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THE SENSITIVITY OF THE DAMAGE INDEX OF THE GENERAL VIBRATION METHOD TO DAMAGE LEVEL FOR STRUCTURAL HEALTH MONITORING

Fergyanto Efendy Gunawan

Industrial Engineering Department, BINUS Graduate Program – Master of Industrial Engineering
Bina Nusantara University
Jl. K. H. Syahdan No. 9, Kemanggisan, Palmerah, Jakarta 11480, Indonesia
fgunawan@binus.edu

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Abstract. *For damage detection, this research article discusses an easy-to-compute damage index derived from the governing dynamic of the structure that has potential practical application in Structural Health Monitoring (SHM). We use simplified structural models to explore the sensitivity of the index to damages, to compare the index performance with a traditional but popular damage detection method, and to understand the local/global predictive capability of the index. We use two simple models, namely, single- and two-degree-of-freedom systems. The results suggest that the damage index is local, that can only monitor damages occurring near the points of measurements, but it is sensitive to damages, unlike the natural frequency, which is global but less sensitive.*
Keywords: Structural health monitoring, General vibration model, Power Spectral Density, Damage index

1. **Introduction.** Determining features of engineering structures that are sensitive to structural damages has become a very active research topic in the area of Structural Health Monitoring (SHM). The features are commonly extracted or derived from the structural deformation or rate of deformation data. The most widely used features are the structural natural frequencies, mode shapes, deformation curvatures, the Power Spectral Density (z and F statistics), and other features derived from those quantities [1, 2].

The natural frequency and mode shape are advantageous and practical for damage detection. The quantities can be estimated from data obtained at a few measurement points and are theoretically capable of changing with damages that occur on any part of the structure. The quantities are global. Our understanding of the quantities and their measurement methods are well established. With those characteristics, the quantities are widely used for damage detections.

During the last decade, we witness the proliferation of the use of machine learning or soft computing techniques for SHM applications. The machine learning techniques are often used in conjunction with those damage sensitive features as the input data [3, 4, 5]. However, we also witness the use of the techniques with raw data directly measured from the structural deformation. In this approach, the features are not explicitly derived but are established as an integral part of the techniques [6, 7]. For example, [7] developed a convolution neural network model that classified the conditions of an engineering structure into healthy and damaged based on the acceleration data recorded on many points on the structure.

In terms of the computational complexity, the traditional methods that use the structural vibration characteristics are simpler. The modern methods based on machine learning techniques tend to require a large amount of data and much higher computational

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complexity. Gunawan et al. [8] pointed out that the machine learning approach was only effective when the training data were available in a significant number.

The traditional methods can achieve high accuracy by applying our theoretical understanding of structural mechanics. Many recent publications proposed detection methods solely based on correlations of the patterns in the data without exploiting much these understandings [3, 5, 6, 7].

However, recently, Zhang et al. [9] discussed a new method, called the general vibration model, where the damage was detected by evaluating the deviation of the structural dynamic equilibrium from the dynamic on the healthy condition. The method was verified experimentally. Some aspects, such as the sensitivity of the method to the damage level had not been discussed and how better or worse the performance of the method was with respect to the traditional methods, had not been addressed. This article intends to fill this gap.

Thus, the contributions of this research are of the following. The first is to present the sensitivity of the damage index of the general vibration method to the damage level. The second is to compare the sensitivity of the damage index with the traditional and widely used damage sensitive features. The third is to identify the local/global damage detection capability of the index.

The article is structured as the following. Section 2, Research Methods, discusses the mathematical models of the structural dynamics and its numerical solution. Section 3, Results, discusses the effects of the damage level to the current damage index, and a comparison to the traditional methods. Finally, Section 4, Conclusions, presents a summary of the current findings and a proposal for the future investigation.

2. **Research Methods.** The governing dynamic of the system consisting of N lumped masses can be obtained by applying the Newton's second law to each mass. The results are the dynamic equilibrium equation of:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$, respectively, are column-wise vectors of displacements, velocities, and accelerations. All vectors have the same size of $N \times 1$. Respectively, the symbols of $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote the mass, damping, and stiffness matrices. All matrices have the size of $N \times N$.

We define a damage index, $\mathbf{d}$, as

$$\mathbf{d}(t) = |\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)|, \tag{2}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are structural properties in the intact or healthy condition. Theoretically, the damage index should be zero when the structural responses, $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$, are obtained from the healthy condition. In the other condition where the structure contains damages, the index is expected to deviate from zero. We also hypothesize that the value of the index will proportionally be with the damage level. This study intends to establish evidences to support these hypotheses.

2.1. **Numerical integration.** We solve Equation (1) numerically for a general structural condition. Firstly, we rewrite the governing dynamic in the state-space form by introducing a new free variable:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \end{bmatrix}.$$

By this introduction, we can express Equation (1) in the form of the standard initial value problem:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(t, \mathbf{x}(t)), \qquad \mathbf{x}(0) = \mathbf{x}_0 = \begin{bmatrix} \mathbf{u}_0 & \mathbf{v}_0 \end{bmatrix}^T, \tag{3}$$

where

$$\mathbf{F}(t,\mathbf{x}(t)) = \mathbf{A}\mathbf{x}(t) + \mathbf{p}(t), \tag{4}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{N\times N} & \mathbf{I}_{N\times N} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \tag{5}$$

$$\mathbf{p}(t) = \begin{bmatrix} \mathbf{0}_{N\times 1} \\ \mathbf{M}^{-1}\mathbf{f}(t) \end{bmatrix}. \tag{6}$$

In Equations (5) and (6), $\mathbf{0}_{N\times N}$ is a zero matrix with the size $N \times N$, $\mathbf{I}_{N\times N}$ is an identity matrix of the same size, and $\mathbf{0}_{N\times 1}$ is a zero vector of the size $N \times 1$.

The problem of Equation (3) is solved by using the higher-order Runge-Kutta method where the data of the initial structural conditions $\mathbf{x}_0$ are propagated through the time domain by a successive computation of

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \frac{1}{90}\left(7\mathbf{k}_1 + 32\mathbf{k}_3 + 12\mathbf{k}_4 + 32\mathbf{k}_5 + 7\mathbf{k}_6\right)\cdot\Delta t, \tag{7}$$

where

$$\mathbf{k}_1 = \mathbf{F}\left(t_i, \mathbf{x}_i\right) \tag{8}$$

$$\mathbf{k}_2 = \mathbf{F}\left(t_i + \frac{\Delta t}{4}, \mathbf{x}_i + \frac{\mathbf{k}_1\Delta t}{4}\right) \tag{9}$$

$$\mathbf{k}_3 = \mathbf{F}\left(t_i + \frac{\Delta t}{4}, \mathbf{x}_i + \frac{\mathbf{k}_1\Delta t}{8} + \frac{\mathbf{k}_2\Delta t}{8}\right) \tag{10}$$

$$\mathbf{k}_4 = \mathbf{F}\left(t_i + \frac{\Delta t}{2}, \mathbf{x}_i - \frac{\mathbf{k}_2\Delta t}{2} + \mathbf{k}_3\Delta t\right) \tag{11}$$

$$\mathbf{k}_5 = \mathbf{F}\left(t_i + \frac{3\Delta t}{4}, \mathbf{x}_i + \frac{3\mathbf{k}_1\Delta t}{16} + \frac{9\mathbf{k}_4\Delta t}{16}\right) \tag{12}$$

$$\mathbf{k}_6 = \mathbf{F}\left(t_i + \Delta t, \mathbf{x}_i - \frac{3\mathbf{k}_1\Delta t}{7} + \frac{2\mathbf{k}_2\Delta t}{7} + \frac{12\mathbf{k}_3\Delta t}{7} - \frac{12\mathbf{k}_4\Delta t}{7} + \frac{8\mathbf{k}_5\Delta t}{7}\right) \tag{13}$$

The symbol $\mathbf{x}_i$ denotes the structural deformation at t_i. We start the computation from $i = 0$ and increasing successively to $i = 1, 2, \ldots$. The variable Δt denotes the time step whose value should be very small for the Runge-Kutta method to estimate the structural dynamic responses accurately. The computation of the recursive formula Equation (7) should be repeated until the final time is reached.

2.2. **Single-degree-of-freedom system.** For the case of the single-degree-of-freedom (SDOF) system without damping, the governing dynamic can be written as:

$$m\ddot{u}(t) + ku(t) = p_0\sin(\Omega t), \tag{14}$$

where $p_0\sin(\Omega t)$ is a harmonic forcing function. To the zero initial velocity and position, the system responds with

$$u(t) = U\sin(\Omega t) - \frac{U\Omega}{\omega_n}\sin(\omega_n t), \tag{15}$$

$$v(t) = U\Omega\cos(\Omega t) - U\Omega\cos(\omega_n t), \qquad \text{and} \tag{16}$$

$$a(t) = -U\Omega^2\sin(\Omega t) + U\Omega\omega_n\sin(\omega_n t), \tag{17}$$

where $U = p_0/(k - m\Omega^2)$, and $\omega_n^2 = k/m$ is the natural frequency. The displacement, in the frequency domain, can be obtained by applying the Fourier transform, resulting in:

$$U(\omega) = U\frac{\sqrt{2\pi}}{2\jmath}\left[\delta(\omega+\Omega) + \delta(\omega-\Omega)\right] + \frac{U\Omega}{\omega_n}\frac{\sqrt{2\pi}}{2\jmath}\left[\delta(\omega+\omega_n) + \delta(\omega-\omega_n)\right]. \tag{18}$$

The frequency response can also be expressed in the form of the Power Spectral Density (PSD):

$$S_{uu}(\omega) = |U(\omega)|^2, \tag{19}$$

from which, the F statistic for SHM application can be established:

$$F(\omega) = \frac{S^H_{uu}(\omega)}{S^U_{uu}(\omega)}. \tag{20}$$

The superscript H denotes the healthy structural condition and U denotes the unknown condition. Theoretically, the value of F statistic is one for the intact condition.

For the damaged condition, we reduce the spring stiffness k by a factor determined by the damage level α. The healthy condition is associated with $\alpha = 0$. The damage index of $\alpha = 1$ means the condition of complete structural failure.

2.3. **Two-degree-of-freedom system.** In addition to the previous SDOF system, we also study the applicability of the general vibration method in the context of two-degree-of-freedom (2-DOF) system. The model allows to study the method on more complex scenarios. The model is depicted in Figure 1.

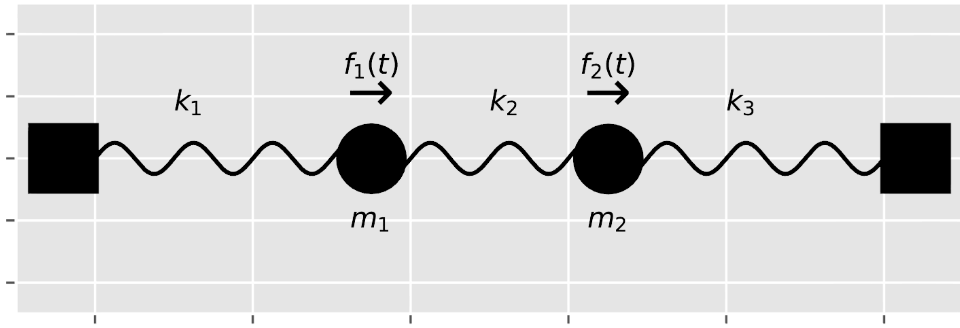

FIGURE 1. A model of two-degree-of-freedom system with the masses m_1 and m_2, the springs with the stiffnesses k_1, k_2, and k_3, and the applied dynamic forces $f_1(t)$ and $f_2(t)$. The left and right ends of the model are fixed.

For the system, the stiffness, damping, and mass matrices, respectively, are:

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}. \tag{21}$$

The applied dynamic forces are $f_1(t)$ on Mass 1 and $f_2(t)$ on Mass 2.

3. **Results.** We start the discussion from the case of the SDOF system, and then, it is followed by the 2-DOF system. As for the first case, the model parameters are $m = 1$, $k = 1$, $\Omega = 0.25$, and $p_0 = 1$ (see Section 2.2). All are in their respective units. We analyze the responses for various damage levels from 0 up to 20% or 0.2 in 30 increments. The model is analyzed for 4096 s with the time step of 0.25 s.

First, we present the displacement-time histories for a few damage levels in Figure 2. It suggests that it is rather difficult to conclude the structural condition from the response data as the change of the structural integrity does not change the displacement-time history remarkably and consistently. The displacement peak around 80 s increases with the damage level. However, at around 70 s, the peak decreases. It is challenging to notice the occurrence of the damage from the displacement time history.

Unlike the displacement time history, the phase plot, where the displacement time history is plotted against the velocity time history, is seriously altered by the damage as shown in Figure 3. For clarity, the figure only shows the conditions without damage and with damage at the level of 0.05517. The time duration is also fixed shortly for a duration

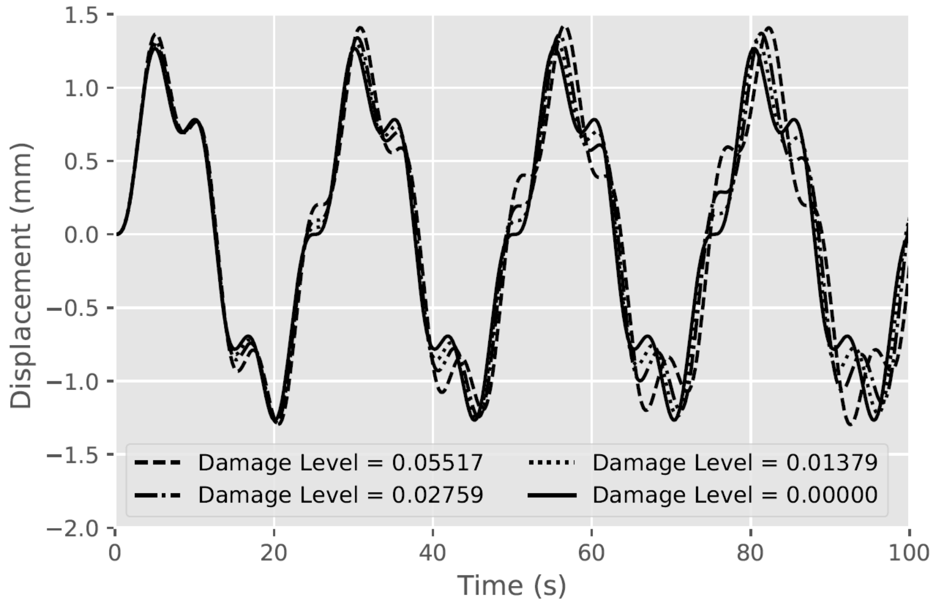

Figure 2. The effects of the damage to the displacement of the mass of the single-degree-of-freedom system

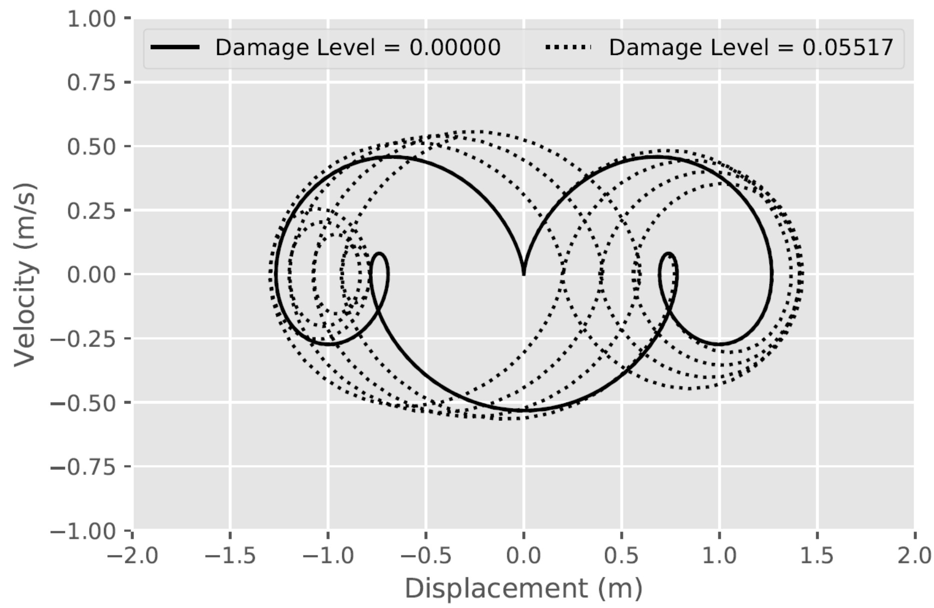

Figure 3. The phase plot of the single-degree-of-freedom system on the healthy and damaged conditions

of 100 s. Higher deviation from the healthy condition exists for a longer time. This finding seems to suggest that the similarity of the phase plot with the healthy condition may be an indicator of structural integrity. However, this assessment still requires further studies by taking into account of the variation of the applied loads and the change of the environmental factors.

Figure 4 shows how the damage alters the displacement time history and the phase plot. In this case, the damage is initiated at the time of 25 s from the initial condition. The figure signifies the previous finding that the change of the response trajectory due to the damage is more visible in the phase plot. The damage level is 0.05517.

The damage index, computed by Equation (2), is also seriously affected by the damage level. Figure 5 shows the evolution of the damage index for the three damage levels, namely, 0.01379, 0.02759, and 0.05517. We note that on the healthy condition, the value

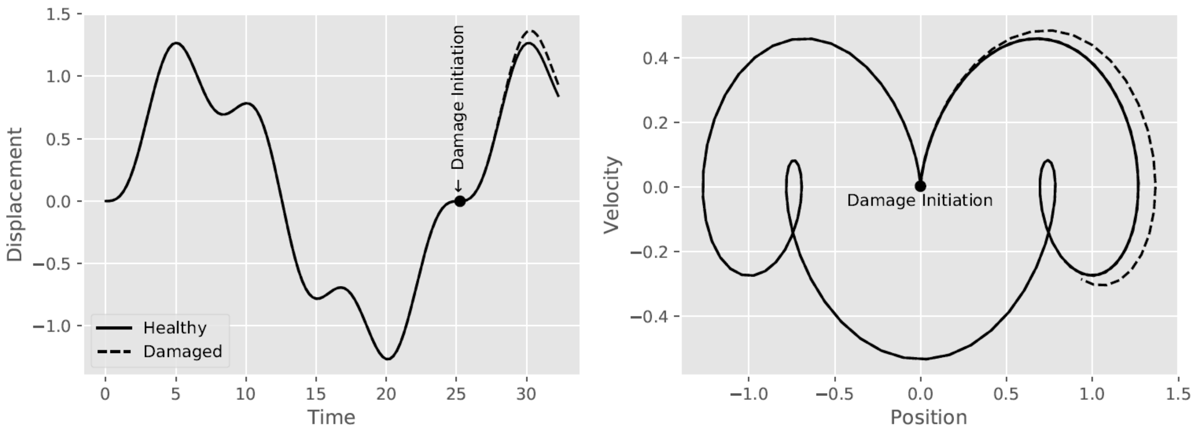

FIGURE 4. The effect of the damage initiation to the displacement history and the phase plot. The damaged case is at the level 0.05517.

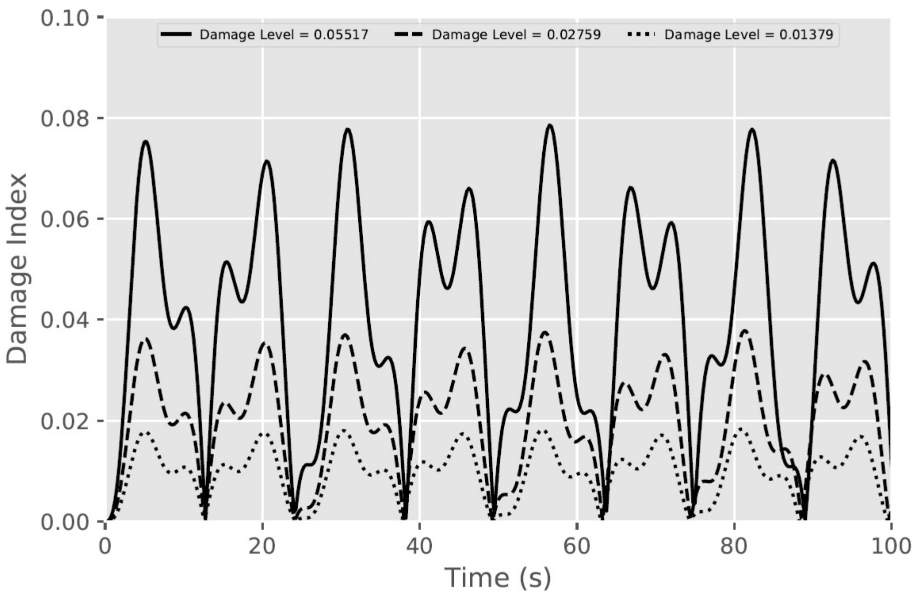

FIGURE 5. The time history of the damage indexes for three damage levels. On the healthy condition, the value of the damage index is zero.

of the index is constant at zero. The damage index seems to be periodical, suggesting that the damage severely affects the structural responses at a certain time, but it may not be observable for other time. It suggests that for practical applications, the damage index should be observed continuously through time.

In Figure 6, we show the effects of the damage level to the three damage sensitive features. The two of them, namely the change of the natural frequency and the Power Spectral Density method (F statistic), are traditional and were discussed in a great length in many references, for example, [10, 11, 12, 13]. As the damage index varies with time, only the maximum damage index is selected for each damage case. As the F statistic varies with the frequency, only the maximum F statistic is selected for each damage case.

Generally, we find that the three damage features are related linearly to the damage level. The fitness to the linear model is at the R^2 statistic, called the coefficient of determination representing the fitness of the model to the data, exceeding 0.99. We note that $R^2 = 1$ indicating a perfect fitness. For this particular case, the rate of change of the F statistic with the damage level is the highest, followed by the damage index, and the change of the natural frequency. It suggests that the F statistic is the most sensitive to the damage, closely followed by the damage index.

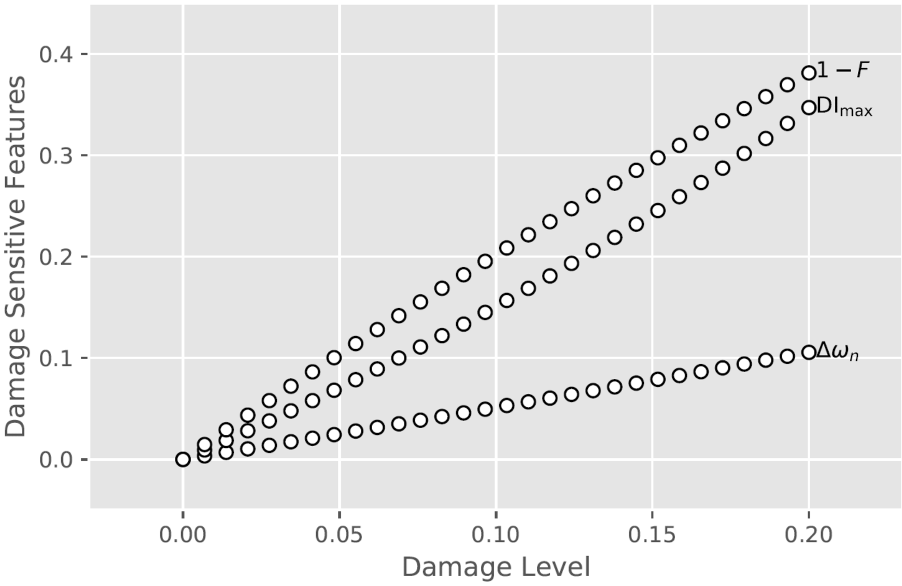

FIGURE 6. For the case of the one-degree-of-freedom system, the effects of the spring damage to the chance of the natural frequencies, in percent, the damage index, and the F statistics

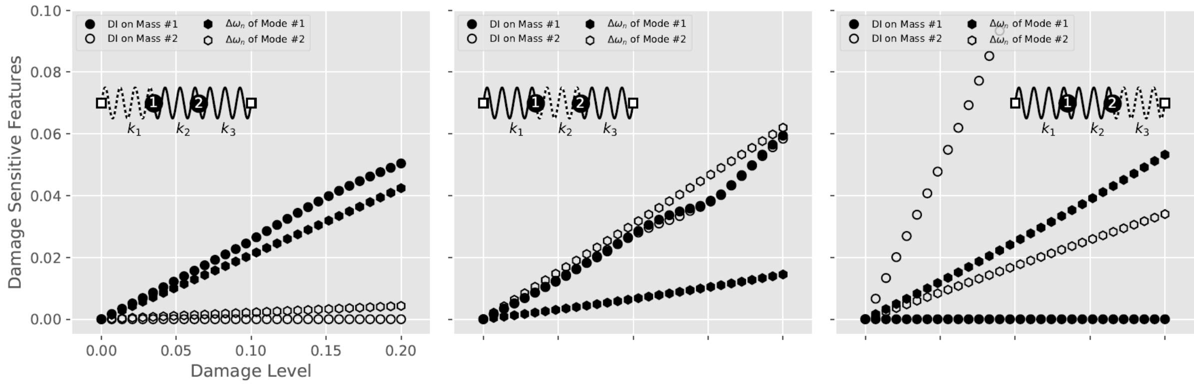

FIGURE 7. The three scenarios of the spring damages and their effects to the damage index and change in the natural frequencies. The damaged springs are in broken line.

Now, we shift our discussion to the case of 2-DOF system with $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, $m_1 = 1$, $m_2 = 1$, $f_1(t) = 0$, and $f_2(t) = \sin(0.25t)$. We study three damage scenarios that occur on each spring separately. In Figure 7, the damaged spring is printed with a broken line. Similar to the previous case, we observe the damage index computed on each mass as the damage grows. The conclusions are of the following.

The change of the natural frequencies, $\Delta\omega_n$, which are marked with the symbol '⬡', is always affected by the damages on Spring 1, Spring 2, or Spring 3. On the left panel of the figure, the damage on Spring 1 strongly affects the change of the natural frequency associated with the first mode, but it weakly affects the second mode. When the second spring has damaged, it affects severely the second vibration mode, as well as the first mode. As for the last scenario, the damage on the third spring, it affects both frequency modes nearly equally.

In the other side, the damage index computed on a mass is only affected by the damaged spring directly connected to the mass. When the damage occurs on Spring 1, which is connected to Mass 1, it affects the damage index computed on Mass 1. The value of the damage index on the Mass 2 is entirely unaffected. In addition, the damage that occurs on Spring 2, which is connecting Mass 1 and Mass 2, affects the damage indexes computed on both masses with the same magnitude. Finally, the damage on spring 3 only affects the damage index computed on the second mass.

The fact illustrates the well-known understanding that natural frequency is global in nature. Generally speaking, the damages on any part of the structure affect the natural frequencies. From this study, we conclude that the damage index, derived from the general vibration model, is local in nature. The index is only affected by the local damage. In terms of sensitivity to the damage level, the damage index is potentially more sensitive than the natural frequency.

4. **Conclusions.** Finding damage sensitive features is one of the research endeavors in the area of structural health monitoring. Recently, we encounter a proposal of the damage index [9] that is essentially describing the deviation of the dynamic equation from an equilibrium condition when the deformation data are obtained from non-intact structural conditions. In this study, we evaluate the sensitivity of the index with respect to the structural damages. We also compare other damage sensitive features, namely, the change of the natural frequencies and the F statistics. Our finding suggests the damage index is local in nature that is only sensitive to the damage that occurs near or at the point of measurement, unlike the change of the natural frequencies which is global. However, the damage index has the potential as it is very sensitive to damages. Based on the facts, we recommend the use of the damage index in conjunction with the traditional methods.

As for future research, the dynamic equilibrium of many structural elements can be compactly presented in the frequency domain for many conditions. Establishing the damage index in the domain may be simpler and more useful. Investigating the index in the frequency domain may be interesting for future research.

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