $\delta(i)$ is smaller, the older data should contain more information on the current state. In comparison with the covariance Q_δ of unknown input noise $\delta(i)$, if the covariance R of the measurement noise $v(i)$ is larger, more data should be used to suppress the influence of the noise by means of averaging the measurement data.

Thus, in this paper, the normalized covariance $Q_\delta R^{-1}$ is considered as a useful design parameter to adjust finely the filtering performance. The noise suppression of the proposed Kalman filtering based unknown input identification scheme might be closely related to the normalized covariance $Q_\delta R^{-1}$. The proposed filtering scheme can have greater noise suppression as the normalized covariance $Q_\delta R^{-1}$ decreases, which can enhance the filtering performance of the proposed scheme. However, too small $Q_\delta R^{-1}$ may yield the long convergence time of filtered estimates, which can degrade the filtering performance of the proposed scheme. And vice versa. This illustrates the proposed scheme's compromise between the noise suppression and the tracking speed of the filtered estimates. Thus, the unknown input noise covariance $Q_\delta R^{-1}$ is a continuous parameter to adjust finely the filtering performance. However, since $Q_\delta R^{-1}$ is an integer, fine adjustment of the properties with $Q_\delta R^{-1}$ is difficult. Moreover, it is not easy to determine the normalized covariance in systematic ways. In applications, one method to determine the normalized covariance is to take the appropriate value that can provide enough noise suppression.

4. **Computer Simulations for DC Motor System.** Computer simulations are performed for the discrete-time state-space model for the speed-sensorless DC motor system as follows [10,11]:

$$A = \begin{bmatrix} 0.8187 & -0.0011 \\ 0.0563 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1813 \\ 1.0069 \end{bmatrix}, \quad E = \begin{bmatrix} -0.0069 \\ 6.3210 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.0006 & 0 \\ 0 & 0.0057 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The armature current and the rotational speed of the DC motor are taken as state variables. The armature voltage is treated as the input and the armature current is chosen as the output.

System and measurement noise covariances are taken as $Q = diag\left([0.01^2 \quad 0.01^2]\right)$ and $R = 0.05^2$, respectively. The unknown input is emulated by the step-type load torque $p(i) = 0.5$ on the interval $100 \leq i \leq 300$. To verify that the normalized covariance can be a useful design parameter to adjust finely the filtering performance, four kinds of values are taken by $Q_\delta R^{-1} = 0.1, 1, 10, 100$. To make a clearer comparison of estimation performances, simulations of 20 runs are performed and each single simulation run lasts 500 samples. Figure 1 shows root mean square (RMS) estimation errors of the rotational speed for 20 simulations. Figure 2 shows estimation error of the rotational speed for one of

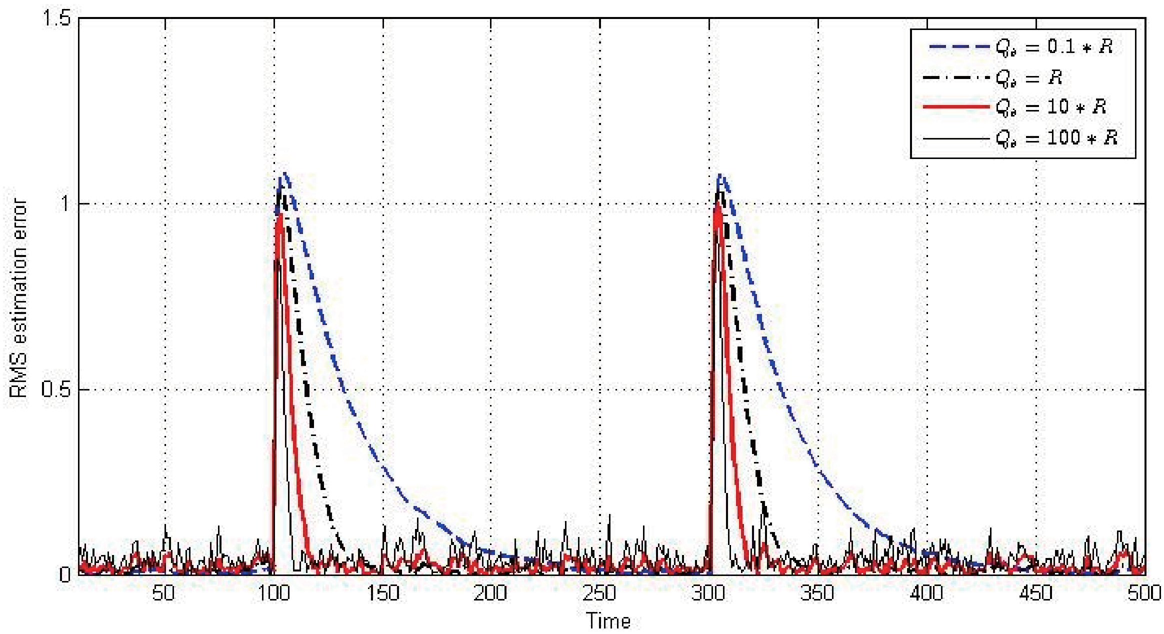

FIGURE 1. RMS estimation error of the 2nd state variable (rotational speed)

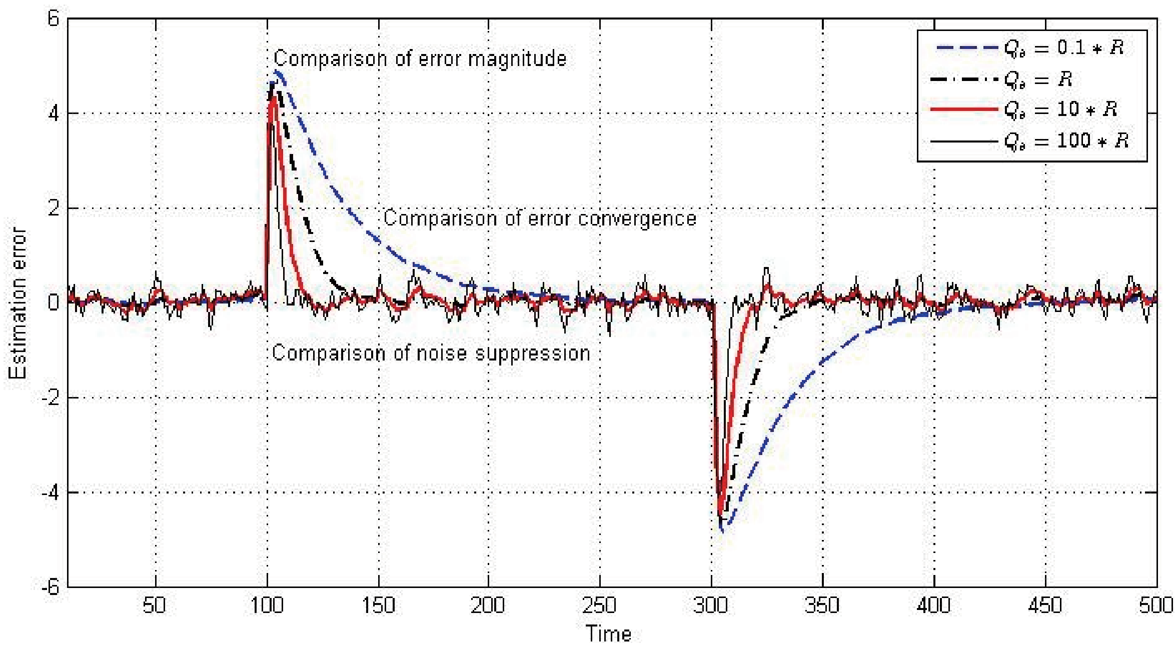

FIGURE 2. Estimation error of the 2nd state variable (rotational speed)

20 simulations. Finally, Figure 3 shows the unknown input estimate, that is, the estimate of the load torque.

As shown in simulations results, the noise suppression of the proposed scheme might be closely related to the normalized covariance $Q_\delta R^{-1}$. The proposed filtering scheme can have greater noise suppression for both state estimate and unknown input estimate as the normalized covariance $Q_\delta R^{-1}$ decreases. However, small $Q_\delta R^{-1}$ may yield the long convergence time of estimation error and the long tracking time of unknown input estimate. These computer simulations show that the normalized covariance $Q_\delta R^{-1}$ can make the tradeoff between the noise suppression and the tracking speed of the state estimation as well as the unknown input identification.

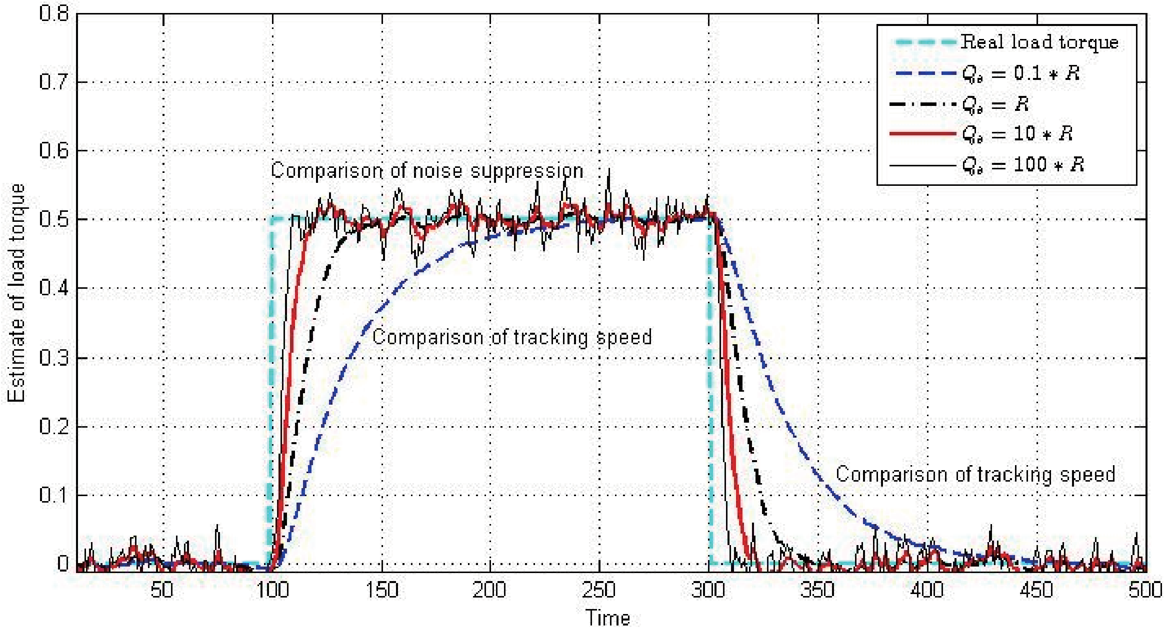

Figure 3. Unknown input estimate (load torque)

5. **Concluding Remarks.** This paper has proposed an alternative Kalman filtering based unknown input identification scheme in order to resolve some issues. The unknown input has been augmented as a state term and then estimated by the Kalman filtering. The unknown input noise covariance has been considered as a useful design parameter to adjust finely the filtering performance of the proposed scheme. Computer simulations for the DC motor system have shown that the unknown input noise covariance can make the tradeoff between the noise suppression and the tracking speed of the state estimation as well as the unknown input identification.

Although a guideline for the choice of the unknown input noise covariance has been described, this could still be somewhat nonsystematic. Therefore, a more systematic approach of determining unknown input noise covariance should be researched in future work.

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