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INTUITIONISTIC FUZZY *I*-CONVERGENT SEQUENCE SPACES DEFINED BY BOUNDED LINEAR OPERATOR

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Abstract. *A fuzzy theory was introduced by Zadeh, and a huge number of research papers have appeared on fuzzy theory and its applications as well as fuzzy analogues of the classical theories. Fuzzy set theory is a powerful hand set for modelling uncertainty and vagueness in various problems arising in field of science and engineering. Motivated by his work we introduce some certain Intuitionistic fuzzy I-convergent sequence spaces* $S^I_{(\mu,\nu)}(B)$ *and* $S^I_{0(\mu,\nu)}(B)$ *defined by bounded linear operator. The purpose of this paper is to construct the basic concepts of the so-called "Intuitionistic fuzzy I-convergent sequence spaces". We study the fuzzy topology and algebraic properties of these spaces. In the last we also make an attempt to prove some inclusion relations involving these spaces.*
Keywords: Ideal, Filter, *I*-convergence, Intuitionistic fuzzy normed spaces, Bounded linear operator

1. **Introduction.** In 1965, the theory of fuzzy sets has advanced in a variety of ways and in many disciplines. Applications of this theory can be found in artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, logic, management science, operations research, pattern recognition and robotics [1,4-6]. Mathematical developments have advanced to a very high standard and are still forthcoming to day. Fuzzy sets were introduced by Zadeh [26] in 1965 as follows: a fuzzy set A in a nonempty set X is a mapping from X to the unit interval $[0,1]$ and A(x) is interpreted as the degree of membership of x in A. Intuitionistic fuzzy sets can be viewed as a generalization of fuzzy sets that may better model imperfect information which is in any conscious decision making. Intuitionistic fuzzy sets take account of both the degrees of membership and of non membership subject to the condition that their sum does not exceed 1. Intuitionistic fuzzy sets (IFS) are applied in different areas. The IF-approach to artificial intelligence includes treatment of decision making and machine learning, neural networks and pattern recognition, expert systems database, machine reasoning, logic programming, etc. The concept of intuitionistic fuzzy normed space [21] and of intuitionistic fuzzy 2-normed space [18] are the latest developments in fuzzy topology.

The concept of statistical convergence at intitial stage was studied by Fast [3], the notion of ideal convergence (I-convergence) was introduced and studied by Kostyrko et al. [16] by using the idea of I of subsets of the set of natural numbers $\mathbb{N}$ and further

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studied in [20]. And also, it was studied by many others [8-11,14,22-25]. Quite recently, Das et al. [2] studied the notion of I and I^*-convergence of double sequences in $\mathbb{R}$.

Recently Khan and Yasmeen [12,13] studied the intuitionistic fuzzy Zweier I-convergent sequence spaces defined by modulus function and Orlicz function. In 2015, Khan and Shafiq [15] in their paper introduced some I-convergent sequence space of bounded linear operator defined by sequence of moduli. Our aim is to generalize this sequence space by introducing fuzzy set. In this paper we study about the convergence of sequences involving fuzzy set and ideal and defined by bounded linear operator. We study some topological and algebraic properties of these spaces such as linearity, solidity, and countability.

The remainder of this paper is organized as follows. Section 2 presents some definitions, preliminaries, lemmas which are used in this paper. Section 3 consists of main results in which we define some sequence spaces and study some fuzzy topology and algebraic properties of these spaces. Finally, brief conclusions are drawn.

2. Definitions and Preliminaries.

Definition 2.1. *Let X be a non empty set. Then, a family of sets $I \subseteq 2^X$ is said to be an* Ideal *in X if*

(i) $\phi \in I$;

(ii) I is additive, that is, $A, B \in I \Rightarrow A \cup B \in I$;

(iii) I is hereditary, that is, $A \in I, B \subseteq A \Rightarrow B \in I$.

An Ideal $I \subseteq 2^X$ is called non trivial if $I \neq 2^X$. A non trivial ideal $I \subseteq 2^X$ is called admissible if $\{\{x\} : x \in X\} \subseteq I$.

A non trivial ideal I is maximal if there cannot exist any non trivial ideal $J \neq I$ containing I as a subset.

Let $I \subset 2^{\mathbb{N}}$ be a non-trivial ideal in $\mathbb{N}$. Then a sequence $x = (x_k)$ is said to be I-convergent to a number L if, for every $\epsilon > 0$, the set $\{k \in \mathbb{N} :\mid x_k - L \mid\geq \epsilon\} \in I$.

Definition 2.2. *Let X be a non empty set. Then $\mathcal{F} \subset 2^X$ is said to be a filter on X if and only if $\phi \notin \mathcal{F}$, for $A, B \in \mathcal{F}$ we have $A \cap B \in \mathcal{F}$ and for each $A \in \mathcal{F}$ and $B \supset A$ implies $B \in \mathcal{F}$, i.e., to each Ideal I there is a Filter corresponding to I, $\mathcal{F}(I) = \{K \subset \mathbb{N} : K^c \in I\}$.*

Definition 2.3. *Let $I \subset 2^{\mathbb{N}}$ be a non-trivial ideal in $\mathbb{N}$. Then a sequence $x = (x_k)$ is said to be I-Cauchy if, for each $\epsilon > 0$, there exists a number $N = N(\epsilon)$ such that the set $\{k \in \mathbb{N} :\mid x_k - x_N \mid\geq \epsilon\} \in I$.*

Definition 2.4. [12] *The five-tuple $(X, \mu, \nu, *, \diamond)$ is said to be an intuitionistic fuzzy normed space (for short, IFNS) if X is a vector space, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and μ, ν are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions for every $x, y \in X$ and $s, t > 0$:*

(a) $\mu(x, t) + \nu(x, t) \leq 1$,

(b) $\mu(x, t) > 0$,

(c) $\mu(x, t) = 1$ if and only if $x = 0$,

(d) $\mu(\alpha x, t) = \mu\left(x, \frac{t}{\mid \alpha \mid}\right)$ for each $\alpha \neq 0$,

(e) $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$,

(f) $\mu(x, .) : (0, \infty) \to [0, 1]$ is continuous,

(g) $\lim_{t\to\infty} \mu(x, t) = 1$ and $\lim_{t\to 0} \mu(x, t) = 0$,

(h) $\nu(x, t) < 1$,

(i) $\nu(x, t) = 0$ if and only if $x = 0$,

(j) $\nu(\alpha x, t) = \nu\left(x, \frac{t}{\mid \alpha \mid}\right)$ for each $\alpha \neq 0$,

(k) $\nu(x, t) \diamond \nu(y, s) \geq \nu(x + y, t + s)$,

(l) $\nu(x,.) : (0,\infty) \to [0,1]$ *is continuous,*
(m) $\lim_{t\to\infty} \nu(x,t) = 0$ *and* $\lim_{t\to 0} \nu(x,t) = 1$.
In this case (μ,ν) *is called an intuitionistic fuzzy norm.*

Definition 2.5. *Let* $(X,\mu,\nu,*,\diamond)$ *be an IFNS. Then a sequence* $x = (x_k)$ *is said to be convergent to* $L \in X$ *with respect to the intuitionistic fuzzy norm* (μ,ν) *if, for every* $\epsilon > 0$ *and* $t > 0$*, there exists* $k_0 \in \mathbb{N}$ *such that* $\mu(x_k - L, t) > 1 - \epsilon$ *and* $\nu(x_k - L, t) < \epsilon$ *for all* $k \ge k_0$*. In this case we write* $(\mu,\nu) - \lim x = L$.

Definition 2.6. *Let* $(X,\mu,\nu,*,\diamond)$ *be an IFNS. Then a sequence* $x = (x_k)$ *is said to be a Cauchy sequence with respect to the intuitionistic fuzzy norm* (μ,ν) *if, for every* $\epsilon > 0$ *and* $t > 0$*, there exists* $k_0 \in \mathbb{N}$ *such that* $\mu(x_k - x_l, t) < \epsilon$ *and* $\nu(x_k - x_l, t) < \epsilon$ *for all* $k, l \ge k_0$.

Definition 2.7. *Let* K *be the subset of natural numbers* $\mathbb{N}$*. Then the asymptotic density of* K*, denoted by* $\delta(K)$*, is defined as*

$$\delta(K) = \lim_n \frac{1}{n} |\{k \le n : k \in K\}|,$$

where the vertical bars denote the cardinality of the enclosed set.

A number sequence $x = (x_k)$ *is said to be statistically convergent to a number* ℓ *if, for each* $\epsilon > 0$*, the set* $K(\epsilon) = \{k \le n : | x_k - \ell | > \epsilon\}$ *has asymptotic density zero, i.e.,*

$$\lim_n \frac{1}{n} |\{k \le n : | x_k - \ell | > \epsilon\}| = 0.$$

In this case we write $st - \lim x = \ell$.

Definition 2.8. *A number sequence* $x = (x_k)$ *is said to be statistically Cauchy sequence if, for every* $\epsilon > 0$*, there exists a number* $N = N(\epsilon)$ *such that*

$$\lim_n \frac{1}{n} |\{j \le n : | x_j - x_N | \ge \epsilon\}| = 0.$$

The concepts of statistical convergence and statistical Cauchy for double sequences in intuitionistic fuzzy normed spaces have been studied by Mursaleen et al. [19].

Definition 2.9. *Let* $I \subset 2^{\mathbb{N}}$ *be a non trivial ideal and* $(X,\mu,\nu,*,\diamond)$ *be an IFNS. A sequence* $x = (x_k)$ *of elements of* X *is said to be* I*-convergent to* $L \in X$ *with respect to the intuitionistic fuzzy norm* (μ,ν) *if for every* $\epsilon > 0$ *and* $t > 0$*, the set*

$$\{k \in \mathbb{N} : \mu(x_k - L, t) \le 1 - \epsilon \text{ or } \nu(x_k - L, t) \ge \epsilon\} \in I.$$

In this case L *is called the* I*-limit of the sequence* (x_k) *with respect to the intuitionistic fuzzy norm* (μ,ν) *and we write* $I_{(\mu,\nu)} - \lim x_k = L$.

Definition 2.10. *Let* X *and* Y *be two normed linear spaces [7] and* $B : \mathcal{D}(B) \to Y$ *be a linear operator, where* $\mathcal{D} \subset X$*. Then, the operator* B *is said to be bounded, if there exists a positive real* k *such that*

$$\|Bx\| \le k\|x\|, \text{ for all } x \in \mathcal{D}(B).$$

The set of all bounded linear operators $\mathcal{B}(X,Y)$ *[17] is a normed linear space normed by*

$$\|B\| = \sup_{x \in X, \|x\|=1} \|Bx\|$$

and $\mathcal{B}(X,Y)$ *is a Banach space if* Y *is a Banach space.*

3. **Main Results.** In this article we introduce the following sequence spaces:

$$S^I_{(\mu,\nu)}(B) = \{(x_k) \in \ell_\infty : \{k \in \mathbb{N} : \mu(B(x_k) - L, t) \leq 1 - \epsilon \text{ or } \nu(B(x_k) - L, t) \geq \epsilon\} \in I\};$$

$$S^I_{0(\mu,\nu)}(B) = \{(x_k) \in \ell_\infty : \{k \in \mathbb{N} : \mu(B(x_k), t) \leq 1 - \epsilon \text{ or } \nu(B(x_k), t) \geq \epsilon\} \in I\}.$$

We also define an open ball with center x and radius r with respect to t as follows:

$$\mathcal{B}_x(r,t)(B) = \{(y_k) \in \ell_\infty : \{k \in \mathbb{N} : \mu(B(x_k) - B(y_k), t) \leq 1 - \epsilon \text{ or } \nu(B(x_k) - B(y_k), t) \geq \epsilon\} \in I\}.$$

Theorem 3.1. *$S^I_{(\mu,\nu)}(B)$ and $S^I_{0(\mu,\nu)}(B)$ are linear spaces.*

Proof: We shall prove the result for $S^I_{(\mu,\nu)}(B)$. The proof for the other space will follow similarly. Let $x = (x_k)$, $y = (y_k) \in S^I_{(\mu,\nu)}(B)$ and α, β be scalars. Then for a given $\epsilon > 0$, we have

$$A_1 = \left\{k \in \mathbb{N} : \mu\left(B(x_k) - L_1, \frac{t}{2\mid\alpha\mid}\right) \leq 1 - \epsilon \text{ or } \nu\left(B(x_k) - L_1, \frac{t}{2\mid\alpha\mid}\right) \geq \epsilon\right\} \in I;$$

$$A_2 = \left\{k \in \mathbb{N} : \mu\left(B(y_k) - L_2, \frac{t}{2\mid\beta\mid}\right) \leq 1 - \epsilon \text{ or } \nu\left(B(y_k) - L_2, \frac{t}{2\mid\beta\mid}\right) \geq \epsilon\right\} \in I.$$

$$A_1^c = \left\{k \in \mathbb{N} : \mu\left(B(x_k) - L_1, \frac{t}{2\mid\alpha\mid}\right) > 1 - \epsilon \text{ or } \nu\left(B(x_k) - L_1, \frac{t}{2\mid\alpha\mid}\right) < \epsilon\right\} \in \mathcal{F}(I);$$

$$A_2^c = \left\{k \in \mathbb{N} : \mu\left(B(y_k) - L_2, \frac{t}{2\mid\beta\mid}\right) > 1 - \epsilon \text{ or } \nu\left(B(y_k) - L_2, \frac{t}{2\mid\beta\mid}\right) < \epsilon\right\} \in \mathcal{F}(I).$$

Define the set $A_3 = A_1 \cup A_2$, so that $A_3 \in I$. It follows that A_3^c is a non-empty set in $\mathcal{F}(I)$. We shall show that for each $(x_k), (y_k) \in S^I_{(\mu,\nu)}(B)$.

$$A_3^c \subset \{k \in \mathbb{N} : \mu((\alpha B(x_k) + \beta B(y_k)) - (\alpha L_1 + \beta L_2), t) > 1 - \epsilon \text{ or } \nu((\alpha B(x_k) + \beta B(y_k)) - (\alpha L_1 + \beta L_2), t) < \epsilon\}.$$

Let $m \in A_3^c$. In this case

$$\mu\left(B(x_m) - L_1, \frac{t}{2\mid\alpha\mid}\right) > 1 - \epsilon \text{ or } \nu\left(B(x_m) - L_1, \frac{t}{2\mid\alpha\mid}\right) < \epsilon$$

and

$$\mu\left(B(y_m) - L_2, \frac{t}{2\mid\beta\mid}\right) > 1 - \epsilon \text{ or } \nu\left(B(y_m) - L_2, \frac{t}{2\mid\beta\mid}\right) < \epsilon.$$

We have

$$\begin{aligned}
&\mu\Big((\alpha B(x_m) + \beta B(y_m)) - (\alpha L_1 + \beta L_2), t\Big) \\
&\geq \mu\left(\alpha B(x_m) - \alpha L_1, \frac{t}{2}\right) * \mu\left(\beta B(x_m) - \beta L_2, \frac{t}{2}\right) \\
&= \mu\left(B(x_m) - L_1, \frac{t}{2\mid\alpha\mid}\right) * \mu\left(B(x_m) - L_2, \frac{t}{2\mid\beta\mid}\right) \\
&> (1 - \epsilon) * (1 - \epsilon) = 1 - \epsilon.
\end{aligned}$$

and

$$\begin{aligned}
&\nu\Big((\alpha B(x_m) + \beta B(y_m)) - (\alpha L_1 + \beta L_2), t\Big) \\
&\leq \nu\left(\alpha B(x_m) - \alpha L_1, \frac{t}{2}\right) \diamond \nu\left(\beta B(x_m) - \beta L_2, \frac{t}{2}\right) \\
&= \mu\left(B(x_m) - L_1, \frac{t}{2\mid\alpha\mid}\right) \diamond \mu\left(B(x_m) - L_2, \frac{t}{2\mid\beta\mid}\right) \\
&< \epsilon \diamond \epsilon = \epsilon.
\end{aligned}$$

This implies that

$$A_3^c \subset \{k \in \mathbb{N} : \mu((\alpha B(x_k) + \beta B(y_k)) - (\alpha L_1 + \beta L_2), t) > 1 - \epsilon \\ \text{or } \nu((\alpha B(x_k) + \beta B(y_k)) - (\alpha L_1 + \beta L_2), t) < \epsilon\}.$$

Hence, $S^I_{(\mu,\nu)}(B)$ is a linear space.

Theorem 3.2. *Every open ball $\mathcal{B}_x(r,t)(B)$ is an open set in $S^I_{(\mu,\nu)}(B)$.*

Proof: Let $\mathcal{B}_x(r,t)(B)$ be an open ball with centre x and radius r with respect to t. That is

$$\mathcal{B}_x(r,t)(B) = \{y = (y_k) \in \ell_\infty : \{k \in \mathbb{N} : \mu(B(x_k) - B(y_k), t) \le 1 - r \\ \text{or } \nu(B(x_k) - B(y_k), t) \ge r\} \in I\}.$$

Let $y \in \mathcal{B}^c_x(r,t)(B)$. Then $\mu(B(x_k) - B(y_k), t) > 1 - r$ and $\nu(B(x_k) - B(y_k), t) < r$. Since $\mu(B(x_k) - B(y_k), t) > 1 - r$, there exists $t_0 \in (0, t)$ such that $\mu(B(x_k) - B(y_k), t_0) > 1 - r$ and $\nu(B(x_k) - B(y_k), t_0) < r$. Putting $r_0 = \mu(B(x_k) - B(y_k), t_0)$, we have $r_0 > 1 - r$, and there exists $s \in (0, 1)$ such that $r_0 > 1 - s > 1 - r$.

For $r_0 > 1 - s$, we have $r_1, r_2 \in (0, 1)$ such that $r_0 * r_1 > 1 - s$ and $(1 - r_0) \diamond (1 - r_0) \le s$. Putting $r_3 = \max\{r_1, r_2\}$.

Consider the ball $\mathcal{B}^c_y(1 - r_3, t - t_0)(B)$. We prove that

$$\mathcal{B}^c_y(1 - r_3, t - t_0)(B) \subset \mathcal{B}^c_x(r,t)(B).$$

Let $z = (z_k) \in \mathcal{B}^c_y(1 - r_3, t - t_0)(B)$, then $\mu(B(y_k) - B(z_k), t - t_0) > r_3$ and $\nu(B(y_k) - B(z_k), t - t_0) < 1 - r_3$.

Therefore,

$$\mu(B(x_k) - B(z_k), t) \ge \mu(B(x_k) - B(y_k), t_0) * \mu(B(y_k) - B(z_k), t - t_0) \\ \ge (r_0 * r_3) \ge (r_0 * r_1) \ge (1 - s) \ge (1 - r)$$

and

$$\nu(B(x_k) - B(z_k), t) \le \nu(B(x_k) - B(y_k), t_0) \diamond \nu(B(y_k) - B(z_k), t - t_0) \\ \le (1 - r_0) \diamond (1 - r_3) \le (1 - r_0) \diamond (1 - r_2) \le s \le r.$$

Thus $z \in \mathcal{B}^c_x(r,t)(B)$ and hence

$$\mathcal{B}^c_y(1 - r_3, t - t_0)(B) \subset \mathcal{B}^c_x(r,t)(B).$$

Remark 3.1. *$S^I_{(\mu,\nu)}(B)$ is an IFNS.*

Define

$$\tau^I_{(\mu,\nu)}(B) = \{A \subset S^I_{(\mu,\nu)}(B) : \textit{ for each } x \in A \textit{ there exists } t > 0 \\ \textit{and } r \in (0,1) \textit{ such that } B_x(r,t)(B) \subset A\}.$$

Then $\tau^I_{(\mu,\nu)}(B)$ is a topology on $S^I_{(\mu,\nu)}(B)$.

Theorem 3.3. *The topology $\tau^I_{(\mu,\nu)}(B)$ on $S^I_{0(\mu,\nu)}(B)$ is first countable.*

Proof: $\left\{\mathcal{B}_x\left(\frac{1}{n}, \frac{1}{n}\right)(B) : n = 1, 2, 3, \ldots\right\}$ is a local base at x, and the topology $\tau^I_{(\mu,\nu)}(B)$ on $S^I_{0(\mu,\nu)}(B)$ is first countable.

Theorem 3.4. *$S^I_{(\mu,\nu)}(B)$ and $S^I_{0(\mu,\nu)}(B)$ are Housedorff spaces.*

Proof: We prove the result for $S^I_{(\mu,\nu)}(B)$. Similarly the proof follows for $S^I_{0(\mu,\nu)}(B)$. Let $x, y \in S^I_{(\mu,\nu)}(B)$ such that $x \neq y$. Then $0 < \mu(B(x) - B(y), t) < 1$ and $0 < \nu(B(x) - B(y), t) < 1$. Putting $r_1 = \mu(B(x) - B(y), t)$, $r_2 = \nu(B(x) - B(y), t)$ and $r = \max\{r_1, 1 - r_2\}$.

For each $r_0 \in (r, 1)$ there exist r_3 and r_4 such that $r_3 * r_4 \geq r_0$ and $(1-r_3) \diamond (1-r_4) \leq (1-r_0)$. Put $r_5 = \max\{r_3, r_4\}$ and consider the open balls $\mathcal{B}_x\left(1-r_5, \frac{t}{2}\right)$ and $\mathcal{B}_y\left(1-r_5, \frac{t}{2}\right)$. Then clearly $\mathcal{B}_x^c\left(1-r_5, \frac{t}{2}\right) \cap \mathcal{B}_y^c\left(1-r_5, \frac{t}{2}\right) = \phi$. If there exists $z \in \mathcal{B}_x^c\left(1-r_5, \frac{t}{2}\right) \cap \mathcal{B}_y^c\left(1-r_5, \frac{t}{2}\right)$, then

$$\begin{aligned} r_1 = \mu(B(x) - B(y), t) &\geq \mu\left(B(x) - B(z), \frac{t}{2}\right) * \mu\left(B(z) - B(y), \frac{t}{2}\right) \\ &\geq r_5 * r_5 \geq r_3 * r_3 \geq r_0 > r_1 \end{aligned}$$

and

$$\begin{aligned} r_2 = \nu(B(x) - B(y), t) &\leq \nu\left(B(x) - B(z), \frac{t}{2}\right) \diamond \nu\left(B(z) - B(y), \frac{t}{2}\right) \\ &\leq (1-r_5) \diamond (1-r_5) \leq (1-r_4) \diamond (1-r_4) \leq (1-r_0) < r_2 \end{aligned}$$

which is a contradiction. Hence $S^I_{(\mu,\nu)}(B)$ is Housedorff.

Theorem 3.5. *$S^I_{(\mu,\nu)}(B)$ is an IFNS and $\tau^I_{(\mu,\nu)}(B)$ is a topology on $S^I_{(\mu,\nu)}(B)$. Then a sequence $(x_k) \in S^I_{(\mu,\nu)}(B)$, $x_k \to x$ if and only if $\mu(B(x_k) - B(x), t) \to 1$ and $\nu(B(x_k) - B(x), t) \to 0$ as $k \to \infty$.*

Proof: Fix $t_0 > 0$. Suppose $x_k \to x$. Then for $r \in (0, 1)$, there exists $n_0 \in \mathbb{N}$ such that $(x_k) \in \mathcal{B}_x(r, t)(B)$ for all $k \geq n_0$,

$$\mathcal{B}_x(r, t)(B) = \{k \in \mathbb{N} : \mu(B(x_k) - B(x), t) \leq 1 - r \text{ or } \nu(B(x_k) - B(x), t) \geq r\} \in I,$$

such that $\mathcal{B}_x^c(r, t)(B) \in \mathcal{F}(I)$. Then $1-\mu(B(x_k)-B(x), t) < r$ and $\nu(B(x_k)-B(x), t) < r$. Hence $\mu(B(x_k) - B(x), t) \to 1$ and $\nu(B(x_k) - B(x), t) \to 0$ as $k \to \infty$.

Conversely, if for each $t > 0$, $\mu(B(x_k) - B(x), t) \to 1$ and $\nu(B(x_k) - B(x), t) \to 0$ as $k \to \infty$, then for $r \in (0, 1)$, there exists $n_0 \in \mathbb{N}$ such that $1 - \mu(B(x_k) - B(x), t) < r$ and $\nu(B(x_k) - B(x), t) < r$, for all $k \geq n_0$. It follows that $\mu(B(x_k) - B(x), t) > 1 - r$ and $\nu(B(x_k) - B(x), t) < r$ for all $k \geq n_0$. Thus $(x_k) \in \mathcal{B}_x^c(r, t)(B)$ for all $k \geq n_0$ and hence $x_k \to x$.

Theorem 3.6. *A sequence $x = (x_k) \in S^I_{(\mu,\nu)}(B)$ is I-convergent if and only if for every $\epsilon > 0$ and $t > 0$ there exists a number $N = N(x, \epsilon, t)$ such that*

$$\left\{N \in \mathbb{N} : \mu\left(B(x_N) - L, \frac{t}{2}\right) > 1 - \epsilon \text{ or } \nu\left(B(x_N) - L, \frac{t}{2}\right) < \epsilon\right\} \in \mathcal{F}(I).$$

Proof: Suppose that $I_{(\mu,\nu)} - \lim x = L$ and let $\epsilon > 0$ and $t > 0$. For a given $\epsilon > 0$, choose $s > 0$ such that $(1-\epsilon) * (1-\epsilon) > 1 - s$ and $\epsilon \diamond \epsilon < s$. Then for each $x \in S^I_{(\mu,\nu)}(B)$,

$$P = \left\{k \in \mathbb{N} : \mu\left(B(x_k) - L, \frac{t}{2}\right) \leq 1 - \epsilon \text{ or } \nu\left(B(x_k) - L, \frac{t}{2}\right) \geq \epsilon\right\} \in I,$$

which implies that

$$P^c = \left\{k \in \mathbb{N} : \mu\left(B(x_k) - L, \frac{t}{2}\right) > 1 - \epsilon \text{ or } \nu\left(B(x_k) - L, \frac{t}{2}\right) < \epsilon\right\} \in \mathcal{F}(I).$$

Conversely let us choose $N \in P$. Then

$$\mu\left(B(x_N) - L, \frac{t}{2}\right) \leq 1 - \epsilon \text{ or } \nu\left(B(x_N) - L, \frac{t}{2}\right) \geq \epsilon.$$

Now we want to show that there exists a number $N = N(x, \epsilon, t)$ such that

$$\{k \in \mathbb{N} : \mu(B(x_k) - B(x_N), t) \leq 1 - s \text{ or } \nu(B(x_k) - B(x_N), t) \geq s\} \in I.$$

For this, define for each $x \in S^I_{(\mu,\nu)}(B)$

$$Q = \{k \in \mathbb{N} : \mu(B(x_k) - B(x_N), t) \leq 1 - s \text{ or } \nu(B(x_k) - B(x_N), t) \geq s\} \in I.$$

Now we have to show that $Q \subset P$. Suppose that $Q \subsetneq P$. Then there exist $n \in Q$ and $n \notin P$.

Therefore, we have

$$\mu(B(x_n) - B(x_N), t) \leq 1 - s \text{ or } \mu\left(B(x_n) - L, \frac{t}{2}\right) > 1 - \epsilon.$$

In particular

$$\mu\left(B(x_N) - L, \frac{t}{2}\right) > 1 - \epsilon.$$

Therefore, we have

$$\begin{aligned} 1 - s \geq \mu(B(x_n) - B(x_N), t) &\geq \mu\left(B(x_n) - L, \frac{t}{2}\right) * \mu\left(B(x_N) - L, \frac{t}{2}\right) \\ &\geq (1 - \epsilon) * (1 - \epsilon) > 1 - s, \end{aligned}$$

which is not possible. On the other hand

$$\nu(B(x_n) - B(x_N), t) \geq s \text{ or } \nu\left(B(x_n) - L, \frac{t}{2}\right) < \epsilon.$$

In particular

$$\nu\left(B(x_N) - L, \frac{t}{2}\right) < \epsilon.$$

Therefore, we have

$$s \leq \nu(B(x_n) - B(x_N), t) \leq \nu\left(B(x_n) - L, \frac{t}{2}\right) \diamond \nu\left(B(x_N) - L, \frac{t}{2}\right) \leq \epsilon \diamond \epsilon < s,$$

which is not possible.

Hence $Q \subset P$. $P \in I$ implies $Q \in I$.

4. **Conclusions.** In this paper we have studied the concept of Intuitionistic fuzzy I-convergent sequence spaces by using bounded linear operator. Quite recently Khan and Yasmeen studied about "Intuitionistic Fuzzy Zweier I-convergent Sequence Spaces defined by Orlicz function" from these space we get an idea and we define some different spaces such as $S^I_{(\mu,\nu)}(B)$ and $S^I_{0(\mu,\nu)}(B)$ for sequence spaces defined by bounded linear operator. This study provides a new tool to deal with ideal convergence and it is very useful in many branches of science and engineering. Future work includes a detailed evaluation of these sequence spaces using different operators and functions such as compact operator, Orlicz function, and modulus function.

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