ICIC Express Letters
Volume **13**, Number **10**, October **2019**

ICIC International ©2019 ISSN 1881-803X
pp. 875–881

BIPOLAR ANTI FUZZY IDEALS OF *K*-ALGEBRAS

Ria Anggraeni, Agus Widodo and Noor Hidayat

Department of Mathematics
University of Brawijaya
Jl. Veteran, Lowokwaru, Malang 65145, East Java, Indonesia
ria.anggraeni22@gmail.com; noorh@ub.ac.id

Received March 2019; accepted June 2019

Abstract. *A mapping which has real number interval* $[-1, 1]$ *on the codomain is called a bipolar fuzzy set. Bipolar fuzzy can be applied in some algebraic structures, for example, K-algebra. An algebraic structure which is built from group G and fulfilling several axioms is called K-algebra. Not only bipolar fuzzy, but also bipolar anti fuzzy can be applied to K-algebra. In this paper, we investigated bipolar anti fuzzy ideals of K-algebras and discussed related properties. We characterize bipolar anti fuzzy ideal of K-algebras by means of positive anti* β*-cut and negative anti* α*-cut.*

Keywords: Bipolar fuzzy set, Bipolar anti fuzzy set, K-algebra, Ideal in K-algebra

1. **Introduction.** The fuzzy set theory was firstly introduced by Zadeh [1]. A mapping which has real number interval $[0, 1]$ on the codomain is called fuzzy set. Fuzzy sets are widely applied to various sciences, including algebra. Zhang [2] introduced the concept of bipolar fuzzy set by developing from the fuzzy. Bipolar fuzzy is a pair of membership functions and non-membership functions, respectively represented by positive and negative values. Bipolar fuzzy set is also applied in algebra, for example, in K-algebra.

K-algebra is a kind of an algebraic structure which is built by groups $(G, *, e)$ with binary operations $(\odot)$ and fulfilling the certain axioms and it is denoted by $\mathcal{K} = (G, *, \odot, e)$. This concept was discussed firstly by Dar and Akram [3], and they also discussed about characterization of K-algebra as *BCI*-algebra. The discussion was continued where Akram and Dar [4,5] wrote about homomorphism in K-algebra and fuzzy ideals of K-algebra. Along with the development of fuzzy set theory, Dar and Akram [5] discussed the bifuzzy ideal of K-algebra, and bifuzzy is a pair of two fuzzy sets. Not only fuzzy theory, but also bipolar fuzzy is applied to K-algebra. In 2010 Akram et al. [6] discussed the application of bipolar fuzzy in K-algebra. Bipolar fuzzy set is not only applied in K-algebras, but also in *BCK*/*BCI*-algebras, for example, Lee [7] discussed about bipolar fuzzy subalgebras and bipolar fuzzy ideals of *BCK*/*BCI*-algebras.

Bipolar anti fuzzy is also applied in algebraic structure, for example, in group and ring as follows. Muthuraj et al. [8-13] wrote some application of bipolar anti fuzzy in *HX* group, subgroup, and ring. Hayat et al. [14] discussed some application of bipolar anti fuzzy in hemirings. Motivated by Hayat et al.'s article about bipolar anti fuzzy, in this paper we will introduce bipolar anti fuzzy ideal of K-algebra. In this research we can expand the view about fuzzy theory and K-algebra. The remainder of this paper is structured as follows. In Section 2 we present some basic theories about K-algebra and fuzzy. In Section 3, we investigate bipolar anti fuzzy ideals of K-algebra and discuss its properties. In Section 4, we characterize bipolar anti fuzzy ideal of K-algebras by means of positive anti β-cut and negative anti α-cut. In the last section, we summarize the conclusion and give advice on some topics for future work.

DOI: 10.24507/icicel.13.10.875

2. **Preliminaries.** In this section we will discuss some basic theories about bipolar anti fuzzy ideals of K-algebra. In the work of [3], discuss about K-algebra and we refer to readers to point [4-7] for more result in this topic.

Definition 2.1. *Let $(G, *)$ be a group and its order more than 2. Define a binary operation on G as follows*

$$\odot : G \times G \to G$$
$$\odot(x, y) = x \odot y = x * y^{-1}$$

If the following axioms are held by G:

i. $(x \odot y) \odot (x \odot z) = (x \odot ((e \odot z) \odot (e \odot y))) \odot x$
ii. $x \odot (x \odot y) = (x \odot (e \odot y)) \odot x$
iii. $x \odot x = e$
iv. $x \odot e = x$
v. $e \odot x = x^{-1}$ *for every* $x, y, z \in G$

then G is called K-algebra which is built by group G and we denote by $\mathcal{K} = (G, *, \odot, e)$. If $(G, *, e)$ is an Abelian group, then we can replace axiom i and ii with

i.* $(x \odot y) \odot (x \odot z) = z \odot y$
ii.* $x \odot (x \odot y) = y$

for every $x, y, z \in G$.

Definition 2.2. *Let $\mathcal{K} = (G, *, \odot, e)$ be a K-algebra. A non empty set H in $\mathcal{K}$ is called K-subalgebra if $e \in H$ and $h_1 \odot h_2 \in H$, for every $h_1, h_2 \in H$.*

Definition 2.3. *Let I be a non empty set in K-algebra $\mathcal{K} = (G, *, \odot, e)$. I is called ideal of $\mathcal{K}$ if the following conditions are satisfied for every $x, y \in G$.*

i. $e \in I$
ii. $x \odot y \in I$, $y \odot (y \odot x) \in I \Rightarrow x \in I$

Definition 2.4. *Let X be a non empty set and μ_A be a mapping*

$$\mu_A : X \to [0, 1]$$

where $[0, 1]$ is a closed interval between 0 and 1. Fuzzy set of A in X is defined by a set

$$A = \{(x, \mu_A(x)) \,|x \in X\}$$

where $\mu_A(x)$ is called a membership function for A.

Definition 2.5. *Let $\mathcal{K} = (G, *, \odot, e)$ be a K-algebra. A fuzzy set A of $\mathcal{K}$ is called fuzzy ideal of $\mathcal{K}$ if the following conditions are satisfied for every $x, y \in G$.*

i. $\mu_A(e) \geq \mu_A(x)$, *and*
ii. $\mu_A(x) \geq \min\{\mu_A(x \odot y), \mu_A(y \odot (y \odot x))\}$.

Definition 2.6. *Let X be a non empty set and λ_B^+ and λ_B^- be a mapping*

$$\lambda_B^+ : X \to [0, 1] \text{ and } \lambda_B^- : X \to [-1, 0]$$

where $[0, 1]$ is a closed interval between 0 and 1. $[-1, 0]$ is a closed interval between -1 and 0. A set B which is defined by

$$B = \left\{x, \left(\lambda_B^+(x), \lambda_B^-(x)\right) \middle| x \in X\right\}$$

is called bipolar fuzzy set B of X, where $\lambda_B^+(x)$ is called a membership function for B and $\lambda_B^-(x)$ is called a non-membership function for B. Furthermore, bipolar fuzzy set will be written by $B = (\mu^+, \mu^-)$.

Definition 2.7. *Bipolar fuzzy set $B = (\mu^+, \mu^-)$ in K-algebra $\mathcal{K}$ is called bipolar fuzzy subalgebra if it satisfies for every $x, y \in G$.*

i. $\mu^+(x \odot y) \geq \min\{\mu^+(x), \mu^+(y)\}$
ii. $\mu^-(x \odot y) \leq \max\{\mu^+(x), \mu^+(y)\}$

3. **Bipolar Anti Fuzzy Ideal of K-algebra.** Before we discuss about bipolar anti fuzzy ideal of K-algebra, firstly we defined about bipolar fuzzy ideal of K-algebras, which is analog with the definition in [7] and [14].

Definition 3.1. *Let $B = (\lambda^+, \lambda^-)$ be a bipolar fuzzy set of K-algebra $\mathcal{K}$ and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$, for every $x \in \mathcal{K}$.*

i. $B(x) \geq t' \Leftrightarrow (\lambda^+(x), \lambda^-(x)) \geq (t^+, t^-) \Leftrightarrow \lambda^+(x) \geq t^+$ *and* $\lambda^-(x) \leq t^-$

ii. $B(x) \leq t' \Leftrightarrow (\lambda^+(x), \lambda^-(x)) \leq (t^+, t^-) \Leftrightarrow \lambda^+(x) \leq t^+$ *and* $\lambda^-(x) \geq t^-$

Definition 3.2. *Let $B = (\lambda^+, \lambda^-)$ be a bipolar fuzzy set of K-algebra $\mathcal{K}$ with*

$$\lambda^+(z) = \begin{cases} t^+, & z = x \\ 0, & z \neq x \end{cases}$$

$$\lambda^-(z) = \begin{cases} t^-, & z = x \\ 0, & z \neq x \end{cases}$$

Then B is called a bipolar value fuzzy point where $t' = (t^+, t^-) \in (0,1] \times [-1,0)$ and support x, written as $x_{t'} = \left(x_t^+, x_t^-\right)$. $x_{t'}$ is said to belong to B, written as $x_{t'} \in B$ if $B(x) \geq t'$, so $\lambda^+(x) \geq t^+$, $\lambda^-(x) \leq t^-$.

Definition 3.3. *Let $B_1 = (\lambda^+, \lambda^-)$ and $B_2 = (\mu^+, \mu^-)$ be two bipolar fuzzy sets of $\mathcal{K}$,*

$$\max\{B_1, B_2\} \textit{ is defined as } \left(\max\{\lambda^+, \mu^+\}, \min\{\lambda^-, \mu^-\}\right)$$

$$\min\{B_1, B_2\} \textit{ is defined as } \left(\min\{\lambda^+, \mu^+\}, \max\{\lambda^-, \mu^-\}\right)$$

Definition 3.4. *A bipolar fuzzy set $B = (\lambda^+, \lambda^-)$ is called a bipolar fuzzy ideal of $\mathcal{K}$ if the following conditions hold.*

i. $\lambda^+(e) \geq \lambda^+(x)$ *and* $\lambda^-(e) \leq \lambda^-(x)$

ii. $\lambda^+(x) \geq \min\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$ *and* $\lambda^-(x) \leq \max\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$

Example 3.1. *Let $G = \{e, a, b, x, y, z\}$ and binary operation $\circ$ in G is defined in Table 1.*

TABLE 1. Binary operation $\circ$ in G

$\circ$	e	a	b	x	y	z
e	e	a	b	x	y	z
a	a	b	e	z	x	y
b	b	e	a	y	z	x
x	x	y	z	e	a	b
y	y	z	x	b	e	a
z	z	x	y	a	b	e

We can prove that $(G, \circ)$ is a group and $\mathcal{K} = (G, \circ, \odot, e)$ is a K-algebra. We defined a bipolar fuzzy set $B = (\mu^+, \mu^-)$ of $\mathcal{K}$ as follows $\mu^+(e) = 0.8$, $\mu^+(t) = 0.06$ for every $t \neq e$ and $\mu^-(e) = -0.7$, $\mu^-(t) = -0.14$ for every $t \neq e$. We can prove that B is a bipolar fuzzy ideal of K-algebra $\mathcal{K}$.

Definition 3.5. *Let I be a non empty subset in K-algebra $\mathcal{K}$. Bipolar fuzzy set $C_{I^c} = \left(C_{I^c}^+, C_{I^c}^-\right)$ defined by*

$$C_{I^c}^+(x) = \begin{cases} 0, & x \in I \\ 1, & x \overline{\in} I \end{cases}$$

$$C_{I^c}^-(x) = \begin{cases} 0, & x \in I \\ -1, & x \overline{\in} I \end{cases}$$

is called bipolar-valued anti characteristic function.

Motivated by Definition 3.2, we will discuss about bipolar anti fuzzy, where $x_{t'}$ is said not to belong to B, written as $x_{t'} \overline{\in} B$ if $B(x) \le t'$, so $\lambda^+(x) \le t^+$, $\lambda^-(x) \ge t^-$.

Definition 3.6. *Let $\mathcal{K} = (G, *, \odot, e)$ be a K-algebra. Bipolar fuzzy set $B = (\lambda^+, \lambda^-)$ is said a bipolar anti fuzzy ideal of $\mathcal{K}$ if the following conditions hold.*

i. $x_{t'} \overline{\in} B \Rightarrow e_{t'} \overline{\in} B$
ii. $(x \odot y)_{t'} \overline{\in} B$, $(y \odot (y \odot x))_{r'} \overline{\in} B \Rightarrow x_{\max\{t',r'\}} \overline{\in} B$

Furthermore, bipolar anti fuzzy ideal is abbreviated by BAF ideal.

Theorem 3.1. *If B is a bipolar fuzzy set in K-algebra $\mathcal{K}$, then axioms in Definition 3.6 are equivalent to the following axioms respectively.*

a. $\lambda^+(e) \le \lambda^+(x)$ *and* $\lambda^-(e) \ge \lambda^-(x)$
b. $\lambda^+(x) \le \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$ *and* $\lambda^-(x) \ge \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$

Proof: Based on Definition 3.2 and definition of $x_{t'} \overline{\in} B$, we can prove that the axioms are equivalent.

Example 3.2. *Let $\mathcal{K} = (G, \circ, \odot, e)$ be a K-algebra where $G = \{e, a, b, x, y, z\}$. If a bipolar fuzzy set $B = (\lambda^+, \lambda^-)$ in $\mathcal{K}$ defined by*

$$\lambda^+(x) = \begin{cases} 0.03, & x = e \\ 0.4, & x \ne e \end{cases} \quad \text{and} \quad \lambda^-(x) = \begin{cases} -0.2, & x = e \\ -0.35, & x \ne e \end{cases}$$

then $B = (\lambda^+, \lambda^-)$ BAF ideal of $\mathcal{K}$.

Theorem 3.2. *A bipolar fuzzy set $B = (\lambda^+, \lambda^-)$ is a BAF ideal of K-algebra $\mathcal{K}$ if and only if*

a. $\lambda^+(e) \le \lambda^+(x)$ *and* $\lambda^-(e) \ge \lambda^-(x)$
b. $\lambda^+(x) \le \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$ *and* $\lambda^-(x) \ge \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$

Proof: Because axioms in Definition 3.6 are equivalent to axioms in Theorem 3.1, this theorem has been proven.

Theorem 3.3. *Let $\mathcal{K} = (G, *, \odot, e)$ be a K-algebra and $I \subseteq \mathcal{K}$ in which $I \ne \emptyset$. $C_{I^c} \in$ BAF ideal of $\mathcal{K}$ if and only if I ideal of $\mathcal{K}$.*

Proof: It is clear that $e \in I$ and for every $(x \odot y) \in I$, $(y \odot (y \odot x)) \in I$ implies $x \in I$. So, it can be concluded that I ideal of $\mathcal{K}$. Conversely,

i. We know that $C^+_{I^c}(e) = 0$ and $C^-_{I^c}(e) = 0$. $C_{I^c} = \left(C^+_{I^c}, C^-_{I^c}\right)$ where

$$C^+_{I^c}(x) = \begin{cases} 0, & x \in I \\ 1, & x \overline{\in} I \end{cases}$$

$$C^-_{I^c}(x) = \begin{cases} 0, & x \in I \\ -1, & x \overline{\in} I \end{cases}$$

so $0 \le C^+_{I^c}(x) \to C^+_{I^c}(e) \le C^+_{I^c}(x)$ and $0 \ge C^-_{I^c}(x) \to C^-_{I^c}(e) \ge C^-_{I^c}(x)$.

ii. We know that I ideal of $\mathcal{K}$, then the following axioms hold

$(x \odot y) \in I$ and $(y \odot (y \odot x)) \in I \to x \in I$, cause

$C^+_{I^c}(x \odot y) = 0$ $\quad$ $C^-_{I^c}(x \odot y) = 0$
$C^+_{I^c}(y \odot (y \odot x)) = 0$ $\quad$ $C^-_{I^c}(y \odot (y \odot x)) = 0$
$C^+_{I^c}(x) = 0$ $\quad$ $C^-_{I^c}(x) = 0$

Or can be written as $x \overline{\in} I \to (x \odot y) \overline{\in} I$ and $(y \odot (y \odot x)) \overline{\in} I$, cause

$C^+_{I^c}(x \odot y) = 1$ $\quad$ $C^-_{I^c}(x \odot y) = -1$
$C^+_{I^c}(y \odot (y \odot x)) = 1$ $\quad$ $C^-_{I^c}(y \odot (y \odot x)) = -1$
$C^+_{I^c}(x) = 1$ $\quad$ $C^-_{I^c}(x) = -1$

We can see relation between x and $x \odot y$, as follows

- $x \in I$ and $(x \odot y) \in I$ then
  $C^+_{I^c}(x) = C^+_{I^c}(x \odot y)$ and $C^-_{I^c}(x) = C^-_{I^c}(x \odot y)$
- $x \in I$ and $(x \odot y) \overline{\in} I$ then
  $C^+_{I^c}(x) \leq C^+_{I^c}(x \odot y)$ and $C^-_{I^c}(x) \geq C^-_{I^c}(x \odot y)$
- $x \overline{\in} I \to (x \odot y) \overline{\in} I$ then
  $C^+_{I^c}(x) = C^+_{I^c}(x \odot y)$ and $C^-_{I^c}(x) = C^-_{I^c}(x \odot y)$

Generally, it can be written as $C^+_{I^c}(x) \leq C^+_{I^c}(x \odot y)$ and $C^-_{I^c}(x) \geq C^-_{I^c}(x \odot y)$.

We can see relation between x and $y \odot (y \odot x)$, as follows

- $x \in I$ and $(y \odot (y \odot x)) \in I$ then
  $C^+_{I^c}(x) = C^+_{I^c}(y \odot (y \odot x))$ and $C^-_{I^c}(x) = C^-_{I^c}(y \odot (y \odot x))$
- $x \in I$ and $(y \odot (y \odot x)) \overline{\in} I$ then
  $C^+_{I^c}(x) \leq C^+_{I^c}(y \odot (y \odot x))$ and $C^-_{I^c}(x) \geq C^-_{I^c}(y \odot (y \odot x))$
- $x \overline{\in} I \to (y \odot (y \odot x)) \overline{\in} I$ then
  $C^+_{I^c}(x) = C^+_{I^c}(y \odot (y \odot x))$ and $C^-_{I^c}(x) = C^-_{I^c}(y \odot (y \odot x))$

Generally, it can be written as $C^+_{I^c}(x) \leq C^+_{I^c}(y \odot (y \odot x))$ and $C^-_{I^c}(x) \geq C^-_{I^c}(y \odot (y \odot x))$.

According to the statement above

- $C^+_{I^c}(x) \leq C^+_{I^c}(x \odot y)$ and $C^+_{I^c}(x) \leq C^+_{I^c}(y \odot (y \odot x))$ so $C^+_{I^c}(x) \leq \max\{C^+_{I^c}(x \odot y), C^+_{I^c}(y \odot (y \odot x))\}$
- $C^-_{I^c}(x) \geq C^-_{I^c}(x \odot y)$ and $C^-_{I^c}(x) \geq C^-_{I^c}(y \odot (y \odot x))$ so $C^-_{I^c}(x) \geq \min\{C^-_{I^c}(x \odot y), C^-_{I^c}(y \odot (y \odot x))\}$

$C_{I^c} \in$ BAF ideal of $\mathcal{K}$.

4. Characterization of BAF Ideal of *K*-algebras by Means of Positive Anti *β*-cut and Negative Anti *α*-cut.

Definition 4.1. *Let $\mathcal{K} = (G, *, \odot, e)$ be a K-algebra. If $B = (\mu^+, \mu^-)$ bipolar fuzzy set of $\mathcal{K}$ and $(\alpha, \beta) \in [-1, 0] \times [0, 1]$, then*

i. $\tilde{B}^+_\beta = \{x \in \mathcal{K} | \mu^+(x) \leq \beta\}$ is called positive anti β-cut of B

ii. $\tilde{B}^-_\alpha = \{x \in \mathcal{K} | \mu^-(x) \geq \alpha\}$ is called negative anti α-cut of B

iii. $\tilde{B}_{(\alpha,\beta)} = \{x \in \mathcal{K} | \mu^-(x) \geq \alpha$ and $\mu^+(x) \leq \beta\}$ is called anti (α, β)-cut of B

For every $\gamma \in (0, 1]$ and $\tilde{B}^+_\gamma \cap \tilde{B}^-_\gamma$ is called anti γ-cut of B.

Theorem 4.1. *Bipolar fuzzy set $B = (\mu^+, \mu^-) \in$ BAF ideal of K-algebra $\mathcal{K}$ if and only if the conditions hold.*

i. For every $\beta \in [0, 1]$, $\tilde{B}^+_\beta$ non empty, then $\tilde{B}^+_\beta$ ideal of $\mathcal{K}$

ii. For every $\alpha \in [-1, 0]$, $\tilde{B}^-_\alpha$ non empty, then $\tilde{B}^-_\alpha$ ideal of $\mathcal{K}$

Proof:

i. Let $x \in \tilde{B}^+_\beta \to \mu^+(x) \leq \beta$. We know that $\mu^+(e) \leq \mu^+(x)$ then $\mu^+(e) \leq \beta$. So we can conclude that $e \in \tilde{B}^+_\beta$. Let $x \in \tilde{B}^-_\alpha \to \mu^-(x) \geq \alpha$. We know that $\mu^-(e) \geq \mu^-(x)$ then $\mu^-(e) \geq \alpha$. So $e \in \tilde{B}^-_\alpha$.

ii. Suppose that $(x \odot y), (y \odot (y \odot x)) \in \tilde{B}^+_\beta$ and $x \overline{\in} \tilde{B}^+_\beta$, then $\mu^+(x) > \beta$ so $\beta < \mu^+(x) \leq \max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\}$. It causes $\mu^+(x \odot y) > \beta$ and $\mu^+(y \odot (y \odot x)) > \beta$ so $(x \odot y), (y \odot (y \odot x)) \overline{\in} \tilde{B}^+_\beta$. It is contrary with presupposition, and it must be $x \in \tilde{B}^+_\beta$. In the same way for $\mu^-(x)$, we can conclude that $(x \odot y), (y \odot (y \odot x)) \in \tilde{B}^-_\alpha$ implies $x \in \tilde{B}^-_\alpha$.

$\tilde{B}^+_\beta$ and $\tilde{B}^-_\alpha$ ideal of $\mathcal{K}$. Conversely,

i. For $x \in \tilde{B}^+_\beta$ and $x \in \tilde{B}^-_\alpha$, then $\mu^+(x) \leq \beta$ and $\mu^-(x) \geq \alpha$. As we know that for every $x \in G \to e \in \tilde{B}^+_\beta$ and $e \in \tilde{B}^-_\alpha$ hold $\mu^+(e) \leq \beta$ and $\mu^-(e) \geq \alpha$.

Suppose that $\mu^+(e) > \mu^+(x)$ and $\mu^+(x) = \beta$ then $\mu^+(e) > \beta$. It is contrary with $\mu^+(e) \le \beta$. It must be $\mu^+(e) \le \mu^+(x)$.

In the same way we obtain that $\mu^-(e) \ge \mu^-(x)$.

ii. If $(x \odot y), (y \odot (y \odot x)) \in \tilde{B}^+_\beta$ then $x \in \tilde{B}^+_\beta$. It causes $\mu^+(x) \le \beta$, $\mu^+(x \odot y) \le \beta$, and $\mu^+(y \odot (y \odot x)) \le \beta$. Suppose $\mu^+(x) > \mu^+(x \odot y)$ and $\mu^+(x \odot y) = \beta$ then $\mu^+(x) > \beta$. It is contrary to $\mu^+(x) \le \beta$, and it must be $\mu^+(x) \le \mu^+(x \odot y)$.

Suppose $\mu^+(x) > \mu^+(y \odot (y \odot x))$ and $\mu^+(y \odot (y \odot x)) = \beta$ then $\mu^+(x) > \beta$. It is contrary with $\mu^+(x) \le \beta$, and it must be $\mu^+(x) \le \mu^+(y \odot (y \odot x))$.

Based on the statement above, we obtained that $\mu^+(x) \le \mu^+(x \odot y) \le \beta$ and $\mu^+(x) \le \mu^+(y \odot (y \odot x)) \le \beta$. It can be concluded that $\mu^+(x) \le \max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\}$.

In the same way for $\tilde{B}^-_\alpha$, until we obtained $\mu^-(x) \ge \mu^-(x \odot y)$ and $\mu^-(x) \ge \mu^-(y \odot (y \odot x))$.

Based on the statement above, we obtained $\mu^-(x) \ge \mu^-(x \odot y) \ge \alpha$ and $\mu^-(x) \ge \mu^-(y \odot (y \odot x)) \ge \alpha$. It can be concluded that $\mu^-(x) \ge \min\{\mu^-(x \odot y), \mu^-(y \odot (y \odot x))\}$.

$B = (\mu^+, \mu^-) \in$ BAF ideal of K-algebra $\mathcal{K}$.

Example 4.1. *According to Example 3.1, we have $\mathcal{K} = (G, \circ, \odot, e)$ is a K-algebra where $G = \{e, a, b, x, y, z\}$ and $B = (\lambda^+, \lambda^-)$ is BAF ideal of $\mathcal{K}$, defined by*

$$\lambda^+(x) = \begin{cases} 0.03, & x = e \\ 0.4, & x \ne e \end{cases} \quad \text{and } \lambda^-(x) = \begin{cases} -0.2, & x = e \\ -0.35, & x \ne e \end{cases}$$

If $(\alpha, \beta) = (-0.45, 0.5)$ then $\tilde{B}^+_\beta = \{x \in \mathcal{K} | \lambda^+(x) \le \beta\} = \{e, a, b, x, y, z\}$ and $\tilde{B}^-_\alpha = \{x \in \mathcal{K} | \lambda^-(x) \ge \alpha\} = \{e, a, b, x, y, z\}$. $\tilde{B}^+_\beta$ and $\tilde{B}^-_\alpha$ ideal of $\mathcal{K}$.

Corollary 4.1. *Let $\mathcal{K} = (G, *, \odot, e)$ be a K-algebra and $B = (\mu^+, \mu^-)$ is bipolar fuzzy set of K. If $B = (\mu^+, \mu^-)$ element BAF ideal of $\mathcal{K}$, then anti γ-cut of B ideal of $\mathcal{K}$ for every $\gamma \in [0, 1]$.*

Proof:

i. Let x element anti γ-cut, so $\mu^+(x) \le \gamma$ and $\mu^-(x) \ge -\gamma$. Because $\mu^+(e) \le \mu^+(x)$ and $\mu^-(e) \ge \mu^-(x)$, it causes $\mu^+(e) \le \mu^+(x) \le \gamma$ and $\mu^-(e) \ge \mu^-(x) \ge -\gamma$. So $\mu^+(e) \le \gamma$ and $\mu^-(e) \ge -\gamma \to e$ element anti γ-cut of B.

ii. Suppose that $(x \odot y)$ element anti γ-cut, $(y \odot (y \odot x))$ element anti γ-cut and x is not an element of anti γ-cut, implies
$\mu^+(x \odot y) \le \gamma$ and $\mu^-(x \odot y) \ge -\gamma$
$\mu^+(y \odot (y \odot x)) \le \gamma$ and $\mu^-(y \odot (y \odot x)) \ge -\gamma$
$\mu^+(x) > \gamma$ and $\mu^-(x) < -\gamma$
It is contrary with axioms BAF ideal of $\mathcal{K}$, and it must be x element anti γ-cut. It can be concluded that $(x \odot y)$ element anti γ-cut, $(y \odot (y \odot x))$ element anti γ-cut $\to x$ element anti γ-cut.

It is proven that anti γ-cut ideal of $\mathcal{K}$.

Example 4.2. *Based on Example 3.2, we know that $\mathcal{K} = (G, \circ, \odot, e)$ is a K-algebra where $G = \{e, a, b, x, y, z\}$ and $B = (\lambda^+, \lambda^-)$ BAF ideal of $\mathcal{K}$, where*

$$\lambda^+(x) = \begin{cases} 0.03, & x = e \\ 0.4, & x \ne e \end{cases} \quad \text{and } \lambda^-(x) = \begin{cases} -0.2, & x = e \\ -0.35, & x \ne e \end{cases}$$

If $\gamma = 0.4$ then $\tilde{B}^+_\gamma = \{e, a, b, x, y, z\}$ and $\tilde{B}^-_\gamma = \{e, a, b, x, y, z\}$, anti γ-cut of B is $\tilde{B}^+_\gamma \cap \tilde{B}^-_\gamma = \{e, a, b, x, y, z\}$.

i. $e \in$ anti γ-cut

ii. For every $x, y \in G$, $(x \odot y) \in$ anti γ-cut, $(y \odot (y \odot x)) \in$ anti γ-cut $\to x \in$ anti γ-cut.

It is proven that anti γ-cut ideal of $\mathcal{K}$.

Corollary 4.2. *If bipolar fuzzy set $B = (\mu^+, \mu^-) \in BAF$ ideal of K-algebra $\mathcal{K}$, then $\tilde{B}_{(\alpha,\beta)}$ ideal of $\mathcal{K}$ for every $(\alpha, \beta) \in [-1, 0] \times [0, 1]$.*

Proof: Based on the proof in Theorem 4.1, it is clear that $\tilde{B}_{(\alpha,\beta)}$ ideal of $\mathcal{K}$.

5. **Conclusion.** In this paper, we introduced the concept of bipolar anti fuzzy ideals of K-algebra and investigated related properties. We hope this discussion can expand the view about fuzzy theory and become a reference for further research, for example, BAF bi-ideals.

REFERENCES

[1] L. A. Zadeh, Fuzzy sets, *Information and Control*, vol.8, pp.338-353, 1965.
[2] W. R. Zhang, *Bipolar Fuzzy Sets*, Master Thesis, Lamar University, 1998.
[3] K. H. Dar and M. Akram, On a K-algebra built on a group, *South. Asian Bull. Math.*, vol.29, pp.41-49, 2005.
[4] M. Akram and K. H. Dar, Fuzzy ideals of K-algebras, *Annals of University of Craiova, Math. Comp. Sci. Ser.*, vol.34, pp.11-20, 2007.
[5] K. H. Dar and M. Akram, On K-homomorphism of K-algebras, *International Mathematical Forum*, vol.2, no.46, pp.2283-2293, 2007.
[6] M. Akram, A. B. Saeid, K. P. Shum and B. L. Meng, Bipolar fuzzy K-algebras, *International Journal of Fuzzy Systems*, vol.12, pp.252-258, 2010.
[7] K. J. Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of *BCK/BCI*-algebras, *Bulletin of the Malaysian Mathematical Sciences Society*, vol.32, pp.361-373, 2009.
[8] R. Muthuraj and M. Sridharan, Bipolar anti fuzzy HX group and its lower level sub HX groups, *Journal of Physical Sciences*, vol.16, pp.157-169, 2012.
[9] R. Muthuraj and M. Sridharan, Homomorphism and anti homomorphism on bipolar anti fuzzy subgroup, *Journal of Engineering Research and Applications*, vol.3, pp.1155-1159, 2013.
[10] R. Muthuraj and M. Sridharan, Bipolar anti fuzzy normal HX subgroups, *International Journal of Mathematical Archive*, vol.7, pp.118-128, 2014.
[11] R. Muthuraj and M. Sridharan, Pseudo bipolar fuzzy cosets of bipolar fuzzy and anti fuzzy HX subgroups, *Applied Mathematics and Sciences: An International Journal*, vol.1, 2014.
[12] R. Muthuraj, M. S. Murhuraman and M. Rajinikannan, Bipolar anti fuzzy subrings, *International Journal on Research Innovations in Engineering Science and Technology*, vol.1, 2016.
[13] R. Muthuraj and G. S. Meena, Bipolar anti L-fuzzy sub ℓ-HX group and its lower level sub ℓ-HX groups, *Annals of Pure and Applied Mathematics*, vol.15, pp.253-269, 2017.
[14] K. Hayat, T. Mahmood and B. Y. Cao, On bipolar anti fuzzy h-ideals in hemi-rings, *Fuzzy Informations and Engineering*, vol.9, pp.1-19, 2017.
[15] W. B. V. Kandasamy, *Smarandache Fuzzy Algebra*, American Research Press, 2003.