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A MAXIMUM SEQUENCE LENGTH MASH DIGITAL DELTA-SIGMA MODULATORS

Xiaoling Yang, Yanhua Huang, Yadong Yin and Long-Zhao Shi

College of Physics and Information Engineering
Fuzhou University
No. 2, Xueyuan Road, University Town, Fuzhou 350108, P. R. China
slz@fzu.edu.cn

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Abstract. *For the problem of serious spurious signal interference in the current MultistAge noise SHaping (MASH) structure, this study proposes an improved structure that absorbs and synthesizes the advantages of Prime-Modulus MASH (PM-MASH) and Song and Park's MASH (SP-MASH) structures. For all initial states and constant digital inputs, the structure produces the maximum sequence length. The proposed structure (we call it SH-MASH) cascades several first-order 1-bit error feedback modulators, such as the SP-MASH structure, but it has prime-modulus quantization interval. SH-MASH structure is simulated on MATLAB. Simulation results show that the proposed architecture has a better noise-shaping capability than the PM-MASH and SP-MASH structures at the same modulator word length. SH-MASH structure is comparable to the Hosseini and Kennedy's MASH (HK-MASH) structure, which is recognized as the best structure in the current industry; however, SH-MASH structure has a simpler structure.*
Keywords: Delta-Sigma Modulator (DSM), MultistAge noise SHaping (MASH), Prime-modulus quantizers

1. **Introduction.** The application of non-contact wireless communication in daily life has been increasing and developing rapidly. The development of wireless communication technology has led to an increasing demand for the efficient performance of frequency synthesizer. The fractional-N frequency synthesizer based on Delta-Sigma Modulator (DSM), which allows a Phase Locked Loop (PLL) to achieve high-frequency resolution and loop bandwidth simultaneously, is widely studied and applied [1,2]. The DSM controls the frequency divider and shape quantization noise to achieve efficient suppression. Among various DSM structures, the MultistAge noise SHaping (MASH) modulator is widely applied in fractional-N synthesizers due to its advantages of simplicity and stability [3]. The DSM yields a periodic sequence for constant inputs, which leads to a periodic quantization error sequence. The sequence length or the quantization error sequence length relies on the initial states, input values, and DSM structures [4]. A short sequence length will produce significant spurious components in the output noise power density spectrum, which will reduce the purity of the synthesizer output spectrum. The output spectra of a 14-bit typical MASH 1-1-1 DSM with different initial states but with an input $X = 1024$ are shown in Figure 1. Plot (i) presents the output spectrum when the DSM has zero initial states, and the corresponding sequence length is 2^5. Plot (ii) displays the output spectrum when the initial conditions of all stages are set to zero, except for the first stage, and the corresponding sequence length is 2^{15} [5]. The spectrum of Plot (i) contains noticeable spurs, whereas Plot (ii) is close to the dashed line, which represents the shaped white quantization noise. Figure 1 illustrates that the output spectrum of typical MASH

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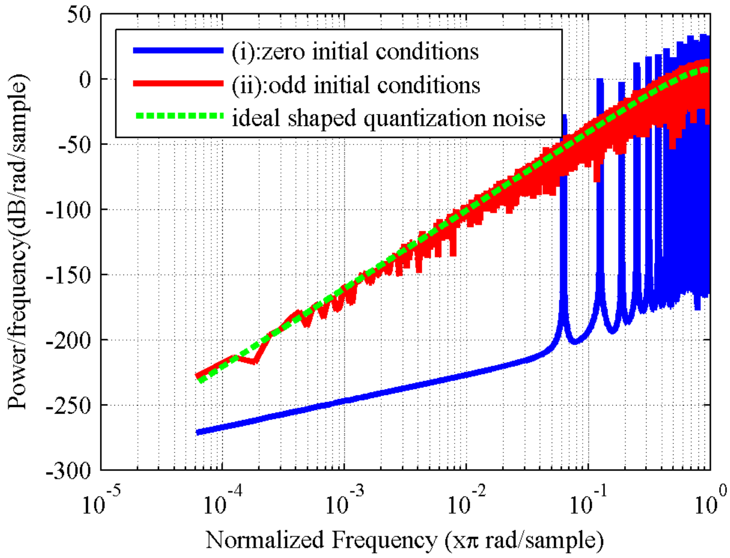


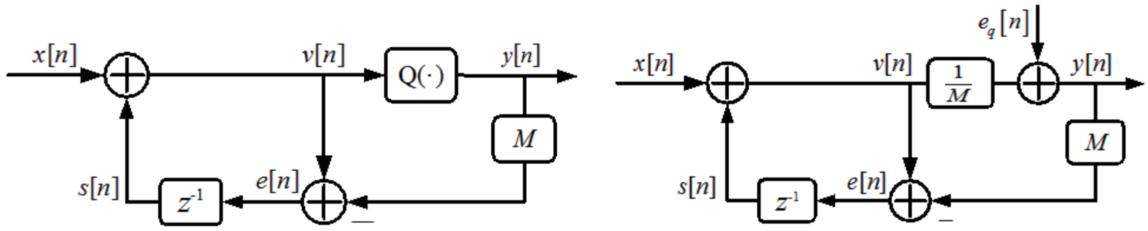
Figure 1. Influence of the sequence length in a 14-bit MASH 1-1-1 DSM

is related not only to the number of digits in DSM, but also to its initial state. These problems make it difficult to apply.

For the problem of short sequence lengths in DSM, two types of methods are adopted. The stochastic approach primarily disrupts the periodic behavior of DSM by superimposing a random dither signal. The sequence length can be increased by superimposing a 1-bit dither signal to the DSM input [6,7]. However, the output noise floor is increased. One method to solve the problem is to shape the dither signals by a high-pass filter [7-9], and another approach is to superimpose a dither signal at the quantizer input [10,11]. Nevertheless, dither signals are also periodic because they are usually produced by a pseudo-random sequence generator. The dither signals consequently have a limited effect [12]. By contrast, the deterministic approach increases the sequence length by structure modification, without the problem of increasing the noise floor. The Prime-Modulus MASH (PM-MASH) structure was proposed in [13], where the modulus of 1-bit Error Feedback Modulator (EFM1) is a prime number (PM-EFM1). The sequence length of PM-MASH always relies on the first stage regardless of the input values and initial states [14]. One method to increase the sequence length is to increase the modulator word length. However, the hardware cost will undoubtedly increase as well. Feeding the output value multiplied by a to the input side was proposed in [15], denoted as HK-EFM1. Hosseini and Kennedy's MASH (HK-MASH) structure HK-MASH is built on the HK-EFM1. HK-MASH can achieve its theoretical maximum value for all inputs and initial conditions. The Song and Park's (SP-MASH) structure was proposed in [16]. Its cascade mode is similar to the conventional MASH architecture but it has an additional link between two contiguous stages. The SP-MASH architecture increases the sequence length effectively. However, the sequence length becomes short for some input values.

We proposed an improved structure called SH-MASH to overcome the shortages of PM-MASH and SP-MASH. This structure significantly increased the sequence length for all input values, where the input values are in the range $[1 \quad M_p - 1]$.

2. **Previous MASH Structures.** A traditional architecture of the EFM1 is shown in Figure 2(a). The input signal $x[n]$ is a constant with n_0-bit resolution. The 1-bit quantizer $Q(\cdot)$ has a threshold value of M, where $M = 2^{n_0}$. The mathematical model of EFM1 is presented in Figure 2(b), where $e_q[n]$ represents an additive noise source, $e_q[n] = -e[n]/M$. The MASH architecture shown in Figure 3 can be obtained by concatenating several



(a) (b)

FIGURE 2. Traditional EFM1 (a) and its mathematical model (b)

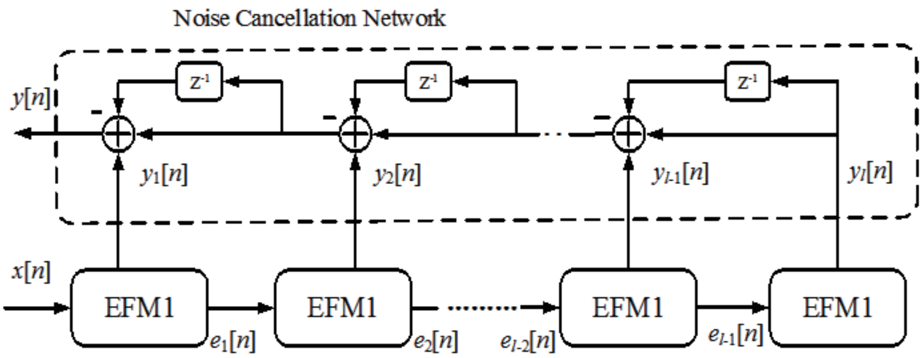
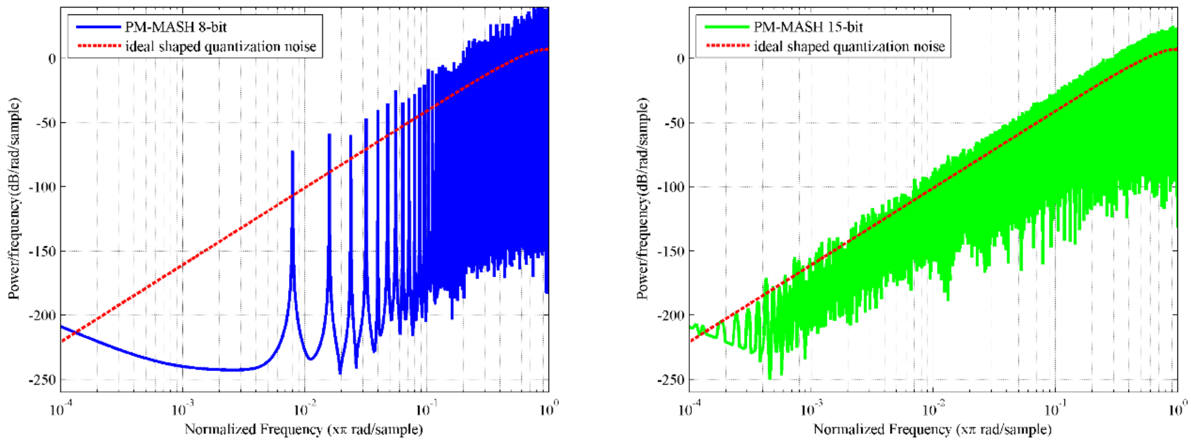


FIGURE 3. Typical MASH structure



(a) PM-MASH 1-1-1 8-bit (b) PM-MASH 1-1-1 15-bit

FIGURE 4. Spectra of the PM-MASH 1-1-1 for different word lengths

EFM1. The input-output relationship in the z-domain is

$$Y(z) = \frac{1}{M} X(z) + \left(1 - z^{-1}\right)^{l} E_{ql}(z) \tag{1}$$

where $Y(z)$ and $E_{ql}(z)$ represent the z-transforms of the output signal and the quantization error of the lth stage, respectively. Although the MASH structures achieve higher-order noise shaping, the sequence lengths become considerably short for some input values.

The PM-MASH structure can be obtained by setting the quantization interval of EFM1 in Figure 3 to a prime number. This structure has a sequence length of M_p when the quantization interval of each stage is M_p (M_p is the highest prime number no larger than M) [14]. Increasing the modulator word length will increase the sequence length, which results in a relatively smooth spectrum. Figure 4 presents the PM-MASH 1-1-1 spectra for different word lengths. The spectrum of the 15-bit PM-MASH 1-1-1 is significantly smoother than that of the 8-bit PM-MASH. However, the spectrum of the 15-bit modulator still contains spurs.

The SP-MASH structure is shown in Figure 5(a). The SP-EFM1 in the figure is built by adding an input signal to the EFM1, as shown in Figure 5(b). In comparison with conventional MASH structure, the SP-MASH has an additional link between two contiguous stages. The connection links the output of the SP-EFM1 to the input of next-stage SP-EFM1. In this case, if the quantization interval of each stage is M, then the sequence length of the lth-order SP-MASH is $N_1 \cdot M^{l-1}$, where the first stage has sequence length of N_1 [16].

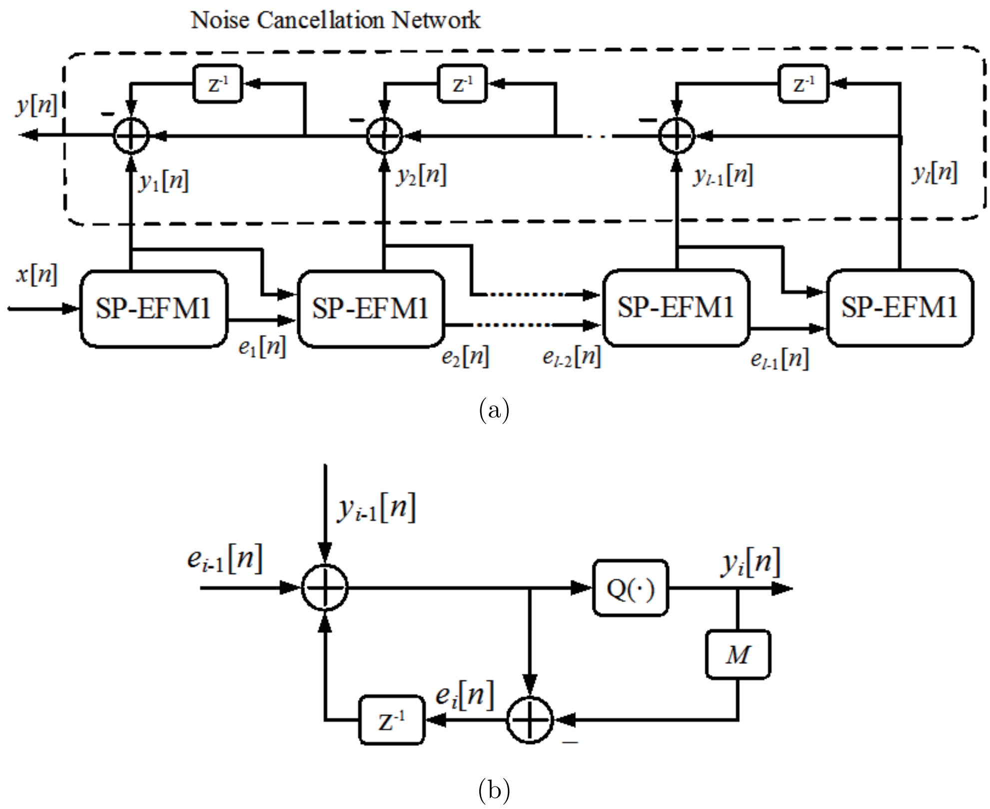


FIGURE 5. SP-MASH (a) and SP-EFM1 (b)

Song and Park increased the bitwidth to increase the sequence length of the SP-MASH structure [16]. Figure 6 shows an example. The bitwidth of the first stage is 5, and the modulus of the later stages is 9. The output of the first stage is left by 4-bit and then fed to the second stage. Therefore, the sequence length can reach at least 2^{19}. Figure 7 presents the spectra of the 5-bit SP-MASH 1-1-1 and the improved SP-MASH (as shown in Figure 6) structure, where the input value is 16. As shown in the figure, the spectrum of the improved SP-MASH is close to the ideal curve and contains minimal spurs.

3. **Proposed MASH Structure.** Shi and Huang proposed a novel MASH structure (Let us call it SH-MASH), as shown in Figure 8(a). The structure is constructed based on an SH-EFM1 presented in Figure 8(b). The second input port of the first stage is 0. In comparison with the quantization interval of SP-EFM1, the quantization interval of SH-EFM1 is expressed by prime number M_p. The transfer function of the SH-MASH

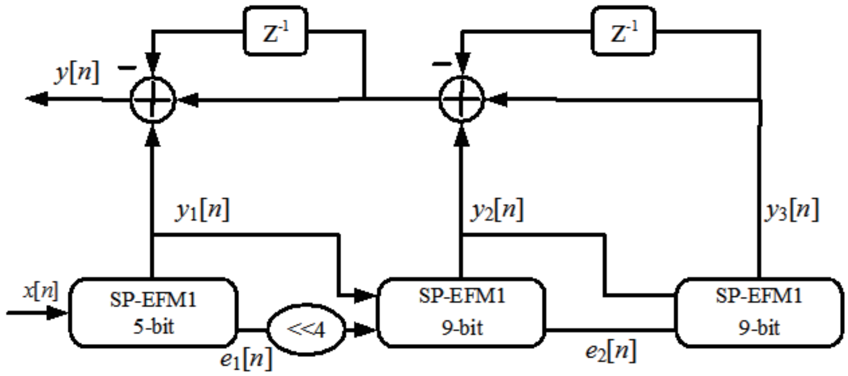
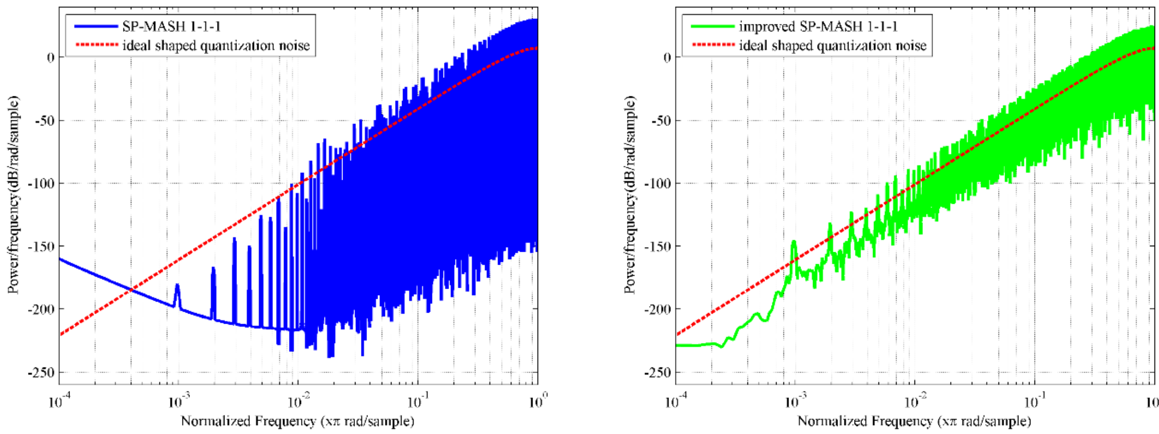
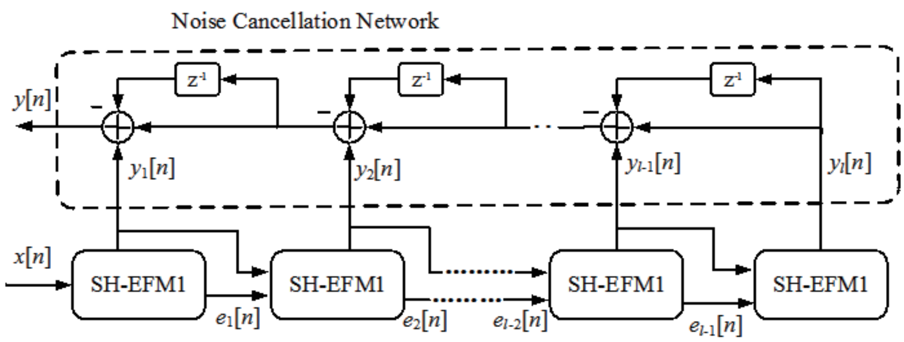


FIGURE 6. Improved SP-MASH structure

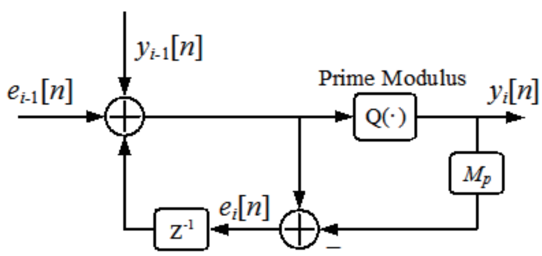


(a) SP-MASH 1-1-1 (b) The improved SP-MASH 1-1-1

FIGURE 7. Spectra of the SP-MASH 1-1-1 and the improved SP-MASH 1-1-1



(a)



(b)

FIGURE 8. SH-MASH (a) and SH-EFM1 (b)

structure is

$$\begin{aligned} Y(z) = {} & \left[\frac{1}{M_p} + \frac{(1-z^{-1})}{M_p^2} + \cdots + \frac{(1-z^{-1})^{l-1}}{M_p^l}\right] X(z) \\ & + (1-z^{-1})^l \left[\frac{E_{q1}(z)}{M_p^{l-1}} + \frac{E_{q2}(z)}{M_p^{l-2}} + \cdots + E_{ql}(z)\right] \end{aligned} \tag{2}$$

Therefore, the Signal Transfer Function (STF) and Noise Transfer Function (NTF) are obtained as follows:

$$STF(z) = \frac{1}{M_p} + \sum_{k=2}^{l} \frac{(1-z^{-1})^{k-1}}{M_p^k} \tag{3}$$

$$NTF(z) = (1-z^{-1})^l \tag{4}$$

The STF consists of all-pass and high-pass filters. The STF can be regarded as an all-pass filter for constant inputs [16]. The NTF is an lth-order high-pass filter.

Using the similarity methods in [15] and [16], the sequence length of the n_0-bit SH-MASH structure is

$$N = M_p^l \tag{5}$$

for all initial states and inputs, where M_p is the largest prime number no larger than 2^{n_0}. n_0 represents the modulator word length.

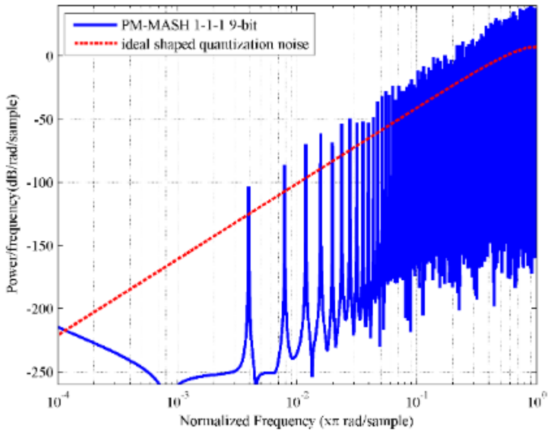
Figure 9 presents the spectra of a 9-bit PM-MASH 1-1-1, 19-bit PM-MASH 1-1-1 [13,14], and 9-bit SH-MASH 1-1-1, all with a constant input $M/2$. The corresponding M values are 2^9, 2^{19} and 2^9, respectively. The spectra of the 19-bit PM-MASH 1-1-1 and the 9-bit SH-MASH 1-1-1 have the same smoothness, whereas that of the PM-MASH 1-1-1 has slightly higher noise power. Thus, the 9-bit SH-MASH 1-1-1 has the same noise-shaping capability as that of the 19-bit PM-MASH 1-1-1. Some spurs are found in the spectrum of the 9-bit PM-MASH 1-1-1, as expected.

Figure 10 presents the spectra of the improved SP-MASH 1-1-1 and 9-bit SH-MASH 1-1-1 with a constant input of 256. The bitwidth of the first stage is 9 bits, and the modulus of the later stages is 13 bits. The SH-MASH 1-1-1 has a smooth spectrum, whereas the spectrum of the improved SP-MASH 1-1-1 contains high power spurs.

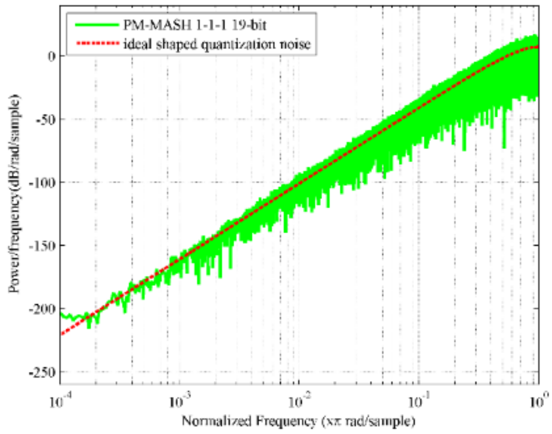
HK-MASH is recognized as the best structure in the current industry. The HK-MASH structure is cascaded by the HK-EFM1, which is shown in Figure 11. The output signal $y[n]$ is multiplied by a and fed to the input side. a is selected to make $(M-a)$ the largest prime number less than M. The HK-EFM1 can be obtained by cascading the HK-EFM1 in accordance with the traditional MASH shown in Figure 3. In comparison with the traditional structure, the advantage of the l-order HK-MASH structure is that the sequence length of the modulator is equal to $(M-a)^l$ for all inputs and initial conditions.

Figure 12 presents the spectra of an HK-MASH 1-1-1 and SH-MASH 1-1-1 with a constant input of 256; both have a word length of 9 bits. The SH-MASH and HK-MASH structures have considerable noise-shaping capability, and no noticeable spurs exist on the spectra. Compared with the HK-EFM1 structure, the SH-EFM1 does not require a feedback loop az^{-1}.

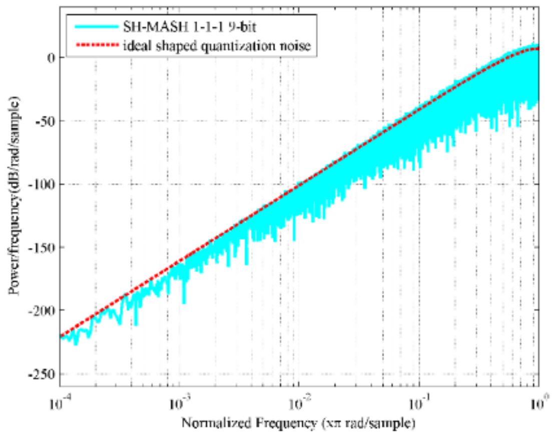
Table 1 compares the sequence lengths of the lth-order PM-MASH, SP-MASH, and SH-MASH. All have an order of 9 bits. The sequence length of the SH-MASH is considerably larger than that of the PM-MASH structure and is comparable to that of the SP-MASH structure. However, compared with the SP-MASH 1-1-1, the SH-MASH 1-1-1 always has a sequence length of M_p^l and independent of input values. The SH-MASH and HK-MASH structures have the same sequence length.



(a) PM-MASH 1-1-1 9-bit

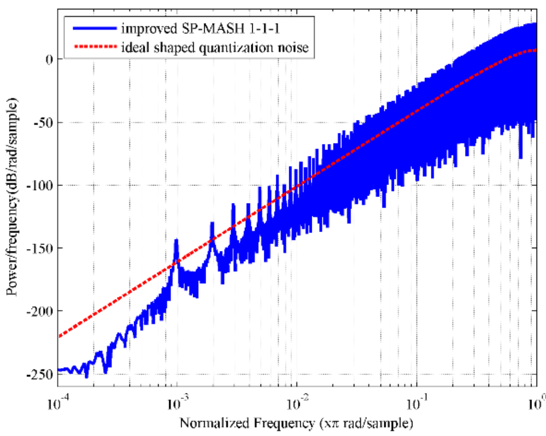


(b) PM-MASH 1-1-1 19-bit

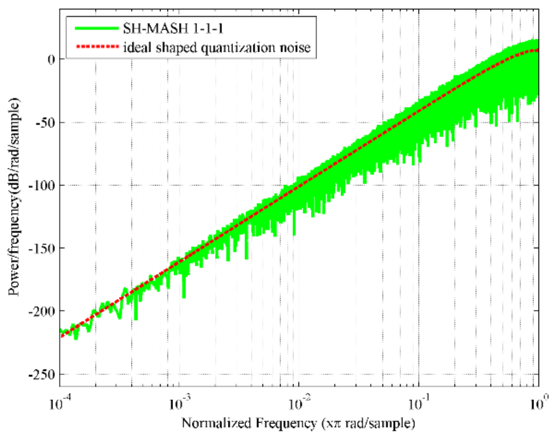


(c) SH-MASH 1-1-1 9-bit

FIGURE 9. Spectra of PM-MASH 1-1-1 and SH-MASH 1-1-1



(a) Improved SP-MASH 1-1-1



(b) SH-MASH 1-1-1

FIGURE 10. Spectra of SH-MASH 1-1-1 and the improved SP-MASH 1-1-1

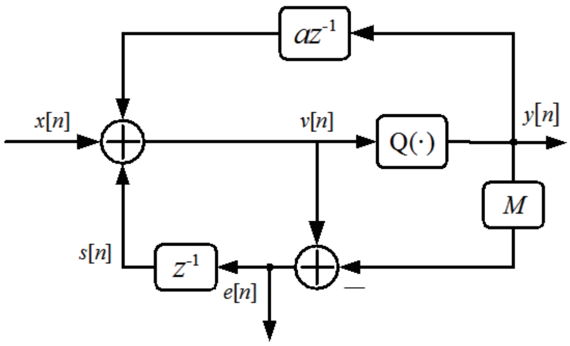
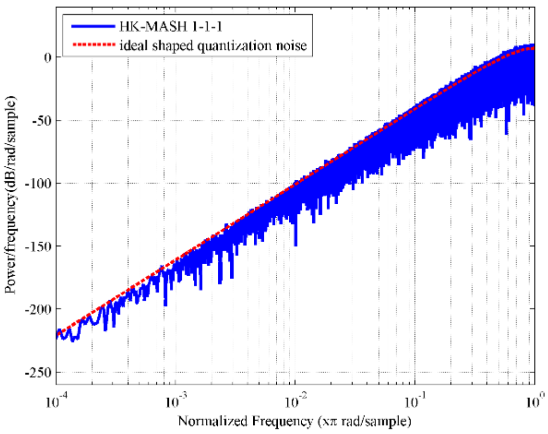
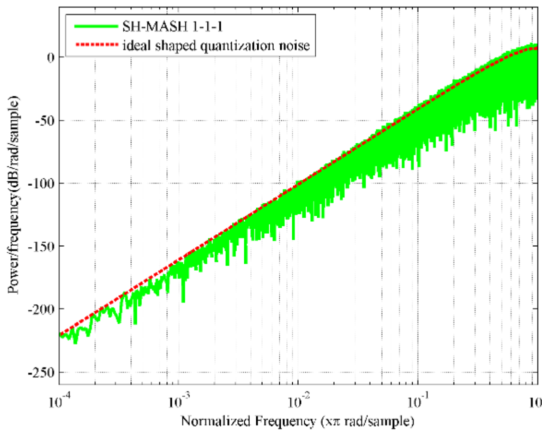


FIGURE 11. HK-EFM1 structure



(a) HK-MASH 1-1-1



(b) SH-MASH 1-1-1

FIGURE 12. Spectra of HK-MASH 1-1-1 and SH-MASH 1-1-1

TABLE 1. Comparison of MASH structures

Structure	Minimum Sequence Length	Maximum Sequence Length
SP-MASH	$2M^{l-1*}$	M^l
PM-MASH	$M_p{}^{**}$	M_p
HK-MASH	$(M-a)^{l***}$	$(M-a)^l$
SH-MASH	M_p^l	M_p^l

* $M = 2^{n_0}$ when the modulator word length is n_0-bit.

** M_p is the largest prime number no larger than M.

*** a is selected to make $(M-a)$ the largest prime number less than M.

4. **Conclusions.** SH-MASH DSM is developed in this study based on the analysis of the principles and advantages of PM-MASH and SP-MASH. This structure is constructed based on SH-EFM1. The lth-order SH-MASH structure has a sequence length of M_p^l and independent of input values and initial states. The mathematical proofs of the results are provided. Three structures are also simulated. The simulations suggest that the SH-MASH has the best noise-shaping capability without significantly increasing the hardware cost. Notably, the SH-MASH structure yields the same sequence length as that of the HK-MASH. The HK-MASH and SH-MASH structures have a considerable noise-shaping effect. However, compared with the HK-MASH structure, the SH-MASH structure does not need a feed loop in the circuit structure. Next, we will apply SH-MASH to specific projects to further verifying its effectiveness.

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