ICIC Express Letters
Volume **13**, Number **11**, November **2019**

ICIC International ©2019 ISSN 1881-803X
pp. 1065–1071

ROBUST PI-D CONTROL OF UNCERTAIN POLYTOPIC CONTINUOUS-TIME DESCRIPTOR SYSTEMS

Vojtech Veselý and Ladislav Körösi

Institute of Robotics and Cybernetics
Faculty of Electrical Engineering and Information Technology
Slovak University of Technology in Bratislava
Ilkovičova 3, 812 19 Bratislava, Slovakia
{ vojtech.vesely; ladislav.korosi }@stuba.sk

Received May 2019; accepted August 2019

Abstract. *The paper is devoted to obtaining a new design procedure to robust PI-D controller design for parameter-dependent uncertain descriptor systems. The obtained results are in the frame of H_2 performance using parameter-dependent Lyapunov function. In the designed PI-D controller in the D part of the controller the designer can use any or all outputs of descriptor systems instead of descriptor outputs. The designed examples show the effectiveness of the proposed method.*

Keywords: Descriptor system, Robust PI-D controller, State derivative feedback, Output feedback, Linear uncertain systems

1. **Introduction.** Descriptor systems allow to incorporate the dynamics which are governed by algebraic and difference equations. Theoretical approach to descriptor systems originated in 1977 [1] thanks to applications in economics as in the Leontief model [2]. Since that time large progress has been made in the field of Linear Time-Invariant (LTI), Linear Parameter-Varying (LPV) and nonlinear descriptor systems, see [1, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 18]. An overview of descriptor stability conditions is given in the papers [6, 7, 9, 10] and references therein. Controller design procedures for descriptor systems are in the papers [3, 4, 8, 11, 13, 14, 15]. The paper [3] considers the design procedure of H_∞ controller for descriptor systems that could have impulse models. The paper [13] is devoted to the robust admissibility analysis and robust controller design for descriptor systems with time varying uncertainties using parameter dependent Lyapunov function. The first author states the new conditions for the admissibility analysis in the form of strict LMI feasibility problem. Then the robust admissibility conditions are obtained for robust controller design. Uncertainties are covered both polytopic and affine parameter-dependent ones. Paper [4] is devoted to a robust H_∞ controller design for a class of descriptor time-delay systems with state feedback. The main idea is split the descriptor system to slow and fast subsystems. The obtained results are in the form of LMI. In the paper [14] the authors consider the problem to design the method for robust stability and stabilization of continuous uncertain descriptor system with state delay. Design procedure is given in strict LMI. The controller feedback is in the form of state feedback. To design of multivariable PID controller via iterative LMI approach the MIMO plant is transformed into descriptor systems [8]. With these new concepts the design of the original PID controllers is transformed to the design of such PID controllers for descriptor systems which ensure admissibility of closed-loop ones. The paper [15] deals to obtain the original controller design procedure with state feedback for linear parameter varying discrete-time descriptor systems. From the above short observation, we obtain the following problem of study:

DOI: 10.24507/icicel.13.11.1065

- design the robust PI-D controller for polytopic uncertain descriptor systems with output feedback and time varying uncertain parameters,
- design the method to robust admissibility analysis of closed-loop descriptor systems with time-varying uncertain parameter of polytopic systems. The obtained results are expressed in the form of LMI.

The designed robust controllers for descriptor systems ensure minimal value of H_2 performance with QSR (state, derivative of state, input) performance matrices.

The paper is organized as follows. In Section 2 the preliminary stability results of descriptor systems and problem formulation are given. Section 3 shows the main results of paper, design procedure to robust PI-D controller for uncertain descriptor systems which ensures H_2 performance and robust admissibility for time varying uncertainty. Examples to design of controllers for turbogenerator are given in Section 4. Section 5 concludes the paper.

Notations. The notations used in this paper are standard in the field of robust controller design. The relation $A > 0$ ($A < 0$) means, that matrix A is positive (negative) definite. I_n stands for identity n dimension matrix. The superscript “T” represents transpose.

2. **Problem Statement and Preliminaries.** Consider the class of uncertain polytopic continuous-time descriptor systems governed by

$$E\dot{x} = A(\xi)x + B(\xi)u, \;\; y = Cz \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input and $y \in R^l$ is the controlled output of the descriptor system (1). The matrices

$$\begin{gathered} (A(\xi), B(\xi)) = \sum_{i=1}^{N} (A_i, B_i)\xi_i, \\ \sum_{i=1}^{N} \xi_i = 1, \;\; \xi_i \geq 0, \;\; \sum_{i=1}^{N} \dot{\xi}_i = 0 \end{gathered} \tag{2}$$

belong to the convex and bounded set a polytope with N-vertices. A_i, B_i, C are constant matrices of corresponding dimensions, ξ_i, $i = 1, 2, \ldots, N$ are constant or possible time-varying but known parameters. The matrix $E \in R^{n\times n}$ has rank $r \leq n$.

In the next, the following definition and lemmas play important roles.

Definition 2.1. [10] *System* (1) *is regular if there exists complex variable* $s \in C$*,* $det(sE - A(.)) = 0$*.*

Definition 2.2. [14] *The pair* $(E, A(.))$ *is said to be robustly stable if the system* (1) *with* $u = 0$ *is regular, impulse free and stable for all admissible uncertainties* ξ*.*

It is well known that regularity conditions guarantee the existence and uniqueness of system solution on $(0, \infty)$. The full degree condition $rank(det(sE - A)) = rank(E)$ ensures that the system is impulse free. The stability of system means that spectrum condition $\lambda(E, A) = s :\ det(Es - A) = 0 < C^-$. The system is called admissible if it is regular, stable and impulse free.

Theorem 2.1. [9] *The equilibrium* $x = 0$ *of a system* (1) *is asymptotically stable if an* $n \times n$ *symmetric positive definite matrix* $P(\xi)$ *exists, such that along the solutions of the system* (1) *the derivative of function* $V(Ex) = (Ex)^{\mathrm{T}} P(\xi)(Ex)$*, is negative definite for the varieties of* Ex*.*

Definition 2.3. [9] *Let in* (1) $u = Fy$*. System* (1) *is admissible if and only if there exist matrices* $P(\xi)$ *and* F *such that*

$$\begin{gathered} E^{\mathrm{T}} P(\xi) = P^{\mathrm{T}}(\xi)E \geq 0, \\ (A(\xi) + B(\xi)FC)^{\mathrm{T}} P(\xi) + P^{\mathrm{T}}(\xi)(A + BFC) < 0 \end{gathered} \tag{3}$$

Note that Definition 2.3 presents a necessary and sufficient stability conditions for output feedback in terms of bilinear matrix inequality. For PI-D controller assume for state derivative feedback control algorithm in the form

$$u = FCx + F_d C_d E\dot{x} \tag{4}$$

and closed loop is

$$(I - B(\xi)F_d C_d)E\dot{x} = (A(\xi) + B(\xi)FC)x \tag{5}$$

where F, F_d – gain matrices of PI and D part of controller, C_d – determines the derivative output feedback. Note that for I part of controller design, the states of system (1) need to be extended. We assume that for (1) the static output feedback can provide proportional (P) and integral (I) parts to design PI robust controller [16].

From (5) one can obtain

$$\begin{gathered} E\dot{x} = A_C x, \\ A_c = (I - B(\xi)F_d C_d)^{-1}(A(\xi) + B(\xi)FC) \end{gathered} \tag{6}$$

assuming that inverse exists.

Definition 2.4. [10] *The system given by* (6) *is asymptotically stable if all roots of* $det(sE - A_c)$, *are in the open left-half complex plane, and system* (6) *is impulse free.*

To assess the closed-loop performance quality, in the frame of H_2 the quadratic cost function is used

$$J_c = \int_{t_0}^{\infty} J(x, \dot{x}, u)dt, \ J = x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru + (E\dot{x})^{\mathrm{T}}S(E\dot{x}) \tag{7}$$

where $Q, \ S \geq 0$ and $R > 0$.

The well-known Bellman-Lyapunov equation is given in the next lemma.

Lemma 2.1. [9] *Consider the system* (6) *with control algorithm* (4)*. Control algorithm* (4) *is the guaranteed cost control law for the* (6) *if and only if there is a Lyapunov function* $V(\xi)$ *such that holds*

$$B_e(\xi) = \max_u \left(\frac{dV(\xi)}{dt} + J(x, \dot{x}, u) \right) \leq 0 \tag{8}$$

Note that for particular structure of Lyapunov function $V(\xi)$ the obtained design procedure may reduce from "if and only if" to "if".

3. **Robust PI-D Controller Design.** To develop the robust controller design procedure Theorem 2.1 and Lemma 2.1 play crucial roles. Let in (8) the Lyapunov function is

$$V(Ex) = (Ex)^{\mathrm{T}}P(\xi)(Ex) \tag{9}$$

where $P(\xi) = \sum_{i=1}^{N} P_i\xi_i$ and $\dot{P}(\xi) = \sum_{i=1}^{N} P_i\dot{\xi}_i$ with time derivative

$$\frac{dV(Ex)}{dt} = \begin{bmatrix} \dot{x}^{\mathrm{T}} & x^{\mathrm{T}} & u^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 & E^{\mathrm{T}}P(\xi)E & 0 \\ E^{\mathrm{T}}P(\xi)E & E^{\mathrm{T}}\dot{P}(\xi)E & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \\ u \end{bmatrix} \tag{10}$$

To split Lyapunov function and system matrices and decrease the conservatism of obtained design procedure we are employing six slack variables. This new LMI based robust admissibility conditions controller design is less restrictive than the existing ones [13]. The following equalities hold

$$\begin{aligned} &v^{\mathrm{T}}\begin{bmatrix} 2N_1^{\mathrm{T}} \\ 2N_2^{\mathrm{T}} \\ 2N_3^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} E & -A(\xi) & -B(\xi) \end{bmatrix} v = 0 \\ &v^{\mathrm{T}}\begin{bmatrix} 2N_4^{\mathrm{T}} \\ 2N_5^{\mathrm{T}} \\ 2N_6^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} -F_d C_d E & -FC & I \end{bmatrix} v = 0 \end{aligned} \tag{11}$$

where $v^{\mathrm{T}} = \begin{bmatrix} \dot{x}^{\mathrm{T}} & x^{\mathrm{T}} & u^{\mathrm{T}} \end{bmatrix}$.

To joint (11) and (10) one could obtain the time derivative of Lyapunov function for descriptor systems in the form

$$\frac{dV(.)}{dt} = v^{\mathrm{T}}\begin{bmatrix} w_{11}(\xi) & w_{12}(\xi) & w_{13}(\xi) \\ w_{21}(\xi) & w_{22}(\xi) & w_{23}(\xi) \\ w_{31}(\xi) & w_{32}(\xi) & w_{33}(\xi) \end{bmatrix} v < 0 \tag{12}$$

Substitute (12) and (7) to Bellman-Lyapunov Equation (8) one obtains

$$B_e(\xi) = v^{\mathrm{T}} W(\xi) v = \sum_{i=1}^{N} v^{\mathrm{T}} W_i v \xi_i < 0 \tag{13}$$

where $W_i = \{w_{ijk}\}_{3\times 3}$, $i = 1, 2, \dots, N$

$$\begin{aligned} w_{i11} &= N_1^{\mathrm{T}} E + E^{\mathrm{T}} N_1 - N_4^{\mathrm{T}} F_d C_d E - E^{\mathrm{T}} C_d^{\mathrm{T}} F_d^{\mathrm{T}} N_4 + E^{\mathrm{T}} S E \\ w_{i12} &= E^{\mathrm{T}} P_i E - N_1^{\mathrm{T}} A_i + E^{\mathrm{T}} N_2 - N_4^{\mathrm{T}} F C - E^{\mathrm{T}} C_d^{\mathrm{T}} F_d^{\mathrm{T}} N_5 \\ w_{i13} &= -N_1^{\mathrm{T}} B_i + E^{\mathrm{T}} N_3 + N_4^{\mathrm{T}} - E^{\mathrm{T}} C_d^{\mathrm{T}} F_d^{\mathrm{T}} N_6 \\ w_{i22} &= E^{\mathrm{T}} \sum_{j=1}^{N} P_j \dot{\xi}_{jm} - N_2^{\mathrm{T}} A_i - A_i^{\mathrm{T}} N_2 - N_5^{\mathrm{T}} F C - C^{\mathrm{T}} F^{\mathrm{T}} N_5 + Q \\ w_{i23} &= -N_2^{\mathrm{T}} B_i - A_i^{\mathrm{T}} N_3 + N_5 - C^{\mathrm{T}} F^{\mathrm{T}} N_6 \\ w_{i33} &= -N_3^{\mathrm{T}} B_i - B_i^{\mathrm{T}} N_3 + N_6^{\mathrm{T}} + N_6 + R \\ w_{i21} &= w_{i12}^{\mathrm{T}},\ w_{i32} = w_{i23}^{\mathrm{T}},\ w_{i31} = w_{i13}^{\mathrm{T}} \end{aligned}$$

where $\dot{\xi}_{jm} > 0$ is the maximal value of uncertainty rate for $j = 1, 2, \dots, N$ polytope vertex.

Inequality (13) implies:

- For robust stability analysis we have obtained the strict LMI conditions. If the LMI condition (13) holds for all $i = 1, 2, \dots, N$ the closed-loop descriptor system satisfies the condition of admissibility, i.e., system is robust asymptotically stable.
- For robust controller design which will guarantee the admissibility conditions in Inequality (13) are in the form of BMI.
- Due to six slack variables the robust controller design procedure for descriptor system is less restrictive than the existing ones.

The obtained results are summarized in the following theorem.

Theorem 3.1. *Consider the uncertain polytopic descriptor system* (1) *with robust control algorithm* (2). *Closed-loop descriptor system* (6) *is parameter dependent quadratically stable with guaranteed cost if there exist slack matrices* $N_1, \dots, N_6$, *symmetric matrices* $P_i, i = 1, 2, \dots, N$ *output feedback gain matrices* F *and* F_D *such that Inequality* (13) *holds for all* $i = 1, 2, \dots, N$.

Proof: Proof of sufficient robust stability conditions immediately follows from Equations (9)-(13).

4. **Example.** The main objective of power system is to continuously supply power to all consumers with an acceptable quality, i.e., ensure the real power balance with acceptable frequency value. The goal of this example is to design a robust PI-D controller for turbogenerator when its model is given in the form of descriptor systems.

The model of turbogenerator is governed by equation [17]

$$\begin{aligned} T_j \frac{d^2\delta}{dt^2} + D\tfrac{d\delta}{dt} &= P_T - P_e \\ \frac{dP_T}{dt} T_T + P_T &= u \\ P_e &= A_m \sin\delta \end{aligned} \tag{14}$$

where [17] parameters of turbo generator are as follows.

T_j – inertia coefficient of turbogenerator (turbine + synchronous generator), $T_j = 0.02245\ \mathrm{s}^2/\mathrm{rad}^2$

D – damper coefficient, $D = 0.2$ s/rad

P_e – total electric active power generated by turbogenerator to the power system is p.u.

P_T – output of turbine power, p.u.

T_T – turbine time constant, $T_T = 6$ s

δ – load angle of turbogenerator [rad]

After linearization of (14) in working points defined by $P_e = \{\ 0.3,\ \ 0.6,\ \ 1\ \}$ p.u. and $A_m = 1.5$, 2 – maximal value of electrical power, one can obtain the following descriptor model

$$E\dot{x} = A_i x + B_i u,\ y = Cx,\ i = 1, 2 \ldots, N$$

where $E = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{bmatrix}$, $A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{D}{T_j} & \dfrac{1}{T_j} & -\dfrac{1}{T_j} \\ 0 & 0 & -\dfrac{1}{T_T} & 0 \\ -A_m\cos\delta_0 & 0 & 0 & -1 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{T_T} \\ 0 \end{bmatrix}$,

$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ – output feedback is taken from electrical power.

For three working points and two values of A_m one could obtain $N = 6$ vertices to polytopic system. State vector is defined as $x^{\mathrm{T}} = \begin{bmatrix} \Delta\delta & \omega & \Delta P_T & \Delta P_e \end{bmatrix}$. For the following parameters $r_0 = 1000 \geq P$, rate of plant parameter changes $\overline{\xi}_{im} = 0.1/s$ one may obtain for PI-D controller parameters as follows.

Case A

Derivative feedback from turbine power ΔP_T. Matrix $C_d = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$. Performance parameters $Q = qI$, $q = 0.1$, $S = sI$, $s = 0$, $R = rI$, $r = 1$. Obtained PI-D controller parameters are

$$R_T(s) = \left(-3.5973 - \frac{1.7121}{s}\right)\Delta P_e - 0.1575 s \Delta P_T$$

Case B

Derivative feedback from rates of load angle δ, rotor speed, derivation ω and turbine power P_T. The output matrix C_d is $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$. Performance parameters are $q = 0.0001$, $s = 0$, $r = 1$. The obtained PI-D controller parameters are

$$R_T(s) = \left(-28.6089 - \frac{18.053}{s}\right)\Delta P_e + 0.0173 s \Delta\delta$$

Dynamic behaviors of turbogenerator with proposed PI-D controllers are given in Figure 1 and Figure 2. The simulation results prove the functionality of the proposed method. In the second example different feedback was chosen (see C_d) with lower performance matrix

Q to demonstrate how these parameters influence the dynamics of the controlled process output. Lower waiting matrix Q leads to oscillations 2.

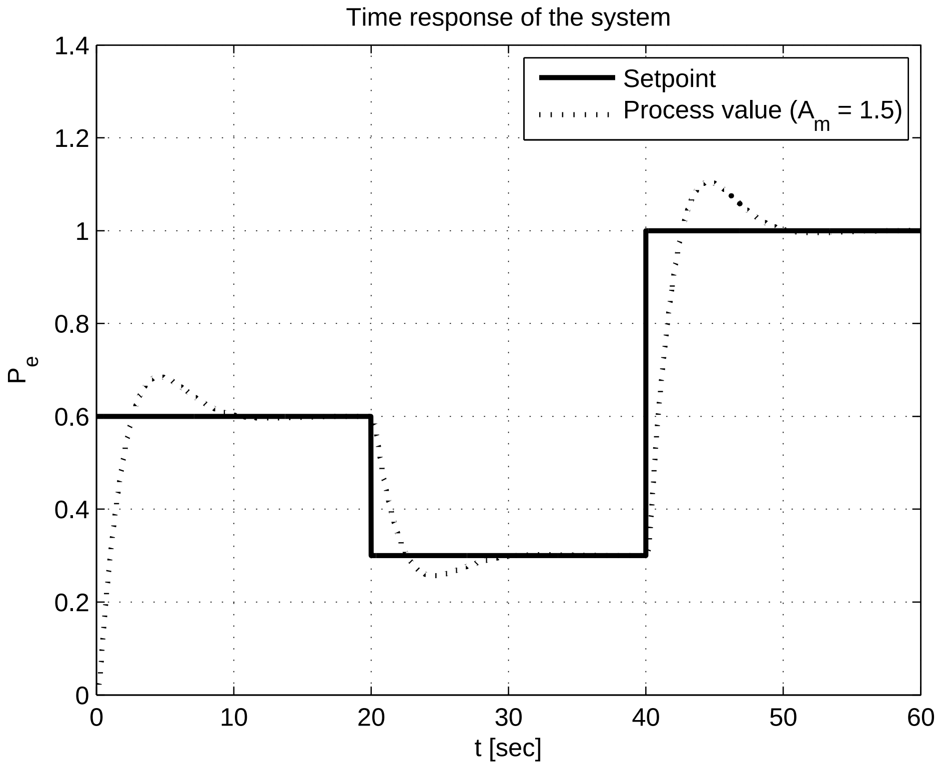

Figure 1. Time response of the controlled output – 1st case

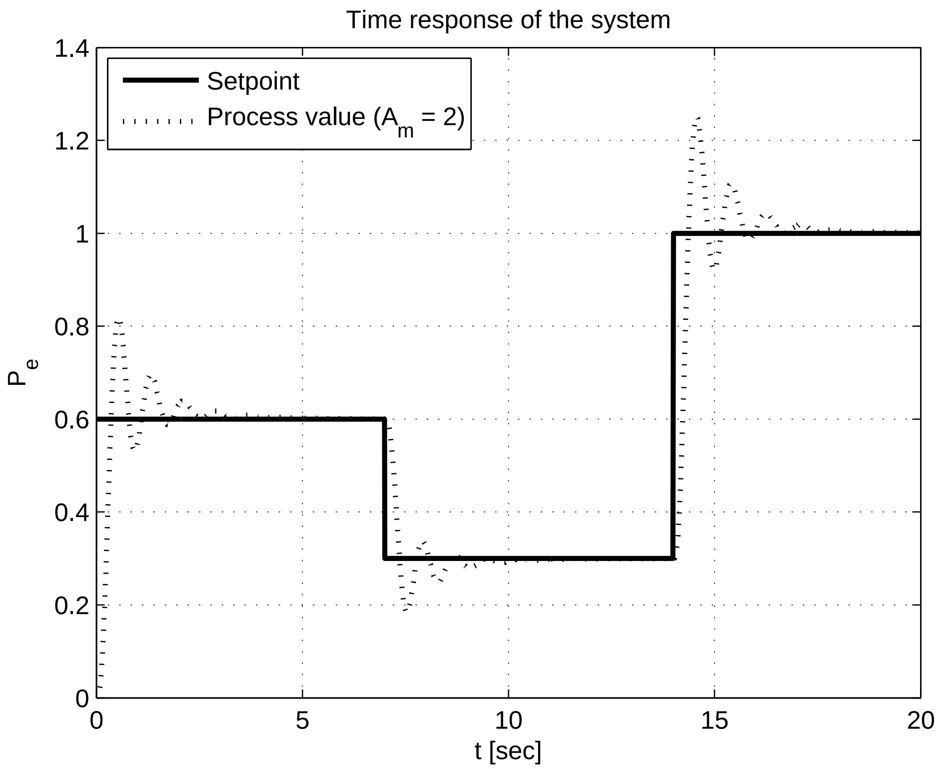

Figure 2. Time response of the controlled output – 2nd case

5. **Conclusion.** A novel approach to obtaining PI-D controller for uncertain polytopic descriptor systems is presented. The proposed design procedure is based on the stability conditions of descriptor systems [9] and Bellman-Lyapunov equations. The obtained results are in the frame of H_2 performance with quadratic criterion (Q, S, R) and parameter dependent Lyapunov function. For the case of robust stability analysis – the robust condition of admissibility to closed-loop descriptor system we have obtained strict LMI condition which is less restrictive, less conservative than existing ones. For D part of PI-D controller the designer can use any or all outputs of descriptor systems instead of algebraic outputs. The designed example for turbogenerator of robust PI-D controller shows the high effectiveness of proposed method. Our research in the field of descriptor systems will continue to design procedures based on L_2 gains and regional pole placement.

Acknowledgment. The work has been supported by the Slovak Scientific Grant Agency VEGA, Grant No. 1/0475/16.

REFERENCES

[1] D. G. Luenberger, Dynamic equations in descriptor form, *IEEE Trans. Automat. Control*, vol.22, no.3, pp.312-321, 1977.
[2] M. S. Silva and T. P. De Lima, Looking for nonnegative solutions of a leontief dynamic model, *Linear Algebra*, vol.364, pp.281-316, 2003.
[3] I. Masubuchi, Y. Kamitane, A. Ohara and N. Suda, H_∞ control for descriptor systems: A matrix inequalities approach, *Automatica*, vol.33, no.4, pp.669-673, 1997.
[4] X. Ji, H. Su and J. Chu, An LMI approach to robust H-infinity control for uncertain singular time-daley systems, *Journal of Control Theory and Applications*, no.4, pp.361-366, 2006.
[5] D. Debeljkovic, N. Višnjić and M. Pješćić, The stability of linear discrete descriptor systems in the sense of Lyapunov: An overview, *Scientific Technical Review*, vol.LVII, nos.3-4, pp.43-57, 2007.
[6] B. Men, Q. L. Zhang, X. Li, C. Yang and Y. Chen, The stabilty of linear descriptor systems, *International Journal of Inforation and System Science*, vol.2, no.3, pp.362-374, 2006.
[7] Z. Su, Q. Zhang, J. Ai and C. Yang, A new approach to uniform practical stability of descriptor systems with infinite time delays in terms of two measurements, *International Journal of Information and Systems Sciences*, vol.6, no.4, pp.345-354, 2010.
[8] C. Lin, Q. G. Wang and T. H. Lee, An improvement on multivariable PID controller design via iterative LMI approach, *Automatica*, vol.40, no.3, pp.519-525, 2004.
[9] C. T. Chen and Y. Q. Liu, Lyapunov stability analysis of linear singular dynamical systems, *IEEE International Conference on Intelligent Processing Systems*, Benjing, China, pp.635-639, 1997.
[10] D. Debeljkovic, N. Višnjić and M. Pješćić, The stability of linear continuous singular systems in the sense of Lyapunov: An overview, *Scientific Technical Review*, vol. LVII, no.1, pp.51-64, 2007.
[11] C. Vorrawan and W. Assawinchaichote, H-infinity fuzzy integral controller for nonlinear descriptor systems, *International Journal of Innovative Computing, Information and Control*, vol.15, no.2, pp.741-756, 2019.
[12] F. L. Lewis, A survey of linear singular systems, *Circuits, Systems and Signal Processing*, vol.5, no.1, pp.3-36, 1986.
[13] G. I. Bara, Robust analysis and control of parameter-dependent uncertain descriptor systems, *Systems & Control Letters*, vol.60, no.5, pp.356-364, 2011.
[14] S. Xu, P. M. Van Dooren, R. Stefan and J. Lam, Robust stability and stabilization for singular systems with state delay and parameter uncertainty, *IEEE Trans. Automatic Control*, vol.47, no.7, pp.1122-1128, 2002.
[15] M. Chadli, J. Daafouz and M. Darouach, Stabilisation of singular LPV systems, *Proc. of the 17th IFAC World Congress*, Seoul, Korea, pp.999-1002, 2008.
[16] V. Veselý and D. Rosinová, Robust PID-PSD controller design: BMI approach, *Asian Journal of Control*, vol.15, no.2, pp.469-478, 2013.
[17] P. Kundur, N. J. Balu and M. G. Lauby, *Power System Stability and Control*, McGraw-Hill, New York, 1994.
[18] M. K. Gupta, N. K. Tomar and M. Darouach, Unknown inputs observer design for descriptor systems with monotone nonlinearities, *International Journal of Robust and Nonlinear Control*, vol.28, no.17, pp.5481-5494, 2019.