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THE OPTIMAL PURCHASING TIME OF AGRICULTURAL PRODUCTS UNDER THE GOVERNMENT'S PRICE INTERVENTION

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Abstract. *This paper provides a mathematical model in which the decision maker should decide whether or not to buy a sequentially appearing agricultural product under the uncertainty of government's intervention over the too high or low price. In the model, if the price of the agricultural product is too high (or low), the government might interfere with the market to lower (higher) the price to a certain level, so the market would be stabilized to some extent. Since the model is formulated from the customer's point of view, the probability of the intervention is assumed to be uncertain. Based on the mathematical model, some numerical experiments are conducted to investigate the optimal decision (purchasing) rules to minimize the expected total payments over the total planning horizon. In addition, the verification of the model provided in the paper is given as well based on the real data for the last 20 years.*

Keywords: Optimal purchasing time, Agricultural products, Optimal stopping strategy

1. **Introduction.** In every Kimchi-making season in Korea, the abruptly changing prices of agricultural products become hot news to not only the Korean people but the government itself. The price of agricultural products depends closely on the climate changes or the production amounts. For instance, in 2016 the price of Chinese cabbage has skyrocketed with over 200% comparing to the price at the same time of the last year due to the heat wave and the heavy rain, while in 2014 the price of agricultural products such as onion and garlic has plunged due to the large amounts of excess production. In case of excessive price changes, the government intervenes in the agricultural market and adjusts the price for the sake of market stability, which makes the current price higher (or lower). Although the government's intervention in market has a significant impact on the price as mentioned above, the critical price on which the government decides whether or not to intervene in market has not known. This kind of uncertainty makes the customer's decision harder on when to make Kimchi at low prices.

This paper formulates mathematically the decision problem on whether or not to buy sequentially changing agricultural products under the uncertainty of government's intervention until the predetermined deadline from the customer's point of view. The objective is to find out the optimal purchasing strategy in order to minimize the expected total cost over the planning horizon.

The related research can be largely categorized as the following two phases: statistical [1-4] or probabilistic approach [5-9]. The statistical researches have been done to examine/estimate the price fluctuation and its related governmental policy with the existing data using the diverse statistical methodology such as time series analysis, ARIMA, and trend analysis. Since our research mainly focuses on the decision under the price uncertainty of the agricultural product, the latter literature can be said to have closer relation.

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Hu et al. [5] proposed the optimal purchase strategy with respect to the optimal amount under stochastic price from the retailer's point of view. Han et al. [6] formulated and analyzed the nonlinear supply chain profit model to derive the optimal order price and quantity maximizing retailer's profit. Zhang and Xu [7] studied the optimal stopping time to invest under the situation with high volatility in order to maximize the decision maker's profit. From the point of view of optimal stopping strategy, Son provided the optimal asset buying time under the budget constraints [8] and optimal admission strategy with respect to state-dependent arrivals [9]. With the similar approach with Son's [8,9] we offer the optimal stopping strategy minimizing the agricultural products purchasing price under a newly introduced observation value, government's intervention, which has not been dealt with in the previous researches as far as authors know.

The rest of the paper is organized as follows. In Section 2, we formulate a mathematical model with respect to the problem dealt with in the paper. In Section 3, we describe how to design the experiment and explain the optimal purchasing strategy. In Section 4, we present the results from the empirical analysis to verify our mathematical model. Conclusions and future studies are given in Section 5.

2. **Model Formulation.** Consider a situation that the decision maker has to make a decision on purchasing agricultural products changing sequentially, the prices of which are denoted by $x_1, x_2, \ldots$, $i.i.d$ random variables from a known distribution function $f(x)$ with expectation μ, until a certain point of time called deadline. By t (≥ 0) let us denote the remaining discrete-time to the deadline. At each point of time the decision maker decides whether or not to buy the agricultural product seeing its changing price. However, the price of agricultural product is influenced by the government's intervention in the market. In this paper, we assume two types of government's intervention: If the price of agricultural product goes up over a predetermined level called upper bound b, then the government intervenes in the market and lowers the price to β $(< b)$, while if the price goes down below the lower bound a, the government then makes the price to be higher than α $(> a)$. The possibility of government's intervention is not known, so we assume it to be p for the both two cases above. Figure 1 shows the distribution function of an agricultural product, the lower (upper) critical price based on which the governmental intervention may happen with a probability p, and the adjusted price by the government's intervention.

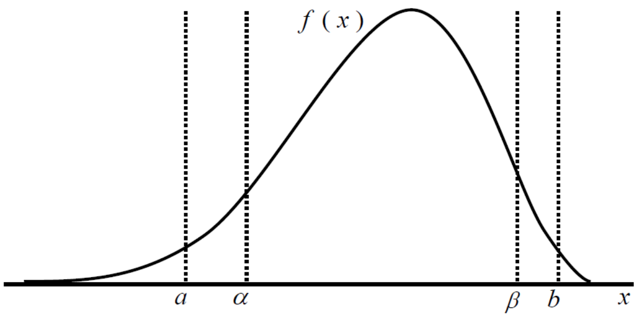


FIGURE 1. The distribution function, the upper (lower) bound of price, and the adjusted price by the governments intervention

Now, by V_t we define the total expected minimum payment for the remaining time t. Then the optimal equation can be expressed as follows.

$$V_t = \int_a^b \min\{x, V_{t-1}\} f(x)dx + \int_0^a \min\{p\alpha + (1-p)x, V_{t-1}\} f(x)dx + \int_b^\infty \min\{p\beta + (1-p)x, V_{t-1}\} f(x)dx, \quad t \geq 2, \tag{1}$$

and

$$V_1 = \int_a^b x f(x) dx + \int_0^a (p\alpha + (1-p)x) f(x) dx + \int_b^\infty (p\beta + (1-p)x) f(x) dx \qquad (2)$$

In Equation (1) the decisions at time t would vary according to the following two cases. 1) When the price is on the range of $a < x < b$ where the intervention never happens, the decision maker might buy the product by paying the price x, or decline to buy it with the expectation that the better one would appear next point of time. So the expected minimum payment will be the minimum of x and V_{t-1}. 2) When the price is higher the upper (lower) bound $b(a)$, if the government intervenes and controls the price with the probability p, then the decision maker should pay the price $\beta(\alpha)$, otherwise, he should pay x. Therefore, the expected payment will be the minimum of $p\beta(\alpha) + (1-p)x$ and V_{t-1}. In Equation (2) with $t = 1$, at the deadline $t = 0$ the decision maker must buy the product, however, the price might be extremely high, so we assume $V_0 = \infty$ without loss of generalization.

Equation (1) can be rearranged as follows.

$$\begin{aligned} V_t = & \int_a^b \min\{x - V_{t-1}, 0\} f(x) dx + \int_0^a \min\{p\alpha + (1-p)x - V_{t-1}, 0\} f(x) dx \\ & + \int_b^\infty \min\{p\beta + (1-p)x - V_{t-1}, 0\} f(x) dx + V_{t-1}, \quad t \geq 2. \end{aligned} \qquad (3)$$

The objective of this paper is to clarify the optimal decision rules to minimize the total expected payment to buy an agricultural product over the whole horizontal periods.

3. **Numerical Experiments.** In this section we investigate the properties of the threshold, V_t, on which the decision is made. In other words, we examine the behavior of V_t as to the time t (≥ 0) by changing the related parameters, $b(a)$, $\beta(\alpha)$, and p. Note that the optima decision for a given time t (≥ 0) can be described as follows: If the appearing price of the agricultural product is greater than V_t, the optimal decision would be not to buy the product and wait for the better one to appear in the next point in time; otherwise, it would be optimal to buy that one immediately. Throughout this section the price of the agricultural products is assumed to follow the uniform distribution i.e., $x \sim Uni\,[1, 2]$.

3.1. **Behavior of V_t related to the lower bound of the price.** In order to investigate the behavior of the decision threshold as to the *lower bound* of the agricultural product, we fix the upper bound and the adjusted upper price as $b = 1.9$ and $\beta = 1.8$, respectively. The both figures in Figure 2 show that the decision threshold V_t decreases over time t, which implies that the more time remains, the more aggressive strategy should be taken by lowering the threshold. In other words, the decision maker would better try to take a cheaper one when he/she has sufficiently enough time until the deadline; however, he/she might have to choose a little higher one as the deadline comes near. In addition, we can see that the decision threshold V_t is increasing in p as well, although the degree of increment is different between the two cases of $a = 1.2$, $\alpha = 1.3$ (left) and $a = 1.3$, $\alpha = 1.4$ (right). This means that if the product price is already sufficiently low and the possibility of government's intervention gets higher, which might make the price go up, hence the decision maker should purchase the product at a higher price that may be influenced by the government's intervention; so it is natural that the decision threshold should increase more severely in case that the lower bound is set higher (like the right side of Figure 2) than the other (left side) as the probability of government's interference gets high.

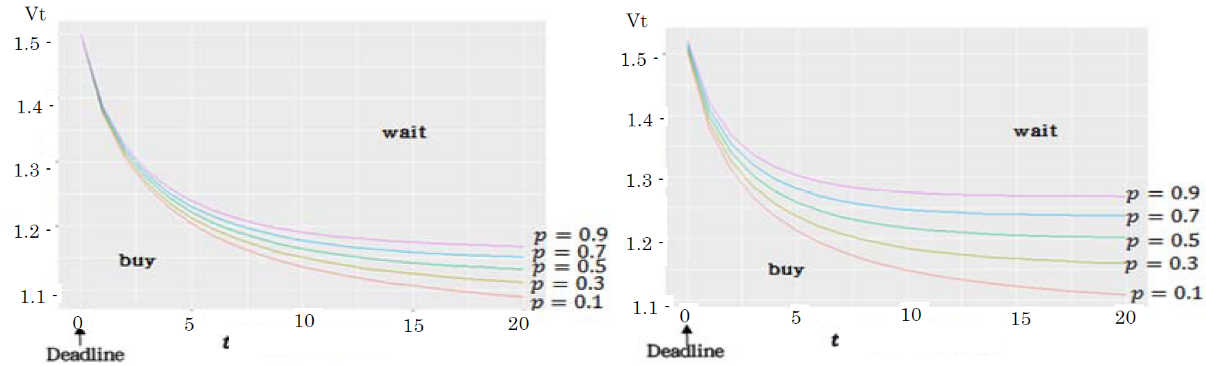


FIGURE 2. The behavior of V_t with respect to the government's intervention probability for both cases of $a = 1.2$, $\alpha = 1.3$ (left) and $a = 1.3$, $\alpha = 1.4$ (right)

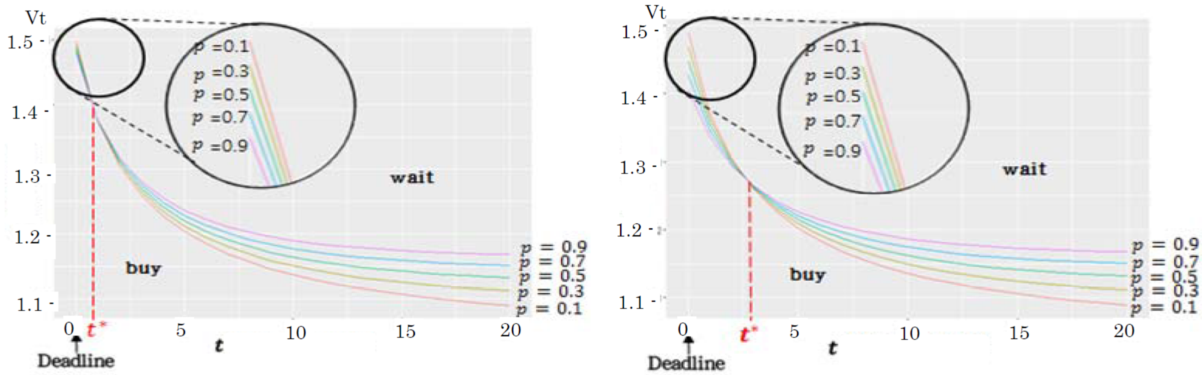


FIGURE 3. The behavior of V_t with respect to the government's intervention probability for both cases of $b = 1.8$, $\beta = 1.7$ (left) and $b = 1.7$, $\beta = 1.6$ (right). The big circle represents magnification of each figure.

3.2. **Behavior of V_t related to the upper bound of the price.** In order to investigate the behavior of the decision threshold as to the *upper bound* of the agricultural product, we fix the lower bound and the adjusted lower price as $a = 1.1$ and $\alpha = 1.2$, respectively.

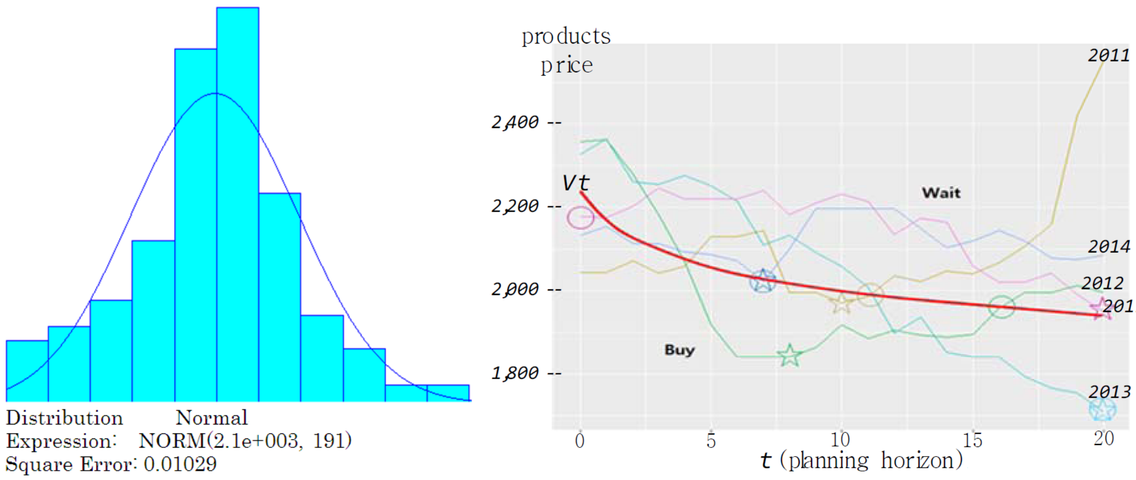
At a glance, Figure 3 is similar to Figure 2 in a sense that V_t decreases over time t in both figures. However, there exists in Figure 3 such t^* that V_t decreases in p for $t < t^*$, but increases for $t^* \leq t$, which is the mostly different point from Figure 2. This means the followings. When there remains sufficiently enough time until the deadline, it would be strategically good to lower the decision threshold to get a cheaper one. In this case the increasing probability p implies that the chances to get the lower price of product gets slim so the product price goes up, and hence the threshold should be set higher as a result. However, if the possibility of the government's intervention is high when the deadline is near, the decision maker might buy the product at a relatively lower price, so he/she would be better to lower the decision threshold, implying that the threshold decreases as the intervening probability increases. This trend might be more prominent as the upper bound decreases (compare the left side of figure to the right one).

4. **Empirical Analysis.** In this section, we verify the mathematical model formulated in this paper by the empirical experiment with the real data from KAMIS (Korea Agricultural Marketing Information Service). We collected Chinese cabbage price data of the last 20 years (1996 $\sim$ 2015) on the Kimchi-making season that is conducted usually from Nov. 15th to Dec. 5th in every year. The total number of data is 420, which are corrected in view of the inflation rate (see Table 1).

TABLE 1. Adjusted Chinese cabbage prices in Kimchi-making season (Nov. 15th ~ Dec. 5th) from 1996 to 2015

Year	Products price for 20 days from Nov. 15th to Dec. 5th																				
	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1996	1,664	1,650	1,739	1,739	1,747	1,700	1,700	1,700	1,708	1,682	1,745	1,761	1,777	1,797	1,971	2,402	2,656	2,656	2,656	2,679	2,778
1997	2,026	2,072	2,034	2,016	2,034	2,048	2,026	2,034	1,999	1,999	1,987	1,987	1,987	1,987	2,118	2,176	2,212	2,212	2,230	2,260	2,312
1998	2,193	2,142	2,163	2,172	2,163	2,163	2,182	2,172	2,212	2,222	2,241	2,260	2,260	2,260	2,260	2,222	2,201	2,193	2,182	2,182	2,153
1999	2,404	2,350	2,271	2,219	2,201	2,165	2,148	2,130	2,130	2,130	2,025	2,025	2,025	2,077	2,077	2,130	2,148	2,148	2,165	2,165	2,165
2000	2,446	2,322	2,239	2,239	2,198	2,177	2,156	2,135	2,135	2,135	2,094	2,094	2,094	2,094	2,094	2,094	2,094	2,094	2,073	2,073	2,073
2001	2,115	2,053	2,053	2,077	2,077	2,040	2,064	1,944	1,944	1,896	1,846	1,846	1,846	1,896	1,896	1,920	2,139	2,139	2,357	2,357	2,357
2002	1,691	1,691	1,684	1,714	1,629	1,678	1,704	1,704	1,840	1,946	1,969	2,017	2,017	2,030	1,981	2,017	2,058	2,105	2,209	2,244	2,399
2003	2,620	2,537	2,538	2,418	2,392	2,420	2,420	2,386	2,334	2,334	2,276	2,286	2,286	2,231	2,169	2,106	2,025	2,029	2,026	1,996	1,961
2004	2,572	2,472	2,452	2,422	2,354	2,320	2,260	2,240	2,188	2,136	2,057	2,007	2,009	1,981	2,017	2,067	2,088	2,067	2,088	2,035	2,025
2005	2,262	2,246	2,098	1,977	1,882	1,789	1,779	1,785	1,762	1,796	1,744	1,724	1,733	1,820	1,916	2,010	2,153	2,215	2,215	2,288	2,295
2006	1,937	1,848	1,871	1,847	1,779	1,758	1,766	1,898	1,885	1,964	2,007	2,076	2,097	2,111	2,103	2,179	2,319	2,396	2,412	2,384	2,384
2007	2,512	2,482	2,411	2,364	2,373	2,196	2,132	2,199	2,179	2,160	2,181	2,202	2,253	2,187	2,193	2,155	2,155	2,128	2,098	2,060	2,080
2008	1,962	2,029	2,009	1,914	2,014	2,095	2,103	2,066	2,019	2,203	2,250	2,250	2,196	2,174	2,114	2,141	2,187	2,187	2,230	2,208	2,225
2009	1,965	2,049	2,049	1,999	1,972	1,976	2,257	2,361	2,214	2,214	2,110	2,076	2,076	2,126	2,176	2,160	2,160	2,160	2,160	2,160	2,160
2010	1,983	2,046	2,121	2,276	2,254	2,358	2,340	2,291	2,151	2,186	2,179	2,113	2,032	2,061	2,142	2,134	2,116	2,116	2,210	2,192	2,164
2011	2,552	2,420	2,159	2,106	2,067	2,040	2,047	2,023	2,036	1,986	1,971	1,995	1,995	2,144	2,130	2,130	2,058	2,042	2,072	2,043	2,043
2012	1,995	2,013	1,996	1,996	1,957	1,895	1,889	1,893	1,905	1,885	1,918	1,864	1,840	1,840	1,840	1,918	2,070	2,182	2,280	2,363	2,358
2013	1,708	1,755	1,767	1,795	1,840	1,840	1,852	1,936	1,899	2,008	2,058	2,092	2,133	2,109	2,215	2,251	2,277	2,255	2,262	2,364	2,327
2014	2,085	2,075	2,079	2,120	2,144	2,119	2,103	2,149	2,198	2,198	2,198	2,198	2,099	2,022	2,071	2,087	2,094	2,113	2,113	2,155	2,133
2015	1,949	1,991	2,042	2,020	2,020	2,058	2,164	2,174	2,134	2,213	2,232	2,211	2,183	2,240	2,221	2,221	2,221	2,247	2,202	2,175	2,178

*source: KAMIS (Korea Agricultural Marketing Information Service), unit: won



(a) (b)

FIGURE 4. (a) The estimated distribution function from the real price data of the last 20 years from 2015, and (b) the plots of products price data during the Kimchi-making season for the last 5 yers. The star denotes the optimal purchasing price based on the optimal decision policy, while the circle stands for the minimum price among the data of each year.

The estimated distribution of the price is shown in Figure 4(a) where the normal distribution with mean 2,100 won is mostly well-fitted. And Figure 4(b) shows the five year's price data (from 2011 to 2015) as well as the decision threshold V_t that is obtained based on the following assumption; the lower bound and the upper bound are, respectively, 1,800 won and 2,400 won, and that the government's intervention probability is 0.2. In Figure 4(b) we can see that V_t decreases as the planning time increases, which is the quite same result as the numerical experiment in the previous section. Table 1 represents the optimal purchasing prices and the minimum price of the data: the former is represented by a circle, and the latter are by star in Figure 4(b). Note that the optimal prices are

Table 2. Summary of the empirical analysis

Year	2011	2012	2013	2014	2015	Average
The optimal purchasing price (y)	1,986	1,957	1,708	2,022	2,178	1,970
The minimum price of the data (z)	1,971	1,840	1,708	2,022	1,949	1,898
Ratio: $(y - z) * 100/z$	0.76%	6.35%	0%	0%	11.7%	3.79%

the first price going down the threshold V_t starting from $t = 20$. Table 2 shows how the mathematical model finds out the best price: Although the biggest difference is seen in 2015, the optimal price is 11.7% higher than the minimum price. The average error between the two prices is 3.79%, i.e., it shows that the mathematical model has 96.2% of accuracy.

5. **Conclusions.** In this paper we presented a mathematical model about the optimal purchasing strategy of agricultural product under the government's intervention from the customer's prospective. Based on this model, we investigated the behavior of the optimal decision strategies over time with respect to the related parameters, especially focusing on the governmental intervention factors. In addition, we showed the validation of our new model through empirical analysis with the real data. We expect that this research can be utilized as a basic purchasing model providing the purchasing strategy in wholesale and retail trade.

In order to make our model more realistic one, we can consider the following topics. 1) The decision maker can buy the products in several times instead of buying at once as in our model. 2) We can consider a model with infinite and continuous planning horizon. 3) The price of the products may depend on the time.

REFERENCES

[1] Y.-S. Lee, J.-J. Kim and S.-J. Noh, *Analysis of Vegetable Price Fluctuations*, Korea Rural Economic Institute, 2012.

[2] B.-S. Yoon and S.-R. Yang, A study on the day-of-the-week, monthly and intra-month effects in spicy vegetable prices, *The Korean Agricultural Economics Association*, vol.45, no.2, pp.187-210, 2004.

[3] S.-H. Yoon, The analysis on statistics of cash price fluctuation of selected agricultural products, *Korea Institute for Industry and Economics*, vol.12, pp.313-326, 1994.

[4] J.-Y. Lim, *Analysis for Forecasting of Mid-Term Agricultural Prices Using Time Series Model*, Chung-Ang University, 2015.

[5] X. Hu, C. L. Munson and S. B. Fotopoulos, Purchasing decision under stochastic prices, *Annals of Operations Research*, pp.201-287, 2012.

[6] S.-G. Han, Y. Xu and J. L. Hu, The stock and the purchase strategy on the nonlinear diversification of price to time with retailer dominating, *International Asia Conference on Industrial Engineering and Management Innovation (IEMI2012) Proceedings*, pp.1369-1387, 2012.

[7] R. Zhang and S. Xu, Optimal stopping time with stochastic volatility, *Economic Modeling*, vol.41, pp.319-328, 2014.

[8] J.-D. Son, Asset buying problem with consideration of the budget constraints and loan, *IE Interfaces*, vol.24, no.4, pp.295-303, 2011.

[9] J.-D. Son, Optimal admission control in a customized production system with state-dependent arrival rates, *Journal of the Korea Management Engineers Society*, vol.16, no.1, pp.113-124, 2011.